COUNTERPARTY RISK EVALUATION IN POWER DERIVATIVES

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Abstract:
Power derivatives are financial risk management tools that have been used over time in the energy sector, based on an underlying energy asset. The remarkable increase of the over-the-counter transactions in this field forces the financial institutions to include the cost of counterparty in the pricing framework. The goal of our research is to present measurement formulas for quoting "completed" power derivatives, i.e. instruments embracing the risk to each party of a contract that the counterparty will not live up to its contractual obligations. Our proposal consists in evaluating derivatives completed of innovative collaterals, such as Credit value adjustment (CVA) and bilateral CVA (BCVA).
We stress the approach by empirical results.

Key words: Counterparty Risk, Power Derivatives, Forward Contract, Credit Value Adjustment (CVA), Bilateral CVA (BCVA), Debt Value Adjustment (DVA)

1. Introduction

Since many year the volatility of the oil price significantly impacts on the balance sheet of the oil companies.

Until the first half of 1960s the oil price was quite steady, indeed many oil companies arranged a long-term contracts with the oil producing countries.

The problems started with the establishment in 1960 of the OPEC (Organization of Petroleum Exporting Countries) that is a permanent intergovernmental organization of 12 oil-exporting developing nations that coordinates and unifies the petroleum policies of its Member Countries.

This organization has affected the price of oil for more than 50 years.

A first substantial increase in oil prices, there has been between 1973 and 1974 when the members of the Organization of Arab Petroleum Exporting Countries (OAPEC) namely the Arab members of the OPEC proclaimed an oil embargo. The oil price increased suddenly from \$2.90 to \$12.00 per barrel. It was called also first crisis.

Between the 1978 and 1979 there was the second crisis characterized by an increase of the oil price from \$ 12 to \$ 30 per barrel.

A further decline in oil price there was in 1997 and 1998 thanks the Asian financial crisis.

Generally, oil market more than other resources is affected by political, economic and environmental events, like financial crisis, terrorist attacks, hurricanes and so on. The figure 1. shows the trend in the Crude oil Brent price in the last 20 years.

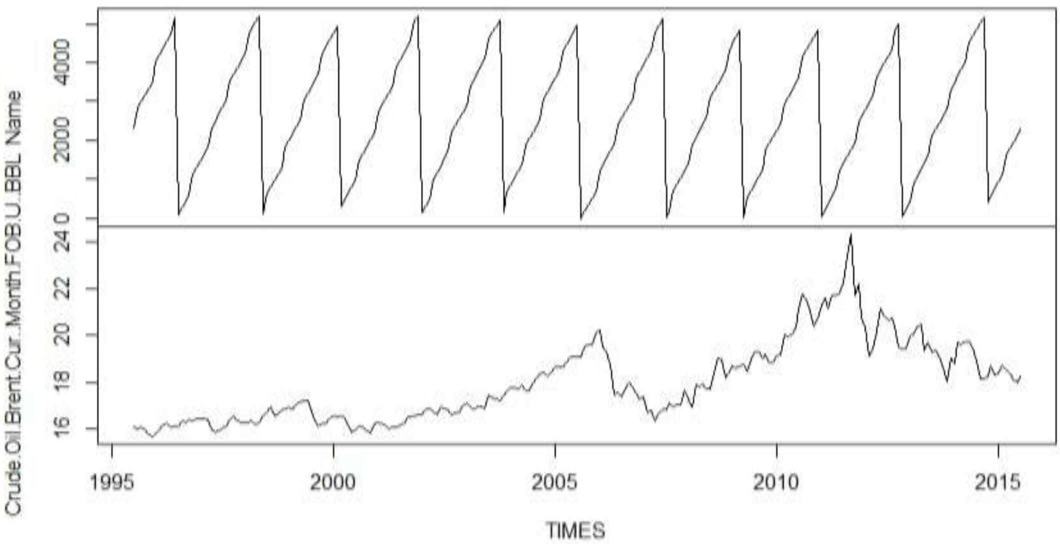

Figure 1. Crude Oil Brent Price from 1995 to 2015

The high volatility in the oil market from the 1960 to nowadays led to the formation of an oil derivatives market and a variety of hedging instruments such as forwards, futures and options written on these commodities.

The paper is organised as follows: in section 2 we investigate the long-term contracts, future and forwards. in section 3 we model counterparty risk in the forward market.

Section 4 provides an empirical application.

2. Long-term Contract, Future and Forward Market

Long-term contract

Long-term contracts are negotiated bilaterally between buyers and sellers. This kind of contract concerns an series of oil deliveries referred to a specified period. Generally the period varies among one and two years. First the parties of the long-term contract specify the method used for calculating the price of an oil cargo. Usually the oil price for each cargo scheduled into the contract is linked to a market (spot) price.

Other information specified into the contract by the parties are: the volumes of crude oil to be delivered, the delivery schedule and the actions to be taken in case of default.

Futures market

The futures markets have been developed after the second oil shock. The future is a derivative contract traded in the exchange, in which two parties agree to buy or sell oil at a certain maturity and certain price. This contract allows to the parties to hedge against the risk of price fluctuations (as for forward and option contracts). When a party agrees to buy an underlying asset on a certain futures maturity for a certain price, it assumes long position, then it hedges against the risk that the price may increase over the time. While it assumes short position if he agrees to sell an underlying asset at certain maturity at certain price. In this case the party hedges the risk that the price of the underlying asset may decrease over the time.

Generally the parties that respectively enter in a long and short position develop divergent forecasts on oil prices.

As aforementioned, the futures contracts are traded into exchange market, therefore they are governed by precise operating rules. Indeed the counterparty risk relies on these contracts is very low because the broker requires that the investor has to deposit fund into a margin account in such way to reduce the potential loss in the case of counterparty default. This account is dearly adjusted.

Generally the intermediation is achieved by a clearinghouse.

Forward market

As well as the futures contract, the forward contracts allow to a party to cover the risk that the oil price may suffer fluctuations. Also the oil forward contracts are derivatives t in the Over-The-Country Market(OTC) traded.

As in the case of futures contract, the oil forward contract provides that a party agrees to buy an certain oil quantity at a future time at a determined price (long position), while the counterparty agrees to sell oil at the same maturity and same price.

Generally the payoff of the buyer (long position) in a forward contract is given by the following equation:

$$S_T - K$$

where S_T is the oil spot price at the maturity and K is the delivery price .

The buyer's payoff will be positive if the spot price is higher than the delivery price agreed by the contract. Vice versa it is negative.

The payoff from a short position in a forward contract is equal to:

$$K - S_T$$

In this case the payoff is positive if the forward price is higher than the oil spot price at the maturity date. Vice versa it is negative.

A problem is to define the oil forward price. According to Hull (2011) the delivery price of a oil forward contract on commodities (in this case the crude oil) is equal to the forward price at the inception, namely at time 0, that is given by the following formula:

$$K = F_0 = S_0 e^{(r+u-y)T}$$

F_0 is the forward price at the inception. The forward price is given by the product of S_0, the spot price of the crude oil at the inception and $e^{(r+u-y)T}$, i.e. the continuous compounding with the rate *r+u-y*, where *r* is the free-risk rate, *u* is the storage cost per annum, *y* is the convenience yield, *T* is the time until delivery in a forward contract. The storage cost, as well as the convenience yield, is a constant proportion of the spot price.

As it is possible to guess at the inception, the value of an oil forward contract should be equal to zero, since only at the inception K and F_0 are equal.

The oil forward price may change over the maturity of the trade, but the delivery price does not change. Then the value of the long oil forward contract over the maturity may change and it is given by the following equation:

$$f = (F_t - K)e^{-r(T-t)}$$

The crude oil forward contract can be assessed also over the duration of the contract. The forward contract value over the maturity may be positive or negative.

In the other side the value of the short oil forward contract over the maturity is given by:

$$f = (K - F_t)e^{-r(T-t)}$$

As we can note both the delivery price and the assessment over the maturity of the forward do not consider the significant counterparty risk typically located in OTC market characterized by higher flexibility of the contractual conditions, but lower price transparency than the exchanged market.

3. Counterparty Risk in the Forward Market

Counterparty risk is the risk that a counterparty in a contract will default prior to the expiration of a transaction and will not therefore fulfill the current and future payments required by the contract.

To evaluate the counterparty risk on a derivative contract we could consider innovative collateralization tools such a Credit Value Adjustment (CVA), Bilateral CVA (BCVA) and Debt Value Adjustment (DVA).

According to Gregory (2012), the CVA is defined as the market price of counterparty risk on a contract obtained by the risk neutral expectation of the loss that could occur for the counterparty default over the term of the contract weighted with the risk-neutral probability of the counterparty default.

The CVA as a stand-alone value is given by the following equation:

$$CVA(t,T) = -\left(1-\bar{\delta}\right)\left[\int_t^T B(t,u)EE(u,T)dS(t,u)\right] \approx$$

$$\left(1-\bar{\delta}\right)\sum_{i=1}^{t} B(t_i)EE(t_i)q(t_{i-1},t_i) \qquad (1)$$

where $(1-\bar{\delta})$ is loss given default, i.e. one minus the recovery rate, δ; $B(t_i)$ denote the risk-free discounting factor at time t_i; $EE(u,T)$ calculated under the risk neutral-measure and $EE(t_i)$ is the expected exposure for the relevant dates in future time given by t_i for *i=0,t*, $S(t,u)$ the survival probability, while $q(t_{i-1},t_i)$ the marginal default probability in the interval between dates t_{i-1} and t_i.

This equation is obtained under the assumption of independence between credit exposure, default probability and recovery rate, of no wrong-way risk and that the party that values the trade cannot default.

The CVA is an innovative tool for easily pricing the counterparty risk, being determined by components that may be obtained from different sources of an institution, in addition you can use it as collateralization charging CVA to the counterparty.

The CVA may be expressed also as a running spread by:

$$CVA_{as\ a\ spread} = \frac{CVA(t,T)}{CVD_{premium}(t,T)} = \frac{CDS_{default}(t,T)}{CDS_{premium}(t,T)} \times EPE = X^{CDS} \times EPE$$

(2)

where X^{CDS} is the fixed periodic premium of a Credit Default Swap (CDS) with same maturity of the instrument in question and it may be defined as a credit spread, while $CDS_{premium}(t,T)$ is the present value of the premiums at time t, $CDS_{default}(t,T)$ being the value of the default component.

The counterparty risk has a bilateral nature. The cost of the counterparty risk considering the bilateral nature may be computed by mean the BCVA formula as following:

$$BCVA \approx (1-\bar{\delta})\sum_{i=1}^{T} B(t_i)EE(t_i)[S_I(t_{i-1})q(t_i,t_{i-1})]$$
$$-(1-\bar{\delta}_I)\sum_{i=1}^{T} B(t_i)NEE(t_i)S(t_{i-1})q_I(t_i,t_{i-1}) \quad (3)$$

where $S_I(.)$ and $S(.)$ respectively represent the survival probabilities of the institution and its counterparty; $q_I(t_i,t_{i-1})$ denotes the default probability of the institution; $\bar{\delta}_I$ the recovery of the institution; while $NEE(t_i)$ is the negative expected exposure, i.e. the EE from the point of view of the counterparty.

The BCVA may be positive or negative according to which counterparty has an higher exposure and higher default probability.

Also the BCVA may be expressed as a running spread by the equation (4):

$$BCVA_{spread} = \frac{BCVA(t,T)}{CDS_{premium}(t,T)} = \frac{CVA(t,T)}{CDS_{premium}(t,T)} + X^{CDS}{}_I \times ENE =$$
$$X^{CDS} \times EE + X^{CDS}{}_I \times ENE \quad (4)$$

where X_I^{CDS} is the CDS fixed periodic premium of an institution and ENE is the expected negative exposure.

Symmetrically the DVA is the price of the counterparty risk obtained under the risk neutral expectation of the loss considering the assumption that the investor that evaluates the derivative may default and his counterparty is default-free. The DVA as a stand-alone value is given by the following formula:

$$DVA = (1-\bar{\bar{\delta}}_I)\sum_{i=1}^{T} B(t_i)NEE(t_i)q_F(t_i,t_{i-1}) \quad (5)$$

$q_I(t_i,t_{i-1})$ denotes the default probability of the institution; $\bar{\delta}_I$ the recovery of the institution; while $NEE(t_i)$ the negative expected exposure, i.e. the EE from the point of view of the counterparty with the difference that the $NEE(t_i)$ is a negative value.

The unilateral DVA as a credit spread is given by:

$$DVA_{as\ a\ spread} = X_F^{CDS} \times ENE \quad (6)$$

where X_F^{CDS} is the periodic premium paid by the investor that enter into the credit default swap to cover his counterparty risk exposure and ENE is the expected negative exposure.

The DVA as a stand-alone value and as a credit spread unlike the CVA are negative values.

For many reasons, the presence of counterparty risk impacts the oil forward contract value. Then for obtaining a fair value of the oil forward contract it is crucial to introduce another dimension in the traditional pricing framework such as the risk under consideration. Generally, the risky value of a derivative contract is given by:

$$risky\ value\ of\ a\ derivative = Free\ risk\ value - CVA$$

We propose pricing formulas of the oil forward derivatives that include the counterparty risk in such a way obtaining risky values of the derivatives (Blake 2014).

In this case we assume the perspective of the seller position in the oil forward contract, i.e the short position.

Under the assumption that the seller is default-free and his counterparty may default, we can use the CVA as a spread for assessing the derivative issued at time 0 at any time t between 0 and the maturity T as following:

$$f_{risky} = (K - F_t)e^{-(r+CVA_{spread})(T-t)} \quad (7)$$

where F_t is the forward price at time t and K is the delivery price.

In this way the oil forward contract value at time t is discounted with a free-risk rate plus the CVA as a spread. Then the risky oil forward contract today is less than its free-risk value. The presence of the counterparty risk for the investor reduces the oil forward values over the duration of the contract.

However, as well known, the value of an oil forward contract at the inception is equal to 0:

$$K = F_0$$

Furthrmore the delivery price namely the forward price at the inception is determined by the product of the spot price and $e^{(r+u-y)T}$. Then you could introduce also the charge of the counterparty risk the delivery price is given by the equation (8):

$$F_0 = S_0 e^{(r+u+CVA_{spread}-y)T} \quad (8)$$

where the CVA as a spread is added to the free-risk rate and the buyer have to pay on delivery a price higher than that of in the case of counterparty risk-free. In this way the cost of the counterparty risk is charged on the buyer.

Under the assumption that the seller, namely the investor that assesses the contract may default and his counterparty is default-free, we can use the DVA as a spread for pricing the derivative issued at a generic time t by mean the equation (9):

$$f_{risky} = (K - F_t)e^{-(r+DVA_{spread})(T-t)} \quad (9)$$

The DVA is a negative value, then it increases the value of the oil forward contract from the point of view of the seller.

Also in this case we include the cost of the counterparty risk into the delivery price of the contract by the expression (10):

$$F_0 = S_0 e^{(r+u+DVA_{spread}-y)T} \quad (10)$$

In formula (10), the DVA as spread reduces the delivery price paid to the seller, charging the cost of the counterparty risk to the seller.

Finally we introduce the evaluation of the oil forward contract at the generic time t from the point of view of the seller, considering the bilateral nature of the counterparty risk by means of the BCVA as a spread:

$$f_{risky} = (K - F_t)e^{-(r+BCVA_{spread})(T-t)} \quad (11)$$

The BCVA as a spread may be positive or negative according to the credit quality of the both parties. If it is positive the risky value of the derivative is lower than the free-risk value, in the other side it is higher.

If the parties of the oil forward contract decide to take in account the bilateral nature of the counterparty risk in the definition of the delivery price, we can write the equation:

$$F_0 = S_0 e^{(r+u+BCVA_{spread}-y)T} \quad (12)$$

In this case the delivery price may be higher or lower than the delivery price determined without the consideration of the counterparty risk according to the credit quality of both party and then the cost of the counterparty risk may be charged on the counterparty with lower credit quality. This cost does not include the *price* of the default.

4. Numerical Applications

Let us consider a 2-year forward contract on WTI (West Texas Intermediate) crude oil agreed between a refinery company, the buyer, and an oil producer, the seller. Generally the WTI is quoted on New York Mercantile Exchange (NYMEX).

In our empirical application, the two parties have agreed on January 1st , 2015 respectively to buy and sell 500'000 barrel of WTI crude oil on January 1st , 2017. Let us suppose that the delivery price was determined by the following equation:

$$K = F_0 = S_0 e^{+(r+u-y)T}$$

The Crude Oil-WTI Spot price at the inception was equal to \$ 53.45. For determining the forward price it was needed also to know the storage cost per annum, the convenience yield and the free-risk rate.

The free risk used was the 2-year treasury rate that at the inception of the trade was 0.6727%.

The storage cost is fitted to a continuous annual rate of 15% on the delivery price and a convenience annual yield fitted to 7% of the delivery price, namely the oil forward price was equal to:

$$K = F_0 = \$53{,}45e^{+(0{,}6727+15-7)2} = \$\ 63.57375$$

On January 1st , 2017 the buyer is going to buy 500'000 barrel of WTI-oil at the price $\$\ 63.57375$, while the seller is going to receive the payment and to deliver the WTI-oil.

In this case the parties of the contract have not considered into the determination of the delivery price the counterparty risk.

We suppose that the seller would quarterly assess the forward contract, according to the following expression:

$$f = (K - F_t)e^{-r(T-t)}$$

where F_t is the oil forward price at the time t considering as maturity of T. Considering that the spot price of the WTI-oil in 31 March 2015 was equal to \$ 47,72 and at the same date the 2-yaer treasury rate was 0,5646% and supposing that the storage cost and convenience yield do not vary over the time the oil forward price for the first quarter was:

$$F_t = S_t e^{+(r+u-y)(T-t)} = 47{,}72 e^{+(0.5646\%+15\%-7\%)} = 55.4122449$$

At this point you can determine the value of the forward contract on 30 March 2015 from the point of view of the seller as following:

$$f = (K - F_t)e^{-r(T-t)} = (63.57375 - 55.4122449)e^{-0.5646\%(2-0.25)} = 8{,}25933439$$

As you can see, from the point of view of the seller the contract had a positive value on March 31, 2015.

As regard the evaluation of the contract for the following quarterly we can achieve a projection of the 2-years treasury interest rate and the WTI-oil price.

For predicting the 2-years treasury interest rate for all the durations of the contract we project by the Cox–Ingersoll–Ross model (or CIR model, 1985) that is given by the following equation:

$$dr_t = \alpha(\beta - r_t)dt + \sigma\sqrt{r_t}dWt$$

where Wt is a Wiener process (modelling the random market risk factor) and α, β, and σ, are the parameters. The parameter α corresponds to the speed of adjustment, β to the mean and σ to volatility.

The table 1 shows the parameters of the CIR model on the aforementioned dataset, while the table 2 shows the simulated annual rate in percentage for the last day of each month until the maturity date.

Table 1. Parameters of the CIR Model

Parameters	values
α	0,205889
β	2,995961
σ	0,804416

Table 2. Evolution of interest rates obtained with the CIR Model

DATE	RATE
01/08/2015	0,680906
01/09/2015	0,390506
01/10/2015	0,380235
01/11/2015	0,45741
01/12/2015	0,55685
01/01/2016	0,372335
01/02/2016	0,252471
01/03/2016	0,22988
01/04/2016	0,330363
01/05/2016	0,468053
01/06/2016	0,571795
01/07/2016	0,466715
01/08/2016	0,374259
01/09/2016	0,268327
01/10/2016	0,370138
01/11/2016	0,205752
01/12/2016	0,154185
01/01/2017	0,208012

For the project of the WTI-oil prices we used the forecasts achieved by The Economist Intelligence Unit that reported in the table 3.

Table 3. Evolution of WTI oil price.

WTI	2015	2016	2017
1 Qtr	47,72[1]	59.40	71.15
2 Qtr	55.09[1]	63.30	74.60
3 Qtr	54.96	65.69	75.33
4 Qtr	56.93	65.89	-

Sources: Haver Analytics; The Economist Intelligence Unit.

Through these data you can observe how change over the time the value of the WTI-oil forward in any time from the point of view of the seller.

The table 4 report the prediction of oil quarterly forward contract values from the point of view of the seller.

Table 4. WTI oil forward contract values at the end of each quarterly from the point of view of the seller

		T-t	u-y	r	(r+u-y)(T-t)	exp+(r+u-y)(T-t)	S_t	F_t	$F_0(K)$	exp-r*(T-t)	f	f*500'000
2015	1 Qtr	1,75	0,08	0,0054	0,14945	1,161195	47,72	55,4122449	63,75	0,990595	8,259334	4129667
	2 Qtr	1,5	0,08	0,005848	0,128772	1,137431	59,48	67,6543816	63,75	0,991266	-3,87028	-1935141
	3 Qtr	1,25	0,08	0,00399	0,104988	1,110697	54,96	61,0438921	63,75	0,995025	2,692645	1346322
	4 Qtr	1	0,08	0,00395	0,08395	1,087575	56,93	61,9156171	63,75	0,996058	1,827151	913575,7
2016	1 Qtr	0,75	0,08	0,0032	0,0624	1,064388	59,4	63,2246481	63,75	0,997603	0,524093	262046,3
	2 Qtr	0,5	0,08	0,005156	0,042578	1,043497	63,3	66,0533883	63,75	0,997425	-2,29746	-1148729
	3 Qtr	0,25	0,08	0,003634	0,020909	1,021129	65,69	67,0779387	63,75	0,999092	-3,32492	-1662458
	4 Qtr	1	0,08	0,002484	0,082484	1,085981	65,89	71,5553077	63,75	0,997519	-7,78594	-3892972

As aforementioned, the forward price of the WTI oil changes over the time. According to how the forward price changes, also the value of the contract varies over the maturity.

The evaluation in this setting does not take in account the typical counterparty risk included in a OTC transaction. Indeed the values reported in the last column of the table 4 are the free-risk.

As seen previously, for taking into account the counterparty risk we can use the CVA, BCVA and DVA, namely the price or the cost of the counterparty risk under different assumptions.

Let us suppose that the seller would assess the forward contract over the time considering also the impact of the counterparty risk under the assumption that only his counterparty may default. To do this, the calculation can be obtained by the following formula:

$$f_{risky} = (K - F_t)e^{-(r+CVA_{spread})(T-t)}$$

where the CVA as a spread is given by the product of the Expected Positive Exposure (EPE) and periodic premium that would be paid if the seller enters into the CDS to cover his counterparty risk exposure (X^{CDS}).

Supposing that the EPE of the seller is 5% and the X^{CDS} is 2% the CVA as a spread is equal to 1%. Then we can compute the risky market value of the WTI-oil forward over the term of the contract, as shown in the table 5.

Table 5. Risky values of WTI oil forward contract at the end of each quarterly from the point of view of the seller under the assumption that only his counterparty may default

		T-t	u-y	r	(r+u-y)(T-t)	exp+(r+u-y)(T-t)	S_t	F_t	$F_0(K)$	CVA	exp-r*(T-t)	f	f*500'000
2015	1 Qtr	1,75	0,08	0,0054	0,14945	1,161195	47,72	55,4122449	63,75	1%	0,97341	8,116053	4058027
	2 Qtr	1,5	0,08	0,005848	0,128772	1,137431	59,48	67,6543816	63,75	1%	0,976508	-3,81266	-1906331
	3 Qtr	1,25	0,08	0,00399	0,104988	1,110697	54,96	61,0438921	63,75	1%	0,982665	2,659196	1329598
	4 Qtr	1	0,08	0,00395	0,08395	1,087575	56,93	61,9156171	63,75	1%	0,986147	1,808971	904485,5
2016	1 Qtr	0,75	0,08	0,0032	0,0624	1,064388	59,4	63,2246481	63,75	1%	0,990149	0,520177	260088,3
	2 Qtr	0,5	0,08	0,005156	0,042578	1,043497	63,3	66,0533883	63,75	1%	0,992451	-2,286	-1143000
	3 Qtr	0,25	0,08	0,003634	0,020909	1,021129	65,69	67,0779387	63,75	1%	0,996397	-3,31661	-1658307
	4 Qtr	1	0,08	0,002484	0,082484	1,085981	65,89	71,5553077	63,75	1%	0,987594	-7,70847	-3854236

The risky values of the WTI oil forward contract, computed under the assumption that only the counterparty may default, are lower than their free-risk values. Indeed as said above the consideration of the unilateral counterparty risk by mean the CVA as a spread reduces significantly the value of the derivative contract.

This kind of evaluation could be achieved also considering the bilateral nature of the counterparty risk through the BCVA or considering that the investor that assesses the contract may default and his is default- free by mean the DVA as a spread.

Fitting the DVA as a spread equal to 0.5% and the BCVA as spread equal to the difference between CVA and DVA, it is possible to calculate the risky value of the contract under the assumption that only seller may default or both parties may default. The risky values of the WTI oil forward over the term of the contract considering the BCVA are reported by the table 6, while the risky values of the forward under the assumption that only the seller may default are reported in the table 7.

Table 6. Risky values of WTI oil forward contract at the end of each quarterly from the point of view of the Seller under the assumption that both parties may default

		T-t	u-y	r	(r+u-y)(T-t)	exp+(r+u-y)(T-t)	S_t	F_t	$F_0(K)$	BCVA	exp-r*(T-t)	f	f*500'000
2015	1 Qtr	1,75	0,08	0,0054	0,14945	1,161195	47,72	55,4122449	63,75	0,5%	0,981965	8,18738	4093690
	2 Qtr	1,5	0,08	0,005848	0,128772	1,137431	59,48	67,6543816	63,75	0,5%	0,98386	-3,84136	-1920682
	3 Qtr	1,25	0,08	0,00399	0,104988	1,110697	54,96	61,0438921	63,75	0,5%	0,988825	2,675868	1337934
	4 Qtr	1	0,08	0,00395	0,08395	1,087575	56,93	61,9156171	63,75	0,5%	0,99109	1,818038	909019,2
2016	1 Qtr	0,75	0,08	0,0032	0,0624	1,064388	59,4	63,2246481	63,75	0,5%	0,993869	0,522131	261065,5
	2 Qtr	0,5	0,08	0,005156	0,042578	1,043497	63,3	66,0533883	63,75	0,5%	0,994935	-2,29172	-1145861
	3 Qtr	0,25	0,08	0,003634	0,020909	1,021129	65,69	67,0779387	63,75	0,5%	0,997844	-3,32076	-1660382
	4 Qtr	1	0,08	0,002484	0,082484	1,085981	65,89	71,5553077	63,75	0,5%	0,992544	-7,74711	-3873555

Table 7. Risky values of WTI oil forward contract at the end of each quarterly from the point of view of the Seller considering the own default

		T-t	u-y	r	(r+u-y)(T-t)	exp+(r+u-y)(T-t)	S_t	F_t	$F_0(K)$	BCVA	exp-r*(T-t)	F	f*500'000
2015	1 Qtr	1,75	0,08	0,0054	0,14945	1,161195	47,72	55,4122449	63,75	-0,5%	0,9993	8,331921	4165960
	2 Qtr	1,5	0,08	0,005848	0,128772	1,137431	59,48	67,6543816	63,75	-0,5%	0,998729	-3,89942	-1949709
	3 Qtr	1,25	0,08	0,00399	0,104988	1,110697	54,96	61,0438921	63,75	-0,5%	1,001263	2,709527	1354763
	4 Qtr	1	0,08	0,00395	0,08395	1,087575	56,93	61,9156171	63,75	-0,5%	1,001051	1,83631	918155
2016	1 Qtr	0,75	0,08	0,0032	0,0624	1,064388	59,4	63,2246481	63,75	-0,5%	1,001351	0,526062	263030,8
	2 Qtr	0,5	0,08	0,005156	0,042578	1,043497	63,3	66,0533883	63,75	-0,5%	0,999922	-2,30321	-1151604
	3 Qtr	0,25	0,08	0,003634	0,020909	1,021129	65,69	67,0779387	63,75	-0,5%	1,000342	-3,32908	-1664538
	4 Qtr	1	0,08	0,002484	0,082484	1,085981	65,89	71,5553077	63,75	-0,5%	1,002519	-7,82497	-3912485

The table 8 summarizes the values of the WTI oil forward contract referred the different hypothesis for a better comparison.

Table 8. Comparison between the free risk values and risky values of WTI oil forward contract

		free risk	CVA	BCVA	DVA
2015	1 Qtr	4129667	4058027	4093690	4165960
	2 Qtr	-1935141	-1906331	-1920682	-1949709
	3 Qtr	1346322	1329598	1337934	1354763
	4 Qtr	913575,7	904485,5	909019,2	918155
2016	1 Qtr	262046,3	260088,3	261065,5	263031
	2 Qtr	-1148729	-1143000	-1145861	-1151604
	3 Qtr	-1662458	-1658307	-1660382	-1664538
	4 Qtr	-3892972	-3854236	-3873555	-3912485

As just said, the risky values under the assumption that only the counterparty may default are lower than the free risk value, since the counterparty's default impacts on the balance sheet of the seller.

They are lower also than the values obtained with the BCVA, because in this last case the impact of the counterparty default to the net of cost of the own default is considered. In addition the risky forward values obtained by mean the DVA are higher than all other values, because it is represented the impact of the seller's default on the balance sheet of the counterparty.

In essence the counterparty risk under different assumptions affects substantially the WTI oil forward values. Then could be needed to contemplate the cost of counterparty risk in the delivery price. The choice of which kind of collateral could depend on which party has a o lower credit quality and bargaining power.

If the seller has a higher credit quality and higher bargaining power than his counterparty, he could require that the delivery price includes compounding of the CVA as a spread. In this way the seller obtains an higher delivery price and could account resources for covering the potential loss in the case of counterparty default.

If the spot price at the inception is equal to $53.45, the 2-years treasury interest rate to 0.6727%, the storage cost and the convenience yield are given respectively by 15% and 7% and a CVA as a spread is equal to 1%, the forward price at the inception, i.e. the delivery price is given by:

$$K_{CVA} = F_{0CVA} = S_0 e^{+(r+u+CVA-y)T} = \$53{,}45 e^{+(0{,}006727+0.15+0.01-0.07)2} = 64{,}85802417$$

In this case, the seller receives a higher price than that he would receive if it is not considered the impact of the unilateral counterparty risk.

If the counterparty of the seller in the WTI oil forward has higher credit quality and higher, he could require that the delivery price include the DVA as a spread. Considering the above data and a DVA as a spread equal to -0.5% the delivery price is:

$$K_{DVA} = F_{0DVA} = S_0 e^{+(r+u+DVA-y)T} = \$53{,}45 e^{+(0{,}006727+0.15-0.005-0.07)2} = 62{,}94118$$

In this case the impact of the counterparty risk of the buyer is charged on the seller that receives un lower delivery price than that he would receive if it is not considered the own default.

However from the point of view of a standardization of WTI oil forward, an Authority could require that the delivery price is determined considering the bilateral nature of the counterparty risk by mean the BCVA. In this case it is considered the impact of the counterparty referred to both parties of the WTI oil forward contract, then the cost of the counterparty risk is charged on the party that has a lower credit quality to the net of the cost of his exposure to counterparty risk. If BCVA as a spread amounts to 0.5% (CVA-DVA), the delivery price of the WTI oil forward contract is given by the following formula:

$$K_{BCVA} = F_{0BCVA} = S_0 e^{+(r+u+BCVA-y)T} = \$53{,}45e^{+(0{,}006727+0.15+0.005-0.07)2} = 64{,}21267605$$

In this case the seller collects an higher delivery price than that one of the counterparty risk-free, but lower than that he receives if it is considered his unilateral exposure to the counterparty risk.

To conclude, no contemplation of the counterparty risk could lead to issues of mispricing in an incomplete assessment of the integrated risks affected the derivative portfolio. On the contrary we propose a *complete* pricing approach for obtaining an adjustment of the evaluation market-oriented.

References

1. Biffis, E. Blake, D. Pitotti, L. Sun, A., **The cost of the counterparty risk and collateralization in longevity swaps**. The journal of risk and insurance, vol. 9999, 2014, pp. 1-33.
2. Cox J.C., Ingersoll J.E. and Ross, S.A., **A Theory of the Term Structure of Interest Rates**, Econometrica, 53, 1985, pp. 385-407
3. Gregory, J., **Counterparty credit risk and credit value adjustment**, Wiley Finance 2nd edition, 2012
4. Hull J. C., **Options, Futures and Other Derivatives**, Pearson 8th edition, 2011

[1] Final values