ROBUST CONTROL CHARTS BASED ON MODIFIED TRIMMED STANDARD DEVIATION AND GINI'S MEAN DIFFERENCE

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Abstract
Control Charts are process control techniques widely used to observe and control deviations and to enhance the quality of the product. Traditional control charts are based on the assumption that process data are independent in nature. Shewhart control charts are well known and are based on the basic assumption of normality. If process parameters are used to construct control limits based on preliminary samples, stability of the limits needs to be established as presence of outliers may affect the setting of control limits. In this paper an attempt has been made to first develop robust control charts based on trimmed mean and modified trimmed standard deviation. Secondly, an estimate of process standard deviation using Gini's Mean Difference (G) is also considered to modify the mean chart. Lastly, a comparative study is carried out to evaluate the performance of these two proposed robust charts with existing robust $\bar{x}$-MAD chart and two classical control charts namely $\bar{x}$-s chart and its modified $\bar{x}$-s_v chart, based on simulated data. Simulation study is also considered for performance evaluation of the proposed charts with other charts based on Average Run Length (ARL) and Operation Characteristic (OC) curves. In addition to the simulation, real data set is also used for setting up of robust control limits.

Keywords: *Control chart, Trimmed mean, Modified Trimmed Standard deviation, Gini's Mean Deviation, Outlier, Average Run Length, Operation Characteristic Curve*

1. Introduction

The Shewhart control chart, used for monitoring industrial processes is the most popular tool in Statistical Process Control (SPC), developed under the assumption of independence and normality. Performance of a control chart is dependent on the stability of the estimates used to construct control limits in phase I of the analysis. If selected estimators for constructing the limits are influenced by extreme values, setting up of limits may affect the control charting procedure. In situations, when the set limits are narrow, risk of a point falling beyond the limit increases thereby false indication of the process being out of control also gets increased. Similarly, if the limits are wider, the risk of points falling within the limits increases and hence falsely indicates the process to be in control.

When extreme values are present, mean and sample standard deviation are not considered to be good representatives of the data. A robust estimator is an estimator that is insensitive to changes in the underlying distribution and also resistant against the presence of outliers. Rockie (1989) suggested that in order to identify outliers, limits of a control chart be set based on robust measures while non-robust measures are plotted on it. There are many robust and non-robust measures of location and scale available in literature, which are used to develop control charts. The benefit of using control charts based on robust control statistics is that it does not have either a very high or a very low false alarm rate whenever the parameters to be controlled are close to the targets, although the data is no longer normal.

A location free, unbiased measure that can be used even with departure from normality is Gini's Mean Difference (*G*) introduced by Corrado Gini (1912). Yitzhaki (2003) introduced *G* as a superior measure of variability for non-normal distributions. Among the robust measures for dispersion, Median Absolute Deviation (*MAD*), introduced by Hampel (1974) is the widely used measure in various applications, as an alternative to sample standard deviation. Yitzhaki and Lambert (2013) showed that *MAD*, Least Absolute Deviation (*LAD*) and absolute deviation from a given quantile (*QUAD*) are actually either Gini's Mean Difference (G) of specific transformations applied to the distribution of the variables, or special cases of the between-group component of Gini's Mean Difference, called *BGMD*.

Riaz and Saghirr (2007) have developed a dispersion control chart based on G. Abu-Shawiesh (2008) introduced *MAD* to develop a robust dispersion control chart. Kayode (2012) used an estimate of process dispersion using *MAD* to improve mean chart and evaluated its performance through a comparative study of control charts.

The location charts use sample mean as an estimate for location parameter which are easily influenced by the extreme values and hence not suitable for heavy tailed distributions. The effect of extreme observations may be reduced with such observations being simply removed or given less weightage.

In this context, trimmed mean [Tukey, 1948] and its standard error are more appealing because of its computational simplicity. Apart from that, these measures are less affected by departures from normality than the usual mean and standard deviation. Wu and Zu (2009) showed that the trimmed mean is much more robust than its predecessor called Tukey trimmed mean. Relative to the mean, trimmed mean is highly efficient for large percentage of trimming at light-tailed symmetric distributions and much more efficient at heavy-tailed ones. Standard error of trimmed mean is not sufficient to estimate process dispersion because of trimming [Dixon and Yuen, 1974]. Huber (1981) obtained a jackknife estimator for its variance. Capéraà and Rivest (1995) derived an exact formula for variance of the trimmed mean as a function of order statistics, when trimming percentage is small. Sindhumol *et al.*(2015) modified trimmed standard deviation and observed it to be relatively more efficient compared to sample standard deviation. The γ-trimmed mean is defined as

$$\mu_t = \frac{1}{1-2\gamma}\int_{x_\gamma}^{x_{1-\gamma}} x\, f(x)dx. \qquad (1)$$

The trimmed mean is both location and scale equivariant. Its influence function is bounded but has jumps at x_γ and $x_{1-\gamma}$. It is qualitatively robust when $\gamma > 0$ and its breakdown point is γ. It shares the best breakdown point robustness of the sample median for any common trimming thresholds.

The principal purpose of this paper is to propose mean charts based on trimmed mean and its modified standard deviation. Moreover, simulation study is carried out to show the robustness at different levels of trimming. Further, a modified mean chart based on *G* is also developed as a robust control chart. Simulation study is carried out to compare the control limits of the proposed robust control charts with robust $\bar{x}$-MAD chart and classical $\bar{x}$-s chart and $\bar{x}-s_v$ chart. Comparison of proposed charts performance using Operation Characteristic (OC) as well as Average Run Length (ARL) for different distributions is also carried out.

2. Classical Mean Charts

If x_{ij} ($i = 1,2,\dots,n$ and $j = 1,2,\dots,m$) represents *m* random subgroups each of size *n* taken from a continuous and identical distribution with σ^2 unknown, an unbiased estimator is obtained using variance $s_j^2 = \frac{1}{n}\sum_{i=1}^{n}(x_{ij}-\bar{x}_j)^2$, where $\bar{x}_j = \frac{1}{n}\sum_{i=1}^{n}x_{ij}$. If the process distribution is normal with parameters (μ, σ) control limits constructed for phase I analysis of mean chart are

$$\text{CL} = \bar{\bar{x}},\ \text{UCL} = \bar{\bar{x}} + 3\frac{\bar{S}}{(\sqrt{n})c_4} = \bar{\bar{x}} + A_3\bar{S},\ \text{LCL} = \bar{\bar{x}} - 3\frac{\bar{S}}{(\sqrt{n})c_4} = \bar{\bar{x}} - A_3\bar{S}. \quad (2)$$

where *n* is the sample size and A_3 is a function of *n* and average of sample mean $\bar{x}$ and standard deviation *s* are taken over *m* subgroups.

Mahmoud *et al.* (2010) considered $\bar{S}_v = (\frac{1}{m}\sum_{i=1}^{m}S_i^2)^{1/2}$ based on unbiased estimator of sample variance and showed that its efficacy in control charting application. Thus, using this estimator, a modification to classical location control limits can be set as

$$\text{UCL} = \bar{\bar{x}} + A\,\bar{S}_v;\ \text{CL} = \bar{\bar{x}};\ \text{LCL} = \bar{\bar{x}} - A\,\bar{S}_v. \quad (3)$$

The above two standard classical control charts is chart is used to draw comparison and evaluate the performance of the proposed charts in the presence of outliers.

3. Robust chart based on Median Absolute Deviation

If $x_1, x_2, \dots, x_n$ is a set of observations, Hampel (1974) defined a robust estimate of dispersion as

$$MAD = 1.4826\ \text{median}\{|x_i - \text{median}(x_i)|\}. \quad (4)$$

Abu-Shawiesh (2008) used MAD to estimate and control process dispersion. Let *m* preliminary samples of size *n* are used to estimate, $\hat{\sigma} = b_n\overline{MAD}$, where average is taken over *m* subgroups. The constant b_n is the small sample correction factor given by Croux and Rousseeu (1992) as given below.

N	2	3	4	5	6	7	8	9
b_n	1.196	1.495	1.363	1.206	1.200	1.140	1.129	1.206

For n>9, b_n = n/(n-0.8). The general model for the control chart based on MAD is introduced by Abu-Shawiesh (2008). Kayode (2012) used MAD to modify limits of mean chart as

$\mathrm{UCL} = \bar{\bar{x}} + \mathrm{A}_6\overline{MAD};\ \ CL = \ \bar{\bar{x}}\ ;\mathrm{LCL} = \bar{\bar{x}} - \mathrm{A}_6\overline{MAD}$ (5)

where $\bar{\bar{x}}$ is the average of the sample mean taken over *m* preliminary samples and $\mathrm{A}_6 = 3\ \frac{b_n}{\sqrt{n}}$. Adekeye and Azubuike (2012) have shown that this robust chart is good even for non-normal population.

4. Robust Control chart based on Gini's Mean Difference (G)

If $x_1, x_2, \dots, x_n$ is a set of observations, an index of variability G is defined as

$G = 2\sum_{j=1}^{n}\sum_{i=1}^{n}\left|x_i - x_j\right|/n(n-2) \quad (i \neq j)$ and $= (\sqrt{\pi}/2)G$. (6)

Riaz and Saghirr (2007) developed a dispersion control chart named *G*-chart based on G and its control limits are

UCL $=\bar{K} + 3b_3\bar{K};$ CL $=\bar{K};$ LCL $=\bar{K} - 3b_3\bar{K}.$ (7)

where $\bar{K}$ is the average of K taken over *m* subgroups and b_3 is the standard deviation of G/σ.

A modified mean chart based on process dispersion estimated using G can be constructed with the control limits as

$\mathrm{UCL} = \bar{\bar{x}} + A\ \bar{K}; \mathrm{CL} = \bar{\bar{x}}; \mathrm{LCL} = \bar{\bar{x}} - A\ \bar{K}.$ (8)

where $A = \ 3\ /\sqrt{n}.$

The proposed modified chart $\bar{x}$-K based on G is considered for performance evaluation with the classical $\bar{x} - sv$ chart as K is an unbiased estimator which can be used as an alternative to sample variance and s_v is based on unbiased estimator.

5. Robust Control Charts based on trimmed mean

Langenberg and Iglewicz (1986) introduced trimmed mean $\bar{x}$ and R chart. Amin and Miller (1993) have developed robust $\bar{x}$ control chart usingVariable Sampling Interval (VSI) schemes and evaluated the behavior of VSI charts where trimmed mean, Winsorized mean, and the median are used. Figueiredo and Gomes (2004 and 2009) considered robust control charts based on the total median and on the total range statistics, for monitoring the process mean value and the process standard deviation, respectively. Schoonhoven.*et.al.* (2011) used a few robust location measures namely Median of means, 20% Trimmed mean of sample means, Hodges–Lehmann, Trimeans and 20% Trimmed mean of sample trimeans to develop mean charts. Schoonhoven and Does (2013) used Adaptively Trimmed Standard deviation denoted by ATS and 20%trimmed mean to improve mean chart.

Let $x_{(1)} \leq x_{(2)} \ldots \leq x_{(n)}$ denote an order statistics sample of size *n*, from a population having symmetric distribution. The *r*-times symmetrically trimmed sample is obtained by dropping both *r*-lowest and *r*-highest values. Here $r = [\alpha n]$ is the greatest integer and trimming is done for $\alpha\%(0 \leq \alpha \leq 0.5)$ of *n*. Trimmed mean is defined as

$\bar{x}_T = \frac{\sum_{i=r+1}^{n-r} x_{(i)}}{n-2r}.$ (9)

Sample standard deviation of observations from trimmed mean is

$s_T = \sqrt{\frac{\sum_{i=r+1}^{n-r}(x_{(i)} - \bar{x}_T)^2}{n-2r-1}}.$ (10)

Modified standard deviation is defined as $\hat{\sigma}_T = s_T^* = 1.4826\, s_T$ where the constant multiplier is used to cover the loss of information due to trimming. As percentage of trimming is increased, this constant gives a control on loss due to trimming. Sindhumol.*et al.*(2015) have introduced this modified standard deviation of trimmed mean, say s_T^* and have introduced it to control process dispersion. If process dispersion is estimated using this robust estimator, its average over m subgroups is, $\bar{s}_T^* = 1.4826\,\bar{\mathrm{s}}_{\mathrm{T}}$ and limits of dispersion chart are

CL $= c_4\bar{s}_T^*$; LCL = $B_3\,\bar{s}_T^*$ and UCL = $B_4\bar{s}_T^*$. (11)

The constants B_3 and B_4 are the same constants as used in a classical chart. The limits of mean control chart can be modified using this modified standard deviation and depending upon the percentage of trimming, these limits have varying width as

CL = $\bar{\bar{x}}$; UCL = $\bar{\bar{x}} + A_3\bar{s}_T{}^*$; LCL = $\bar{\bar{x}} - A_3\bar{s}_T{}^*$. (12)

If location parameter is also estimated using trimmed mean, one can get control limits of robust mean chart for a particular level of trimming as

CL = $\bar{\bar{x}}_T$; UCL = $\bar{\bar{x}}_T + A_3\bar{s}_T{}^*$; LCL = $\bar{\bar{x}}_T - A_3\bar{s}_T{}^*$. (13)

The constants A_3 and A_6 are functions of sample sizes and the advantage of proposed control chart is the usage of the same constants A_3 as used in the classical control chart.

The proposed robust control charts based on two levels of trimming of 10% and 20%, namely $\bar{x} - s_T^*$ charts, $\bar{x}_T - s_T^*$, and the proposed robust control chart based on Gini, $\bar{x}$-*K* chart and the other robust chart $\bar{x}$-*MAD* discussed earlier along with the classical $\bar{x} - s$, $\bar{x} - s_v$ charts are considered for comparison.

6. Comparison of Robust charts for performance evaluation

The performance evaluation of the proposed robust charts is carried out as an empirical study based on Monte Carlo simulation conducted with 5000 runs using SAS software. Random samples generated from N(0,1) with sample sizes (n) of 10 and 20 with m=20 as number of samples for each cases are considered for simulation. Two levels of symmetric trimming at10% ($r = 1$) and 20% ($r = 2$) are considered for each subgroup. Clean samples are considered from N(0,1) to analyze type-1 error and contaminated samples are included to study the effect of detection of outliers or assignable causes of variation. The study considered out-of-control situation based on samples taken from N(1,1), N(2,1) and N(4,1). Samples from normal distribution with high location parameter, as is in the case of N(4,1), detection of shifts are easily studied in all the above charts. When small deviation in process location happened, as from samples N(1,1), even though false alarm is almost equal for all charts, charts based on 20% trimming detects contaminated sample more efficiently, compared to other charts. The performance of this chart is more clear and confirmed for the case of N(2,1). Simulation study also showed that the performance of mean chart is improved compared to the classical mean chart, when modified standard deviation of 20% trimming is used to estimate process dispersion.

A classical way of illustrating the effect of departure from normality is to consider contaminated normal distribution. Contamination of 40% is made for 5% of the data, which are the last four subgroups among 20. Hence sample mean is calculated so that 10% and 20% trimming will be meaningful for a contaminated subgroup. Thus, the contaminated models are 0.60 N(0,1) with 0.40 N(1,1) and 0.40 N(2,1).

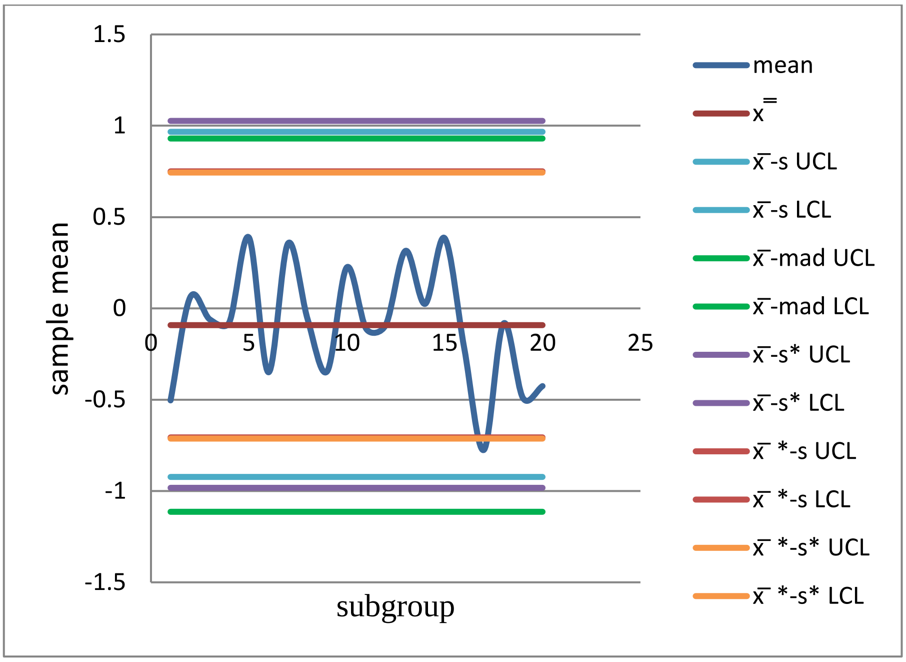

Figure 1: Control limits of modified trimmed mean charts, $\bar{x}$-s_T^* and $\bar{x}_T$-s_T^* and other charts based on MAD and s with n=10, m=20 and 10% contamination using N(2,1)

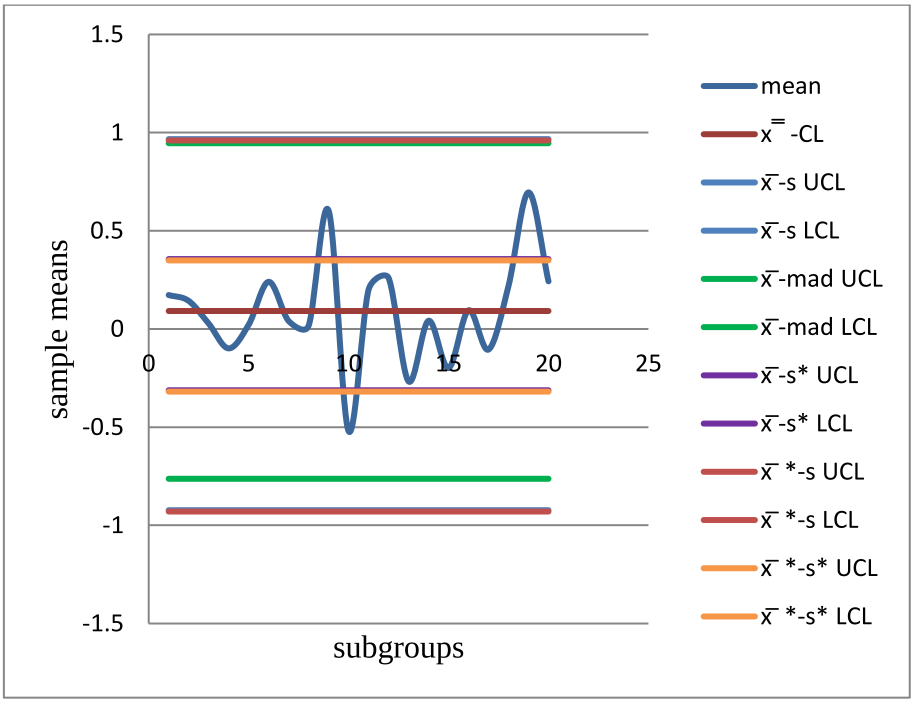

Figure 2: Control limits of modified trimmed mean charts, $\bar{x}$-s_T^* and $\bar{x}_T$-s_T^* and other Charts based on MAD and s with n=10, m=20 and 20% Contamination using N(1,1).

When charts $\bar{x}$-s, $\bar{x}$-s_v, $\bar{x}$-MAD, $\bar{x}$-s_T^*(with 10% and 20% trimming) and $\bar{x}_T$-s_T^*(with 10% and 20% trimming) are compared in presence of contamination, more contaminated samples are detected by $\bar{x}_T$-s_T^* chart than other charts having almost same type I error. Also, for 10% level of trimming, charts $\bar{x}_T$-s and $\bar{x}_T$-s_T^* perform equally well and have the ability to detect small variations too. Though the chart $\bar{x}$-$s_T^*$is bit wider compared to $\bar{x}$-s chart due to the effect of multiplier, chart $\bar{x}_T$-s_T^* has smaller width compared to $\bar{x}$-MAD. Figure-1 shows performance of modified trimmed mean chart compared to other charts including chart based on MAD, in terms of detection of samples when contaminated with N(2,1) ($\bar{x}_T = \bar{x}^*$and $s_T^* = s^*$in all the figures).

At 20% level of trimming, width of the charts reduces respectively for charts $\bar{x}$-s, $\bar{x}$-MAD and $\bar{x}$-$s_T^*$and performance of charts $\bar{x}$-s_T^* and $\bar{x}_T$-s_T^* are same. Increase in trimming makes loss of data as well as an increase in type I error. Still these charts have the ability to detect smaller variation and can even act as warning limits in case of larger trimming levels. Selection of dispersion measure influences performance of mean chart and the simulation study shows that $s_T^*$is a better choice. Figure-2 shows performance of the modified trimmed mean chart compared to other charts including chart based on MAD, in terms of detection of samples when contaminated with N(1,1) respectively. The charts $\bar{x}$-s_T^* and $\bar{x}_T$-s_T^* have same and smaller width which helped in the early detection of variations.

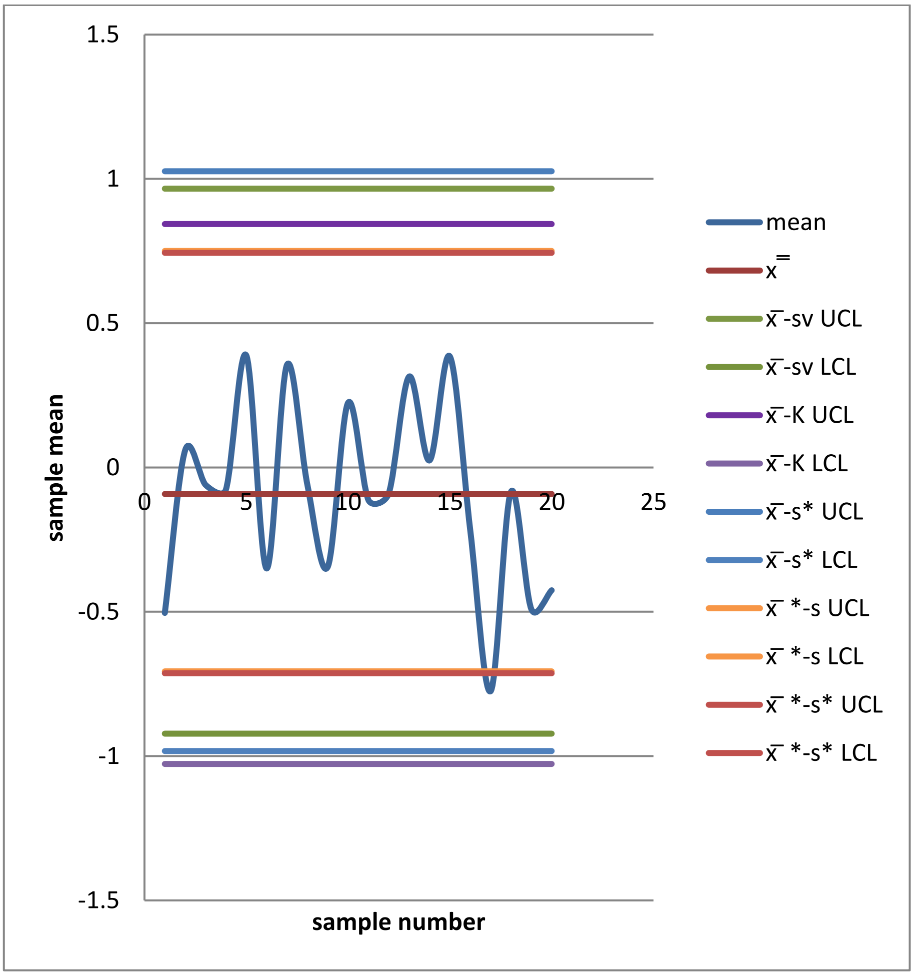

Figure 3: Control limits of modified trimmed mean charts, $\bar{x}$-s_T^* and $\bar{x}_T$-s_T^* and $\bar{x}$-K Charts with other charts based on MAD and s_v with n=10, m=20 and 10% contamination using N(2,1).

When charts $\bar{x}$-s_v, $\bar{x}$-K, $\bar{x}$-s_T^* (with 10% and 20% trimming) and $\bar{x}_T$-s_T^* (with 10% and 20% trimming) are compared in presence of contamination, more contaminated samples are detected by $\bar{x}_T$-s_T^* chart than other charts having almost same type I error. At 10% level of trimming, chart $\bar{x}$-s_T^*showed almost same width as $\bar{x}$-s_v due to the effect of multiplier s_T^*. The charts with smaller width namely, $\bar{x}_T$-s and $\bar{x}_T$-s_T^* perform equally well and have ability to detect small variations too. Mean chart with dispersion measure K is a good alternative to that of s_T^* as width of $\bar{x}$-K chart is smaller in width than that of $\bar{x}$-$s_T^*$and $\bar{x}$-s_v, for small percentage of trimming. Figure -3 showed performance of modified trimmed mean chart compared to other charts in terms of detection of samples when contaminated with N(2,1).

At 20% level of trimming, chart $\bar{x}$-s_T showed almost same performance and width as $\bar{x}_T$-s_T^* and have the smallest width among other charts. All other charts, including charts based on K, have almost the same width. Mean chart with dispersion measure K is a good alternative to mean chart with s_v, though both are based on unbiased estimators. Figure -4 shows performance of trimmed mean chart compared to other charts including $\bar{x} - K$ in terms of detection of samples when contaminated with N(1,1).

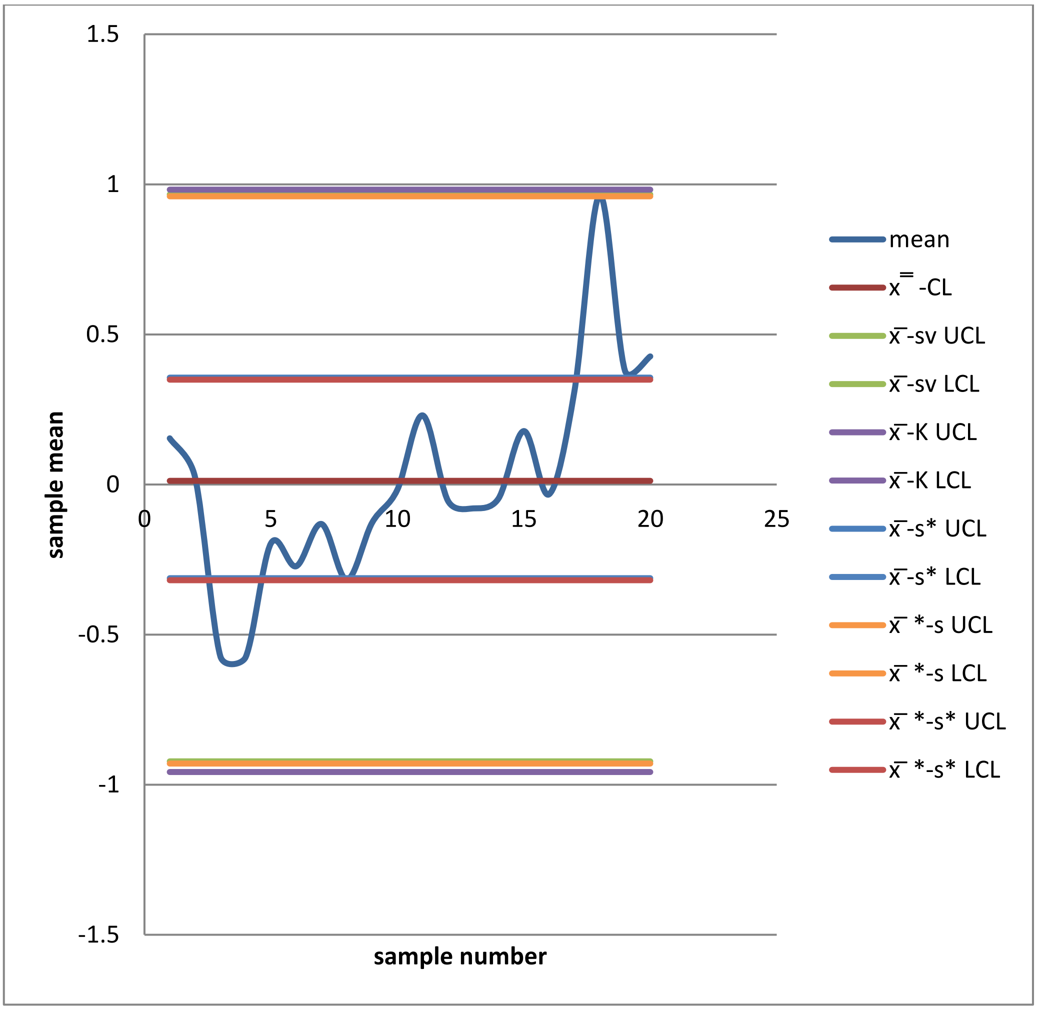

Figure 4: Control limits of modified trimmed mean charts, $\bar{x}$-s_T^* and $\bar{x}_T$-s_T^* and $\bar{x}$-K Charts with other charts based on MAD and s_v with n=10, m=20 and 20% contamination using N(1,1).

6.1. Performance using Operating Characteristic (OC) curves

The ability of the chart to detect shift in quality due to assignable causes of variation is accessed by Operating Characteristic curve. The OC curve of the $\bar{x}$-chart for phase II analysis is studied here. The OC function of $\bar{x}$–chart, is the probability of not detecting the shift in the process mean μ on the first subsequent sample of size n taken after the shift has happened. If the mean in the in-control state is μ_0 and the shift is $\mu_1 = \mu_0 + k\sigma$, β-risk is defined under standard normal distribution function φ as

$$\beta = P(LCL \le \bar{x} \le UCL \,/\, \mu = \mu_1) = \varphi(3 - k\sqrt{n}) - \varphi(-3 - k\sqrt{n}). \quad (14)$$

The OC curve for the chart is drawn by plotting the β -risk against the magnitude of the shift in quality which is expressed in standard deviation units for various sample sizes *n*. Process s.d is considered to be known or to be estimated before considering β–risk. Steeper the curve better the probability of detection of shift, which is variation in the process quality. Figure 5 shows that charts $\bar{x}_T$-s and $\bar{x}_T$-s_T^* have the smallest β-risk compared to all other charts. Mean charts modified with robust measures MAD and G have almost same β-risk and OC curves are overlapping.

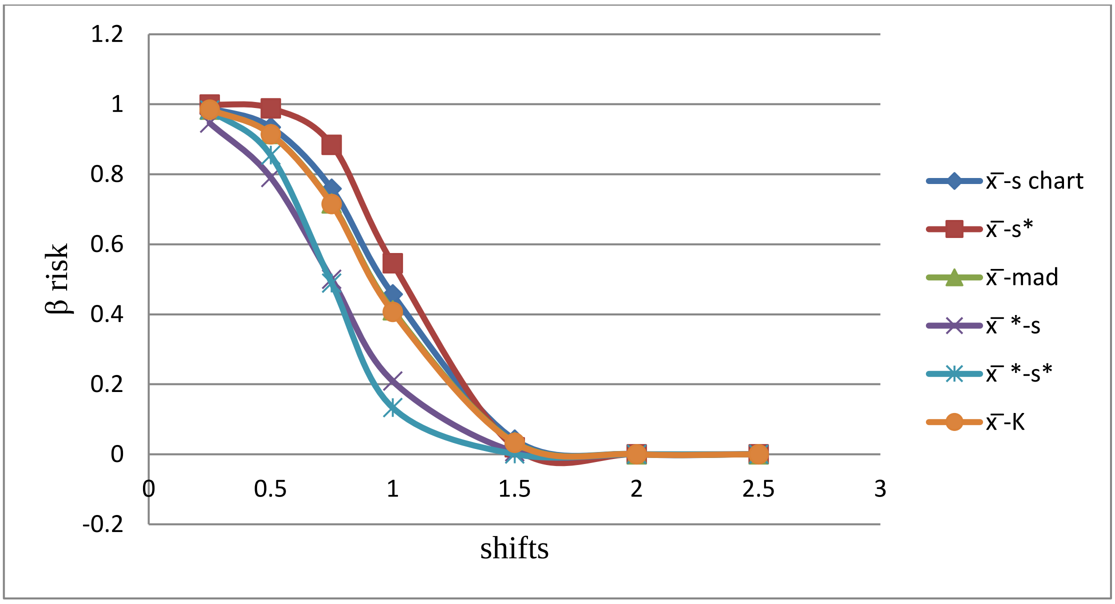

Figure 5: Comparison of OC curves for modified trimmed mean charts and $\bar{x}$-K chart with other charts based on s and MAD.

6.2. Performance Using Average Run Length

Average Run Length is defined as reciprocal of the probability that any point exceeds the control limits. It shows how often false alarms occur or how often the chart detects a shift in quality. If the process is in-control, Average Run Length, ARL_0, should be large and if the process is out-of-control ARL should be very small. The number of subgroups taken before an out-of-control $\bar{x}$ happened is recorded as a run length observation, RLi. The process is repeated 10000 times and the results of this simulation study are given in Table 1. The ARL_0 was calculated as $ARL_0 = \frac{\sum_{i=1}^{10000} AR_i}{10000}$. The same procedure is used to compare the out-of-control ARL_1, that is, in presence of assignable causes also.

Table 1: ARL for sample size n=10 and m=20

Distributions	$\bar{x}-s$ chart	$\bar{x}-S_{var}$ chart	$\bar{x}$ –MAD chart	$\bar{x}-K$ chart	$\bar{x}-S_T$ chart		$\bar{x}_T-S_T$ chart	
					r=1	r=2	r=1	r=2
N(0,1)	323	322	319	230	506	64	527	63
N(1,1)	544	544	581	620	470	0.283	470	0.275
N(2,1)	0.0008	0.0008	0.006	0.004	0.0008	0	0.0008	0
N(0,1)+40%	317	317	331	441	256	0.74	261	0.72

N(1,1)								
N(0,1)+40% N(2,1)	66	66	68	83	65	6.43	65	6.14

Table 1 shows that among the mean charts, dispersion estimated using s_T^* with 10% trimming ARL_0 calculated on N(0,1) is larger than that of other charts. Hence small percentage of trimming can be advisable even for a controlled process. In presence of assignable causes, ARL_1 is better for 20% trimming. The ARL comparison supports mean chart modified with robust MAD than that of G. The study shows the need of using robust measure of dispersion in place of sample standard deviation, especially s_T^* while using mean chart to control a process.

The ARL study supports MAD as a better choice as compared to G to modify mean charts. The study has established use of trimmed mean and modified trimmed standard deviation to estimate process mean and standard deviation and mean charts modified on these estimate as better choices in quality control applications.

7. Charts performance on real data

The performance of charts based on trimmed means at two levels has shown to be a better performer compared to other robust charts and classical charts based on simulation studies, OC curves and ARL. The performance of the charts is validated on real data shown in Figure 6.

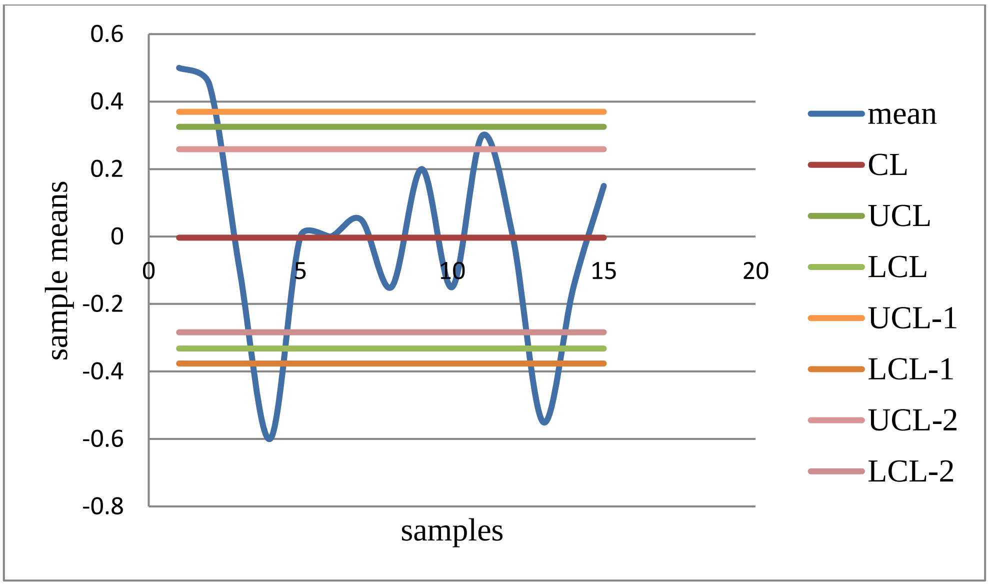

Figure 6: Comparison of modified trimmed mean charts for trimming 10% and 20% respectively with classical mean chart based on data from D.C.Montgomery, Table 6E.5

The fill volume of soft drink beverage bottles is considered as a quality characteristic. The volume is measured (approximately) by placing a gauge over the crown and comparing the height of the liquid in the neck of the bottle against a coded scale. On this scale a reading of zero corresponds to the correct fill height. Fill heights are shown by Montgomery (2009) in Table 6E.5 and fifteen samples of size n=10 have been analyzed using mean charts with dispersion measures s (in figure LCL, UCL), modified trimmed standard deviation s_T^* for 10%

(in figure LCL-1, UCL-1) and for 20% (in figure LCL-2, UCL-2). The s_T^*-chart for 20% trimming detected 10th sample also as out of control point.

8. Conclusion

Trimmed mean and standard deviation based on it are robust measures of location and scale. Trimmed mean is already exposed to control charting process. However, actual variance of trimmed mean which is a function of order statistics or variance of trimmed mean modification based on Winsorization, are not helping to represent process variance. The modified standard deviation presented by Sindhumol.et.al. (2015) is a better alternative in this regard.

Selection of dispersion measure as well as location measure to estimate process parameters influence performance of mean chart. In this study two robust location control charts, one based on measures on trimmed data and the other based on G are proposed. Four types of trimmed mean charts including trimmed mean (10% and 20% of trimming), modified trimmed standard deviation for two levels of trimming (10% and 20% of trimming), two robust mean charts one modified with G and the other with MAD, two classical mean charts based on unbiased sample variance and biased sample standard deviation, are considered and a simulation study is conducted for comparing performance of charts in terms of false and correct detection of outliers. In presence of assignable causes of variation, charts based on trimmed mean and modified trimmed standard deviation, the results are outstanding even for small shifts. The chart can be even used as warning limits for early detection of assignable causes, for large trimming percentage. The chart $\bar{x}_T$-s_T^* has smaller width compared to $\bar{x}$-MAD and $\bar{x}$-K charts, especially in large trimming. Mean chart with dispersion measure K is a good alternative to that of s_v (Mahmoud et al. , 2010) though both are based on unbiased measures of dispersion, as width of $\bar{x}$-K chart is smaller. The study shows that to modify to mean chart, MAD is slightly a better choice than G. It is noted that, both $\bar{x}$-s_T^* and $\bar{x}_T$-s_T^* (at10% trimming) perform better than all other charts when there is no contamination. The comparative study of limits in terms of false detection shows that all charts are alike except for 20% trimming.

The OC curves show that charts $\bar{x}_T$-s and $\bar{x}_T$-s_T^* have the smallest β-risk compared to all other charts and hence have the largest power to detect assignable causes of variations. Mean charts modified with robust measures G and MAD have almost same β-risk.

The ARL study shows that a small percentage of trimming is advisable even for an in-control process. Robust chart based on trimmed mean and modified trimmed standard deviation chart excel in performance for small percentage of trimming and can even be used as warning limits for large percentage of trimming, in the presence of assignable causes. The ARL comparison supports mean chart modified with robust MAD than that of G while OC curve study gives all most equal β-risk.

Robust location chart based on trimmed mean and modified trimmed standard deviation can be used as an effective tool to control and improve process performance. This robust limits help to detect assignable causes or outliers in the phase II analysis if statistical control of process location parameter is tested using sample means collected. Both ARL and OC study show that, $\bar{x}$-$s_T^*$and $\bar{x}_T$-s_T^* (at10% trimming) perform better than all other charts under contamination or otherwise. The study also that in order to frame warning limits, one can increase the trimming percentage. Hence study supports usage of data with small percentage of trimming to frame control limits in phase I analysis to get a better control in phase II analysis of a process.

Charts are tested on real data also, with two levels of trimming. The robust chart for controlling process location parameter using trimmed mean and modified trimmed standard deviation is a better option than classical mean chart in real data too. This modified trimmed mean chart with 20% trimming identifies more outlier points.

Study shows that the performance of classical mean chart is increased if process dispersion is estimated using s_v and the robust measure G is a good alternative to s_v. Among the G and MAD, for constructing a robust control chart, MAD seems to be a better estimate but trimmed mean and modified trimmed standard deviation are better pairs of estimates in quality control applications.

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