COMPOSITE INDEX: METHODS AND PROPERTIES

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Abstract

Composite Index (CI) depends on method of combining several variables or indicators to reflect overall assessment. Each method of combining the component indicators results in different values of CI and different rankings from a given dataset.

The paper describes problems for construction of CI at various stages and proposes a number of methods for obtaining CI along with desired properties which a good CI will satisfy. Existing and proposed methods to construct CI can be compared with respect to those desired properties. The Geometric Mean approach satisfies all the desired properties and avoids calculation of weights or variance-covariance matrix or correlation matrix. The Geometric Mean approach is applicable for situation even where only the two vectors X and T are given for the current year and the previous year. Thus, the Geometric Mean approach is well applicable for assessment of impact. The approach also helps to identify relative importance of the component indicators in terms of values of the ratios and also identify the critical areas and facilitate initiation of corrective measures. Such identification is important from a policy point of view. The GM method reduces the level of substitutability between component indicators and facilitates statistical test of significance of equality of two geometric means. Thus, the Geometric Mean approach and may be taken as best among the methods discussed.

Keywords:

Composite index; Geometric mean; Monotonicity; Time-reversal test; Chain indices

1. Introduction

Composite Index is constructed by combining several variables or indicators together. It is essentially an attempt to find a function *f* from $R_n \to R$ corresponding to *n*-number of component indicators/variables. Indicators are functions of one or more variables to measure the extent to which a specified objective or outcome has been achieved. An indicator provides direct measure of a specified aspect of the objective. However, for longitudinal data, an indicator may be treated as a variable. Similarly, an indicator for several zones, organizations under a particular industry, etc. for a particular time period (say year) may also be treated as a variable. Ghai (2003) opined that there is rarely one single measure of the desired outcome and a combination of several indicators may give a more accurate measure of a specified objective. Saisana, Tarantoorbakhshola, and Saltelli (2005), argued that composite indices can be used to summaries complex or multi-dimensional issues and facilitate ranking of countries on complex issues, etc.

Objective of Composite Index is to find measure combining all the identified variables or indicators to reflect overall current status/ progress or overall distance from the set of quantifiable targets. The well known Human Development Index (HDI) developed by the United Nations Development Programme (UNDP) is an example of such an overall index combining indicators of health, education and income. Status of employment opportunities in a country at a time period may consider combination of well defined indicators like labour force participation rate (LFPR), employment-to-population ratio (EPR) and unemployment rate (UR) (ILO 1999). Other examples are Physical quality of life index (Ram, 1982), Logistics Performance Index (LPI) by World Bank, etc. To supplement multiple set of indicators adopted by monetary authority, Kannan, Sanyal and Bhoi (2006) constructed Monetary Condition Index (MCI) of India considering weighted sum of (i) difference between short term real interest rate at a time period *t* (r_t) and interest rate in a given base period (r_0)and (ii) difference between logarithm of the real effective exchange rate at a time period t (e_t) and exchange rate in a given base period (e_0). It is well known that logarithm transformations are used primarily to correct right skewness which may tends to increase correlations. Bahadori et.al (2011) investigated performance of six hospitals using indicators like Bed Occupancy Rate (X_1), Bed Turnover (X_2), Average Length of Stay (X_3). Chakrabartty (2005) proposed Single Index measure of Port efficiency by Geometric mean of ratios of current value and corresponding target for each chosen indicator. HDR 2010 considered a new HDI based on the geometric mean of the scaled indicators where normalization was done by usual$\frac{X-X_{min}}{X_{max}-X_{min}}$. The 2014 HDR also considered geometric mean approach where normalization was done using fixed maximum and minimum values separately for each chosen indicator (UNDP 2015). Construction of Composite Index (CI) involves stages like:

- Selection of component indicators or variables – There is no thumb rule. Component indicators may depend on purpose, conceptual relevance, consistency and availability of data, etc. Set of quality criteria for the selection of variables and indicators suggested in the literature (Booysen, 2002, Nardo et. al. 2005). However, relevant question is whether component indicators will have strong correlations with CI or whether component indicators will have insignificant correlations amongst themselves? The higher the correlation between the indicators, the fewer statistical dimensions will be present in the dataset. Ravallion (1997), Srinivansan, 1994, observed high correlation of all the HDI components among themselves as well as with the HDI, Ogwang (1994) argued that the HDI doesn't reveal anything beyond that portrayed by the GDP or by the life expectancy alone. McGillivray (1991) found value of Spearman rank correlation between the rankings generated by the per-capita GDP and the rankings generated by the HDI was 0.893. High correlation of one indicator with the Composite Index may imply no need to construct a Composite Index since in that case; one can use the former instead of CI. Similarly, the CI may not be really multidimensional in case the component indicators have high correlations among themselves.
- It is suggested that high correlation between two component indicators may be avoided. For example, Port Performance Index should not cover Container traffic and number of TEUS as they are almost perfectly correlated. Similarly, Turn Round Time (TRT) per thousand tonnes of cargo is a better indicator that TRT. For instance, if normalization is done as $\frac{X-X_{min}}{X_{max}-X_{min}}$, Cochin Port with the lowest TRT among Indian Major Ports, may get the highest possible score of 1.00, while Kandla with the highest TRT among Major Ports,

will get the least possible score of zero. This distortion becomes glaring when we note that both the ports have comparable productivity levels. TRT per thousand tonnes of cargo will not give rise to such distortion. Multi-cargo ports and predominantly single commodity ports may not be measured by the same yardstick say Container traffic. A better way could be to consider proportion of containers in the total cargo of the port.

- Scaling of chosen indicators – Chosen variables or indicators are often transformed or normalized. Method of normalization or transformations will have effects on the CI. For example, Kovacevic (2011) found correlation between Life expectancy (LE) and HDI was maximum (r = 0.92) and same for GDP was least (r = 0.71). However, when logarithmic transformations were used, $r_{HDI,\ln(GDP)} > r_{HDI,GDP}$ and $r_{HDI,LE}$ was least. Transformation may change shape of the original distribution and may give noisy results if attention is not given to properties of such transformations. Primary purpose of scaling is to make the indicators unit free and to have a desirable range.
- Popular method of normalizing an indicator considers $Z = \frac{X - X_{min}}{X_{max} - X_{min}}$. Such normalization depends heavily on the extreme values and on $X_{max} - X_{min}$. For such normalization, Sava (2016) opined that score of country A are relative to the performances of other countries and therefore a positive evolution of scores for the country A does not necessarily imply an increase (it might be just due to a decrease in other countries' performances). However, gain in Z per unit increase in X is not uniform for such a normalization procedure as can be seen from the following hypothetical example: Let an indicator X takes values: 90, 85, 80, 58, 96, and 70. Clearly, X_{max} = 96; X_{min}=58; and $X_{max} - X_{min} = 38$. Gain in normalized value (Z) from increasing X from 80 to 90= 0.2632. Similar gain from increasing X from 85 to 90 after normalization = 0.1032 which is less than $\frac{0.2632}{2} = 0.1316$. In other words the X – Z curve is not linear.
- Methods of combining or functional form - Weighted sum appears to be most popular method of combining the selected indicators/variables though other methods of combining are also available. Selection of weights is a central issue for weighted sum approach. Decancq and Lugo (2009) came out with three approaches to find weights to component indicators namely normative (determined in subjective fashion), data-driven (determined objectively) and hybrid. The approach taken by the HDI is normative. Weighted sum approaches assume additive models. Further use of CI and interpretation of CI depends heavily on the method of combining. Methods of obtaining Composite Index based on variance, covariance is not invariant under change of scale. Presence of outliers can affect interpretations arising from Principal Component Analysis. Methods based on Principal Component Analysis tend to ignore (or poorly weigh) those component variables/indictors which do not have strong correlation with the Composite index even if they are theoretically and practically important. Thus, there is a need to construct a Composite Index that is more inclusive in nature.

2. Problem areas:

Major problems relating to scoring, deciding weights, properties of the weighted sum or other methods of combining etc. for measurement of a Composite Index are illustrated below:

2.1 Scoring of Likert type items: Scores of a criterion on a five points or *n*- points scale obtained from Likert type items are in fact ranks (i.e. in Ordinal level) and averaging of such scores may not be meaningful. For example, if 50% respondents prefer an item most (say score of 5 in a five points scale) and rest 50% dislike the item most (say score of 1 in a five points scale), the average score will indicate that the sample was "Neutral" whereas; the sample actually was bi-polarized. This is because distance in terms of the underlying variable/criterion between two successive response-categories in a Likert type item is not same. Chakrabartty (2014) suggested scoring of Likert test as weighted sum where weights are computed from the data.

2.2 Properties of indicators: For composite Index, a set of indicators scores are combined to a single score which is considered in decision making. The method of combining such criteria scores and mathematical properties of such combined score play vital role in decision making as can be seen from the following two examples:

Example – 1: **Invariant properties of Composite Index**

For a particular assignment, Party "A" wanted 8 weeks' time and quoted a price of Rs. 8,000. The party "B" quoted a price of Rs. 10,000 and wanted 5 weeks' time for the same assignment. For evaluation, it was decided to be based on CI in terms of distance of offers from the hypothetical Ideal Point i.e. Zero time with Zero rupees. Quotations of A could be considered as a point with co-ordinates (8, 8) and the same for B as (5, 10) in the Weeks – Rs. in thousand space.

Square of distance between the point A and origin (D_A^2) was 128 and D_B^2 was 125. So, the party B was preferred because it was closer to the Ideal point.

However, if instead of Rs. in thousand, exact amount in Rs. is considered in the Week- Rs. Space, then $D_A^2 = 64{,}000{,}064 \;\; and \;\; D_B^2 = 1000{,}00{,}025$ which implie**s** party A is preferred

The example highlights that minor change of scale in one criterion variable can reverse the decision. This is because; distance is not invariant under change of scale, which is a mathematical property of distance.

Example – 2: Subjective weights

Consider a hypothetical Two-part Tender scheme where a bidder has to submit Technical proposal and Cost proposal. Decisions made for evaluation under Combined Qualification cum Cost Based System (CQCCBS) are:

i) Financial proposals to be converted to Evaluated Cost (EC) as follows. Proposal with lowest cost is given financial score of 100 and other proposals are given financial scores that are inversely proportional to their price.
ii) Total Score is weighted sum of score on Technical Proposal and Evaluated cost.
iii) The Proposal with highest Total Score to be marked as H_1 and will be selected.

Situation- 1. (Tech.Qualification: 70% & Fin. Proposal: 30%)

Firm	Tech. qualification marks	Evaluated Score(EC)	$\frac{LEC}{EC} X\,100$	Combined Score
A	90	120	$\frac{100}{120} X\,100 = 83$	90(0,7) + 83(0.3) = 87.9 Highest Combined Score(H_1)
B	75	100	100	52.5 + 30 = 82.5(H_3)
C	80	110	91	56+27.3 = 83.3 (H_2)

So, Party A gets selected followed by Party C and the Party B
Let us see what happens if weights are changed and rest remain unchanged

Situation- 2. (Tech.Qualification: 60% & Fin Proposal: 40%)

Firm	Tech. qualification marks	Evaluated Score(EC)	$\frac{LEC}{EC} X\ 100$	Combined Score
A	90	120	83	90(0.6)+83(0.4) = 87.2 (H_1)
B	75	100	100	45 + 40 =85 (H_2)
C	80	110	91	48 + 36.4 = 84.4 (H_3)

Ranks of B and C got changed with change in weights of Tech. qualification and Fin. Proposal though all others conditions were kept unchanged.

Situation- 3. (Tech.Qualification: 40% & Fin. Proposal: 60%)

Firm	Tech. qualification marks	Evaluated Score(EC)	$\frac{LEC}{EC} X\ 100$	Combined Score
A	90	120	83	36 + 49.8 = 85.8 (H_3)
B	75	100	100	30 + 60 = 90 (H_1)
C	80	110	91	32 + 54.6 = 86.6 (H_2)

So, Party B gets selected followed by Party C and the Party A.
The example establishes that for the same set of bids involving several parties, probably any party could be selected if weights to Technical qualification and Financial Proposals are changed, even if sum of weights is one. The example also highlights that subjective weights may not be sound and often may lead to noises.

Cases where sum of weights are not equal to one, the convex property of measurement is violated and keep us in dark about mathematical properties of the total combined score.

3. Objective

To describe various methods of combining component indicators to find an overall index or Composite index and discusses the properties of such combinations along with identification of key indicators for corrective actions or policy decisions to facilitate monitoring, etc.

4. Set up

Let $X_1, X_2, \ldots \ldots .., X_n$ be *n*-variables or indicators with different units. $X_1, X_2, \ldots \ldots .., X_n$ could be independent or dependent to various degrees. Problem is to find Composite Index $Y = f(X_1, X_2, \ldots \ldots .., X_n)$ such that Y satisfies a set of desired properties. Thus, Y is a function from *n*-dimensional space to Real line i.e. $R^n \to R$. Let us assume that each component indicator is positively related to the Composite Index. i.e. higher value of X_i imply higher value of Y,$\forall i = 1,2, \ldots .. n,$ keeping all others unchanged. Thus, to find progress in School Education, an indicator like "Drop out rate" or for overall efficiency of a Port, indicators like "Average Turn-round time" or "Operating Cost" whose lower values imply improvement or higher efficiency, reciprocal of such indicators may be considered.

Since measurement is comparison with a given standard, one can consider the current situation/performance as a vector $\boldsymbol{X} = (X_{1c}, X_{2c}, \ldots\ldots.., X_{nc})^T$ and another vector called target vector (or last year's performance) as $\boldsymbol{T} = (X_{10}, X_{20}, \ldots\ldots, X_{n0})^T$. To make the indicators unit free, one can take ratios i.e $\frac{X_{1c}}{X_{10}}, \frac{X_{2c}}{X_{20}}, \ldots\ldots\ldots, \frac{X_{nc}}{X_{n0}}$ and combine the ratios suitably to find Composite Index.

5. Methods of combining

5.1. Weights obtained from data and Weighted sum

Here attempts are made to find weights from the data so that they are determined in objective fashions i. e. $Y = \frac{1}{n}\sum \frac{w_i X_{ic}}{X_{i0}}$ Determination of weights depends on desired outcomes of the weighted sum or the Composite Index.

5.1.1. Minimum Variance of the Composite Index:

To find the vector of weights $W = (w_1, w_2, \ldots..., w_n)^T$ such that $W'e = 1$ i.e $\sum_{i=1}^{n} w_i = 1$ and $var(Y)$ is minimum where the Composite Index $Y = \sum_{i=1}^{n} w_i X_i$ and $var(Y) = W'SW$ where S_{nxn} is the var-covariance matrix of $X_1, X_2, \ldots\ldots.., X_n$
Consider $A = W'SW + \lambda(1 - W'e)$ where λ is a Lagrangian multiplier
Clearly, $\frac{\partial A}{\partial W} = 2SW - \lambda e = 0$ and $\frac{\partial A}{\partial \lambda} = 1 - W'e = 0$
Solving the above two equations assuming S is non-singular, one can find

$$W = \frac{\lambda S^{-1}e}{2} \implies W = \frac{S^{-1}e}{e'S^{-1}e} \quad \text{and} \quad \lambda = \frac{2}{e'S^{-1}e}$$

Properties:

- Weights obtained from the above method minimizes variance of the Composite Index and $var(Y) = W'SW = \frac{1}{e'S^{-1}e}$
- Covariance between the composite index Y and any component indicator X_i is constant for all *i*= 1,2,….n.
- However, possibility of one or more $W_i's \leq 0$ cannot be ruled out. To ensure that each $W_i \geq 0$, the problem needs to be formulated as $Minimise\, W'SW$ subject to $W'e = 1$ and $W \geq 0$. Theoretically, the method may fail to yield meaningful result if too many $W_i = 0$.

Chakrabartty *et al.* (2015) obtained reliability of a battery of test by combining reliability of the component tests by the above method.

5.1.2. Weights proportional to Covariance of variables and the Composite Index

To find $w_1, w_2, \ldots.., w_n$ such that $0 \leq w_i \leq 1, \sum_{i=1}^{n} w_i = 1$ and $w_i \propto cov(Y, Z_i)$ where $Z_i = \frac{X_i - \bar{X}}{SD(X)}$ i.e. variables which are highly associated with the composite Index linearly to get higher positive weights.

It can be proved that weights vector in this context are eigenvectors corresponding to the maximum eigen values of the variance-covariance matrix S on the assumption of S is positive definite, since$|S - \lambda I|W = 0$, where λ is an eigen value of S.

5.1.3. Weights proportional to Covariance and inversely proportional to SDs

To find $w_1, w_2, \ldots\ldots, w_n$ such that $0 \le w_i \le 1, \sum_{i=1}^{n} w_i = 1$ *and* $w_i \propto \frac{cov(Y,Z_i)}{var(Z_i)}$ Here, W satisfies $|R - \mu I|sW = 0$ where R is the correlation matrix, μ is the maximum eigenvalue of R and s is the diagonal matrix of SDs of Z_i's. It can be proved that $W = \frac{s^{-1}u}{e's^{-1}u}$ where u is eigenvector of R corresponding to its largest eigen value.

5.1.4. CI as an unobservable latent variable

Here, Y is not observable but weights can be found through Principal Component Analysis of the standardized component indicators.

The estimate of $E(Y/X_1, X_2, \ldots\ldots, X_n)$ is $\frac{\lambda_1 P_1 + \lambda_2 P_2 + \ldots\ldots + \lambda_n P_n}{\lambda_1 + \lambda_2 + \cdots\ldots + \lambda_n}$ where λ_1 is the first characteristic root of the correlation matrix **R**. Other $\lambda's$ are defined accordingly. Note that here $\lambda_1 > \lambda_2 > \lambda_3 > \cdots\ldots\ldots > \lambda_n$ and corresponding characteristic vectors of **R** are $\boldsymbol{\alpha_1}, \boldsymbol{\alpha_2}, \ldots\ldots, \boldsymbol{\alpha_n}$ where $\boldsymbol{\alpha_1} = (\alpha_{11}, \alpha_{12}, \ldots\ldots\ldots, \alpha_{1n})^T$. $\boldsymbol{\alpha_2}, \ldots\ldots\ldots, \boldsymbol{\alpha_n}$ are defined accordingly.

The n – principal component of the indicators are $P_1, P_2, \ldots\ldots, P_n$

where $P_1 = \frac{\alpha_{11}(X_1 - \overline{X_1})}{S_1} + \ldots\ldots\ldots + \frac{\alpha_{1n}(X_n - \overline{X_n})}{S_n}$. $P_2, \ldots\ldots, P_n$ are defined accordingly

For n=2, Nagar and Basu (2004) gave the asymptotic distribution of the estimate of Y when number of observations tend to be large.

5.1.5. Multiplicative model

We can have multiplicative models where $Y = \sum \frac{w_i X_{ic}}{X_{i0}}$ with $\sum_{i=1}^{n} w_i = 1$ and $w_i > 0$. Taking log on both sides, one can get additive model

5.2. Geometrical approaches

5.2.1. Based on Inner Product

Consider $\boldsymbol{X} = (X_{1c}, X_{2c}, \ldots\ldots, X_{nc})^T$ and target vector $\boldsymbol{T} = (X_{10}, X_{20}, \ldots\ldots, X_{n0})^T$. $\boldsymbol{X}$ and $\boldsymbol{T}$ are two points in R^n. Let θ be the angle between $\boldsymbol{X}$ and $\boldsymbol{T}$.

Now $\cos\theta = \frac{X'T}{\|X\|\|T\|}$

Lower value of θ indicates that two vectors are close i.e. overall achievements (as reflected by **X**) and the targets (as reflected by **T**) are close. In addition to closeness of the two vectors, one needs to consider length of each such vector. Better approach could be to define Composite Index as $Y = \frac{\|X\|}{\|T\|}\cos\theta$ i.e. ratio of length of vectors **X** and **T** multiplied by cosine of the angle between the two vectors.

The method does not require scaling of the chosen indicators and avoids computation of weights.

5.2 2. Based on Generalized Variance:

It is well known that area of *n*-dimensional parallelogram formed by the two vectors **X** and **T** is $\sqrt{n-1}|S|$ where S denotes the variance-covariance matrix is and determinant of S is the generalized variance, multivariate analog of variance.

Composite Index may be defined as $Y = \frac{\sqrt{n-1}}{|S|}$

The method also does not require scaling of the chosen indicators and avoids computation of weights.

5.3. Geometric Mean approach:

Instead of arithmetic mean of ratios, Composite Index may be defined as the Geometric mean of the ratios of component variables i.e.

$Y = \sqrt[\frac{1}{n}]{\frac{X_{1c}}{X_{10}} \frac{X_{2c}}{X_{20}} \ldots\ldots\ldots \frac{X_{nc}}{X_{n0}}}$ or to avoid n-th root, one may consider just the product of the ratios

i.e. $Y = \prod \frac{X_{ic}}{X_{i0}}$

Note (i) unit of X_{ic} is same as unit of X_{i0} $\forall\, i = 1,2, \ldots\ldots\ldots, n$ and the ratios are unit free;

(ii) $GM\left(\frac{X_i}{Y_i}\right) = \frac{GM(X_i)}{GM(Y_i)}$.

<u>Properties:</u>

The index has the following properties:

- Simple, can consider all chosen component indicators and depicts overall improvement/decline in the current year (c) with respect to base year (0) or finite set of targets;
- Value of a ratio exceeding Unity will indicate improvement in that indicator from the target or base year. If the value of a ratio is less than one, it will imply decline in that indicator and thus identifies critical areas requiring attention;
- Finds relative importance of the chosen indicators;
- Applicable even if financial indicators like Operating Income or Operating Surplus are included in the set of chosen indicators. In other words, the method avoids computation of correlations between a pair of indicators or variance – covariance matrix;
- The Composite Index is independent of change of scale;
- Does not require scaling of component indicators;
- Avoids calculation of weights;
- Avoids computation of variance – covariance matrix or correlation matrix;
- Represents a continuous function which is also monotonically increasing;
- Symmetric over its arguments i.e. independent of order of the chosen indicators;
- To have parity to general convention of Index value = 100 in the base year, the index may be multiplied by 100 to reflect readily percentage changes;
- Satisfies Time –Reversal test i.e. $Y_{c0}.Y_{0c} = 1$;
- Facilitates formation of chain indices;
- Increase of say 1% in *i*-th indicator (i.e. 1% increase in X_{ic}) results in1% increase in the Composite Index if all others remains unchanged. In other words, curve showing gain in an indicator and gain in CI is linear;
- Introduction of new indicator requires estimation of value of that indicator in the Target vecto;
- The method also helps to find solution of the two examples mentioned in Para 2.2 above. Solution to the Example – 1 could be obtained if the ideal point or target vector is taken as $T = (1, 1\,)^T$ either in Weeks – Rs. in thousand space or in Weeks – Rupees space since nothing is free and nothing can be obtained in zero time. In both the cases, $\frac{X_{1A}}{X_{10}}.\frac{X_{2A}}{X_{20}} < \frac{X_{1B}}{X_{10}}.\frac{X_{2B}}{X_{20}}$ implying Party A is preferred. Similarly, Example – 2 could be solved if the Composite Index Y is taken as $Y = \frac{Tech.qualification\ Marks}{\frac{LEC}{EC}X100}$. In that

case, values of the index for A, B and C works out to be respectively $Y_A = \frac{90}{80} = 1.125$, $Y_B = \frac{75}{100} = 0.75$ and $Y_C = \frac{80}{91} = 0.879$ implying selection of party A;

- Assumes positive value for each indicator for all periods. If a particular indicator attains zero or negative value, the method fails.

6. Comparison among the methods:

Desirable properties of a Composite Index as a function from $R^n \rightarrow R$ are as follows:

i. The function should be continuous. All the proposed methods satisfy the condition
ii. The function should be monotonically increasing. All the proposed methods satisfy the condition except 5.2.1 and methods requiring Principal Component Analysis.
iii. Gain in an indicator to be linearly related gain in CI. Method proposed in 5.3 satisfies the condition.
iv. The function to be symmetric over its arguments. All the proposed methods satisfy the condition
v. The index should satisfy Time Reversal Test i.e. $Y_{c0}.Y_{0c} = 1$. Method proposed in 5.3 satisfies the condition. For the method proposed in 5.2.1, $Y_{c0}.Y_{0c}$ tends to unity as θ tends to zero
vi. The index to facilitate formation of chin indices i.e. $Y_{20} = Y_{21}.Y_{10}$ Method proposed in 5.3 satisfies the condition
vii. The index to facilitate statistical test of significance of equality of two geometric means. Method proposed in 5.3 satisfies the condition
viii. Edward and Jhon (1979) have shown that the distribution of GM will approach the lognormal form, even though the parent distribution of X may not be lognormal. Thus, significance tests for hypotheses regarding the difference between geometric means can be performed using conventional t –tests on the logarithms of the observations. Since the geometric mean is a monotonic function of the mean of the logarithms, significance of the difference between the means of the logarithms implies significance of the difference between geometric means

Thus, the Geometric Mean approach i.e. the method proposed in 5.3 satisfies all the above said desired properties and may be taken as the best among the methods discussed. In addition, poor performance in any component indicator gets directly reflected in GM. In other words, a low value of one chosen indicator does not get linearly compensated by high values in another indicator. Thus, GM reduces the level of substitutability between component indicators.

7. Conclusions

Various methods of obtaining Composite Index have been discussed. Each method of combining the component indicators results in different rankings from a given dataset. However, designing of Composite Index should not focus on rank robustness which may be attained if component indicators have high association among themselves. Attempts were also made to compare the proposed methods with respect to a set of desired properties. The Geometric Mean approach satisfies all the desired properties and may be taken as the best among the methods discussed. The method avoids calculation of weights or variance-covariance matrix or correlation matrix. The Geometric Mean approach is applicable for situation even where only the two vectors ***X*** and ***T*** are given like measurement of overall efficiency of an organization or Industry in the current year over the previous year. Thus, the Geometric Mean approach is well applicable for assessment of impact. The approach also helps to identify relative importance of the component indicators in terms of values of the ratios and also identify the critical areas in terms of lower values of the ratios and facilitate initiation of corrective measures. Such identification are important from a policy point of view. The GM- method reduces the level of substitutability between component indicators and facilitates statistical test of significance of equality of two geometric means.

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