JOURNAL OF APPLIED QUANTITATIVE METHODS



# ANALYZING FACTORIAL EXPERIMENTS WITH A SINGLE COMMON CONTROL GROUP

**Soner YIĞIT**
Çanakkale Onsekiz Mart University, Faculty of Agriculture,
Department of Animal Science, Biometry and Genetics Unit , Çanakkale-Turkey

**Corrado DIMAURO**
Department of Agraria, University of Sassari, Sassari, Italy

**Mehmet MENDEŞ**
Çanakkale Onsekiz Mart University, Faculty of Agriculture,
Department of Animal Science, Biometry and Genetics Unit , Çanakkale-Turkey

**E-mail:** mmendes@comu.edu.tr

## Abstract
*Researchers generally put a common control group into their experiments in order to determine how effective the treatments are or to compare the effect of their treatments with a baseline. In this study, classical statistical analysis of factorial experiments and a solution way which has been proposed by Winer et al. (1991) have been compared in terms of type I error rate and test power under different experimental conditions. Results of 100,000 simulation study revealed that performing Winer et al. (1991) test is more appropriate in terms of getting reliable results when there is a single common control group in factorial experiments.*

**Keywords:** *Control group; factorial design; type I error; test power; simulation*

## 1. Introduction

In practice, researchers especially in the field of medicine, agriculture, pharmacy, genetics, social science and some other related sciences commonly put a control group (or baseline) in their experiments in order to investigate if the treatments make a significant affect on interested variable(s) (Kinser and Robins, 2013). The reason is that, there is no other way to see the effect of treatments on the response correctly and reliably. If the researchers do not have a control group, in this case it will not be possible to determine if the treatments have a significant impact. The control group establishes a baseline that the experimental units are compared to and thus, without a control group, researchers will not have anything to compare the experiment's results to. Therefore, since the control group does not receive a treatment, it allows the researchers to eliminate and isolate the effect of the other factors which cannot be able to control or consider (Winer *et al.*, 1991; Pithon, 2013; Bate and Karp, 2014). Considering that the factorial experiments which have a single control group are commonly designed in practice, especially in the fields of agriculture, medicine, biology, aquaculture, forestry etc. It is obvious that statistical analysis of these experiments will be different from that of the analysis of the classical factorial experiments (Winer *et al.*, 1991; Kramer and Font, 2015). However, it is noticed that many researchers have still been performing the classical statistical analysis of the factorial designs even they have a single common control group (Table 2 and 3). Although Winer *et al.* (1991) proposed a different statistical analysis test for such cases, many researchers especially non-statisticians still do not aware of this test. From the light of this point, a comprehensive Monte Carlo Simulation study has been carried out to investigate the performance of the Winer *et al.* (1991) method for analyzing factorial experiments when there is a single common control group. The performance of the Winer *et al.* (1991) test has been compared to the statistical analysis of the classical factorial experiments. An illustrative example has also been put into the manuscript in order to show how data sets of factorial experiments can be analyzed when there is a single common control group.

## 2. Material and Methods

Pseudo random numbers have been generated from normal (0, 1) and different non-normal distributions (Beta (10, 10), Beta (5,10), Beta (10, 5), and Chi-Sq (3)) for four different types of factorial experiments with 2x2, 3x3, 4x4, and 2x4 under both homogeneity of variances assumption is met and not met. Number of replications in each sub-group have been determined as 3, 4, 5, 10, and 20. Each experimental condition has been simulated 100,000 times. In order to estimate the test power of the Winer *et al.* (1991) test and classical statistical analysis of factorial experiment, two different constant numbers with standard deviation form have been added to the numbers in the control group and the last sub-group in the factorial part of the study. Experimental conditions which have been simulated have been presented in Table 1.

**Table 1.** Experimental conditions which have been simulated

| Distributions | Effect Size | Variance Ratio | Sample Size |
|---|---|---|---|
| Normal (0,1)<br>t (10)<br>Beta (10, 10) | 0.0, 0.75, 1.50 | 1:…:1<br>1:…:10<br>1:…:20 | 3,4,5,10,20 |

### 2.1. Statistical Analyses

#### 2.1.1. Classical Analysis of Factorial Designs (CAM)

Experimental design, computational steps of the classical analysis of the factorial design, and degrees of freedom have been presented in Table 2, Table 3, and Table 4 (Winer et al., 1991). Suppose there are two factors namely A and B. If both factors have two levels ($a_1$, $a_2$ and $b_1$, $b_2$) and there is a single common control group in the experiment, in this case, the experimental design will be as in the Table 2.

**Table 2.** Experimental design for classical analysis of factorial design

| | Control | $b_1$ | $b_2$ |
|---|---|---|---|
| $a_1$ | $Y_{001}$<br>$Y_{002}$<br>$\vdots$<br>$Y_{00n}$ | $Y_{111}$<br>$Y_{112}$<br>$\vdots$<br>$Y_{11n}$ | $Y_{121}$<br>$Y_{122}$<br>$\vdots$<br>$Y_{12n}$ |
| $a_2$ | $Y_{001}$<br>$Y_{002}$<br>$\vdots$<br>$Y_{00n}$ | $Y_{211}$<br>$Y_{212}$<br>$\vdots$<br>$Y_{21n}$ | $Y_{221}$<br>$Y_{222}$<br>$\vdots$<br>$Y_{22n}$ |

**Table 3.** Computational steps for classical analysis of factorial designs

| | | |
|---|---|---|
| (1) | $1 = \left(r\sum_{k=1}^{n_0} Y_{00k} + \sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{n} Y_{ijk}\right)^2/(n_0+nrc)$<br>$2 = r\sum_{k=1}^{n_0} Y_{00k}^2 + \sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{n} Y_{ijk}^2$<br>$3 = \sum_{i=1}^{r}(\sum_{k=1}^{n_0} Y_{00k} + \sum_{j=1}^{c}\sum_{k=1}^{n} Y_{ijk})^2/(n_0+nc)$<br>$4 = (\sum_{k=1}^{n_0} Y_{00k})^2/n_0 + \sum_{j=1}^{c}(\sum_{i=1}^{r}\sum_{k=1}^{n} Y_{ijk})^2/nr$<br>$5 = r(\sum_{k=1}^{n_0} Y_{00k})^2/n_0 + \sum_{i=1}^{r}\sum_{j=1}^{c}(\sum_{k=1}^{n} Y_{ijk})^2$ | |
| (2) | Source of Variation | Computational formula for SS |
| | A | $SS_A = (3) - (1)$ |
| | B | $SS_B = (4) - (1)$ |
| | AxB | $SS_{AB} = (5) - (3) - (4) + (1)$ |
| | Within cell | $SS_{Error} = (2) - (5)$ |
| | Total | $SS_{Total} = (2) - (1)$ |

__Note:__ r and c denote the numbers of row and column, $n_0$ and n are the number of replications in the control group and each-sub group

**Table 4.** Source of variation and degrees of freedom for classical analysis of factorial design

| **Source of variation** | **df** |
|---|---|
| Between cell | (rc+r)-1 |
| A | r-1 |
| B | (c+1)-1 |
| AxB | (r-1)c |
| Within cell | rc(n-1)+r($n_0$-1) |

### 2.1.2. Analyzing Factorial Experiments When There Is a Single Common Control Group by Using Winer et al (1991) Method (WM)

Experimental design, computational steps of Winer *et al.* (1991) method, and degrees of freedom of that experiment have been given in Table 5, Table 6, and Table 7.

**Table 5.** Experimental design for Winer *et al.* (1991) method

| **Control** |
|---|
| $Y_{001}$<br>$Y_{002}$<br>$\vdots$<br>$Y_{00n}$ |

| | $b_1$ | $b_2$ |
|---|---|---|
| $a_1$ | $Y_{111}$<br>$Y_{112}$<br>$\vdots$<br>$Y_{11n}$ | $Y_{121}$<br>$Y_{122}$<br>$\vdots$<br>$Y_{12n}$ |
| $a_2$ | $Y_{211}$<br>$Y_{212}$<br>$\vdots$<br>$Y_{21n}$ | $Y_{221}$<br>$Y_{222}$<br>$\vdots$<br>$Y_{22n}$ |

**Table 6.** Computational steps for Winer *et al.* (1991) method

| | | |
|---|---|---|
| (1) | $1 = \left(\sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{n} Y_{ijk}\right)^2/nrc$<br>$2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{n} Y_{ijk}^2$<br>$3 = \sum_{i=1}^{r}(\sum_{j=1}^{c}\sum_{k=1}^{n} Y_{ijk})^2/nc$<br>$4 = \sum_{j=1}^{c}(\sum_{i=1}^{r}\sum_{k=1}^{n} Y_{ijk})^2/nr$<br>$5 = \sum_{i=1}^{r}\sum_{j=1}^{c}(\sum_{k=1}^{n} Y_{ijk})^2/n$ | |
| (2) | Source of Variation | Computational formula for SS |
| | A | $SS_A = (3) - (1)$ |
| | B | $SS_B = (4) - (1)$ |
| | AxB | $SS_{AB} = SS_{b.cell} - (SS_{cont.vs.\ all} + SS_A + SS_B)$ |
| | Within Cell | $SS_{Error} = (2) - (5) + SS_0$ |
| | Total | $SS_{Total} = (2) - (1)$ |

**Table 7.** Source of variation and degrees of freedom Winer *et al.* (1991) method

| Source of variation | df |
|---|---|
| Between cell | (rc+1)-1 |
| Control vs. all others | 1 |
| A | r-1 |
| B | c-1 |
| AxB | (r-1)(c-1) |
| Within cell | rc(n-1)+($n_0$-1) |

$$SS_0 = \sum_{k=1}^{n_0} Y_{00k}^2 - \frac{\left(\sum_{k=1}^{n_0} Y_{00k}\right)^2}{n_0}$$

$$C = \frac{rcC_0}{n_0} - \frac{\sum_{i=1}^{r}\sum_{j=1}^{c} Y_{ij.}}{n}$$

Where,

$C_0$: Sum of the observations in the control group

$$SS_{cont.vs.\ all} = \frac{C^2}{[(rc)^2/n_0] + (rc/n)}$$

$$SS_{b.cell} = \frac{C_0^2}{n_0} + \frac{\sum_{i=1}^{r}\sum_{j=1}^{c} Y_{ij.}^2}{n} - \frac{\left(\sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{n} Y_{ijk} + C_0\right)^2}{nrc + n_0}$$

### 2.2. Illustrative Example.

A data set from an experiment which was carried out in 2007 to investigate the effect of two different feeding programs (R20, NF6) and two lighting programs (16L:8D and 12L:12D) on slaughter weights of Ross 308 broiler chickens was used. There is also a single common control group in this study. The data set has given in Table 8. This data set has been analyzed both by using Classical Analysis Method (CAM) and Winer *et al.* (1991) Method (WM) in order to show differences in the computation steps of two methods.

**Table 8.** Data set which has been considered for this study

| | Feeding Programs | | Control Group |
|---|---|---|---|
| Lighting Programs | R20 | NF6 | |
| 16:8 | 2.216 | 2.209 | |
| | 2.043 | 1.865 | |
| | 2.021 | 2.452 | 2.503 |
| | 2.311 | 2.490 | 2.738 |
| | 1.910 | 1.919 | 2.701 |
| | 1.887 | 2.215 | 2.711 |
| 12:12 | 2.484 | 2.316 | 2.297 |
| | 2.312 | 1.957 | 2.085 |
| | 2.250 | 1.894 | |
| | 2.201 | 2.047 | |
| | 1.864 | 1.816 | |
| | 2.443 | 1.836 | |

#### 2.2.1. Analyzing Data Set by Using Classical Analysis of Factorial Design (CAM)

Experimental Design, computational steps and results of the CAM have been given in Table 9, 10, 11 and 12 respectively.

**Table 9**. Experimental design for classical analysis of factorial design

| | | Feeding Programs | |
|---|---|---|---|
| Lighting Programs | Control | R20 | NF6 |
| 16:8 | 2.503 | 2.216 | 2.209 |
| | 2.738 | 2.043 | 1.865 |
| | 2.701 | 2.021 | 2.452 |
| | 2.711 | 2.311 | 2.490 |
| | 2.297 | 1.910 | 1.919 |
| | 2.085 | 1.887 | 2.215 |
| 12:12 | 2.503 | 2.484 | 2.316 |
| | 2.738 | 2.312 | 1.957 |
| | 2.701 | 2.250 | 1.894 |
| | 2.711 | 2.201 | 2.047 |
| | 2.297 | 1.864 | 1.816 |
| | 2.085 | 2.443 | 1.836 |

**Table 10.** Slaughter weight sums of sub-groups

| | | Feeding Programs | | |
|---|---|---|---|---|
| Lighting Programs | Control | R20 | NF6 | Σ |
| 16:8 | 15.035 | 12.388 | 13.150 | 40.573 |
| 12:12 | 15.035 | 13.554 | 11.866 | 40.455 |
| Σ | 30.070 | 25.942 | 25.016 | 81.028 |

**Table 11.** Computational steps for classical analysis of factorial designs

| | | |
|---|---|---|
| (1) | $1 = 182.376$<br>$2 = 185.443$<br>$3 = 182.376$<br>$4 = 183.583$<br>$5 = 183.833$ | |
| (2) | Source of Variation | Computational formula for SS |
| | Lighting Programs (A) | $SS_A = (3) - (1) = 0.000$ |
| | Feeding Programs (B) | $SS_B = (4) - (1) = 1.207$ |
| | Interaction (AxB) | $SS_{AB} = (5) - (3) - (4) + (1) = 0.250$ |
| | Experimental Error (within cell) | $SS_{Error} = (2) - (5) = 1.610$ |
| | Total | $SS_{Total} = (2) - (1) = 3.067$ |

**Table 12.** Results for classical analysis of factorial designs

| Source of variation | df | SS | MS | F | P-Value |
|---|---|---|---|---|---|
| Between cell | 5 | 1.457 | | | |
| Lighting Programs (A) | 1 | 0.000 | 0.000 | 0.000 | 1.000 |
| Feeding Programs (B) | 2 | 1.207 | 0.604 | 11.185 | **0.000**** |
| Interaction (AxB) | 2 | 0.250 | 0.125 | 2.315 | 0.116 |
| Within cell | 30 | 1.610 | 0.054 | | |

****P<0.01**

### 2.2.2. Analyzing Data Set by Using Winer et al (1991) Method (WM)

Experimental Design, computational steps and results of the WM have been given in Table 13, 14, 15 and 16 respectively.

**Table 13.** Experimental design for Winer *et al.* (1991) method

| Control | Lighting Programs | Feeding Programs R20 | Feeding Programs NF6 |
|---|---|---|---|
| 2.503 | 16:8 | 2.216 | 2.209 |
| 2.738 | | 2.043 | 1.865 |
| 2.701 | | 2.021 | 2.452 |
| 2.711 | | 2.311 | 2.490 |
| 2.297 | | 1.910 | 1.919 |
| 2.085 | | 1.887 | 2.215 |
| | 12:12 | 2.484 | 2.316 |
| | | 2.312 | 1.957 |
| | | 2.250 | 1.894 |
| | | 2.201 | 2.047 |
| | | 1.864 | 1.816 |
| | | 2.443 | 1.836 |

**Table 14.** Sum of sub-groups in terms of slaughter weight

| | | Feeding Programs | | |
|---|---|---|---|---|
| Control (Σ) | Lighting Programs | R20 | NF6 | Σ |
| 15.035 | 16:8 | 12.388 | 13.150 | 25.538 |
| | 12:12 | 13.554 | 11.866 | 25.420 |
| | Σ | 25.942 | 25.016 | 50.958 |

$$SS_0 = \sum_{k=1}^{n_0} Y_{00k}^2 - \frac{(\sum_{k=1}^{n} Y_{00k})^2}{n_0} = 38.030 - \frac{15.035^2}{6} = 0.355$$

Where , $SS_0$ is sum of squares for Control group.

**Table 15.** Computational steps for Winer *et al.* (1991) method

| | | |
|---|---|---|
| (1) | | $1 = 108.197$<br>$2 = 109.383$<br>$3 = 108.197$<br>$4 = 108.232$<br>$5 = 108.483$ |
| (2) | Source of Variation | Computational formula for SS |
| | Lighting Programs (A) | $SS_A = (3) - (1) = 0.000$ |
| | Feeding Programs (B) | $SS_B = (4) - (1) = 0.035$ |
| | Interaction (AxB) | $SS_{AB} = SS_{b.cell} - (SS_{cont.vs.all} + SS_A + SS_B) = 0.252$ |
| | Within cell | $SS_{Error} = (2) - (5) + SS_0 = 0.900 + 0.355 = 1.255$ |

$$C = \frac{rcC_0}{n_0} - \frac{\sum AB_{ij}}{n} = \frac{(2)(2)(15.035)}{6} - \frac{50.958}{6} = 1.530$$

$$SS_{cont.vs.\ all} = \frac{C^2}{[(rc)^2/n_0] + (rc/n)} = \frac{1.530^2}{(4^2/6) + 4/6} = 0.702$$

$$SS_{b.cell} = \frac{C_0^2}{n_0} + \frac{\sum (AB_{ij})^2}{n} - \frac{(G + C_0)^2}{nrc + n_0} = \frac{15.035^2}{6} + 108.483 - \frac{(15.035 + 50.958)^2}{(6)(2)(2) + 6} = 0.989$$

**Table 16**. Results for Winer *et al.* (1991) method

| Source of variation | df | SS | MS | F | *P-Value* |
|---|---|---|---|---|---|
| Between cell | 4 | 0.989 | | | |
| Control vs. all others | 1 | 0.702 | 0.702 | 14.040 | **0.001**** |
| Lighting Programs (A) | 1 | 0.000 | 0.000 | 0.000 | 1.000 |
| Feeding Programs (B) | 1 | 0.035 | 0.035 | 0.700 | 0.411 |
| Interaction (AxB) | 1 | 0.252 | 0.252 | 5.040 | **0.003**** |
| Within cell (0.900+0.355) | 25 | 1.255 | 0.050 | | |

****P<0.01**

## 3. Results

### 3.1. Results of Simulation Study

Type I error estimates have been given in Table 17-21, respectively. As it can be seen from Table 17-21, as long as the variances are homogenous, the Winer et al (1991) Method (WM) has given more reliable results in terms of retaining the Type I error rates at the nominal alpha level (0.05) regardless of number of replications, number of factor levels, and distribution shapes. These results are also valid for testing the effect of control group, main and interaction effects. All Type I error estimates have been found very close to 0.05 when the Winer et al (1991) Method is used for analyzing data sets. On the other hand, the preferring the usage of the Classical Statistical Analysis Method (CAM) for analyzing factorial experiments which have a single common control group has led to get much more deviated estimates even when normality and homogeneity of variances assumptions are met. In other word, performing CAM caused to flactuation in Type I error rate and in test power. When the effect of deviations in the homogeneity of the variances on Type I error and test power estimates is examined, it is noticed that non-fulfillment of the homogeneity of variances assumption has caused to not to retain the type I error rates at 5.00%. Both methods have given obviously deviated estimates under these experimental conditions. The Type I error estimates of WM have varied between 6.6 and 17.0% for testing main and interaction effects while the Type I error estimates for testing effect of the control group have been varied between 0.4 and 3.7%. As it is expected, the effect of heterogeneity of variances on the Type I error estimates has been become more obvious especially when the samples are taken from non-normal populations. The effect of the number of factor levels on the Type I error estimates is negligible level as long as the variances are homogeneous.

When both analyses methods (CAM and WM) are compared in terms of test power, it has observed that the test power estimates of both methods have been mainly affected by the number of replications in each sub-group, effect size, number of the factor levels, and whether the variances are homogenous or not. As it is expected, the test power values increased as the number of replications and effect size increased. The test power estimates have not been obviously affected by the deviations from normality as long as the variances are homogeneous. For example, when distributions are normal, both factors have two levels (2x2), variances are homogenous (1:1:..:1), effect size is 0.75, and number of replication is 10, the test power estimates for WM have been found as 34.3%, 21.4 %21.4%, and 21.5% for effect of control group, main effect-A, main effect-B, and interaction effect (AxB) respectively. Under the same conditions, when both factors have three (3x3) and four (4x4) levels, the test power values have been estimated as 51.0%, 15.0%, 15.3%, 19.9% and 57.6%, 11.6%, 11.8%, 17.5% respectively. Under the same conditions when samples are taken from Beta (10, 10), the test power estimates have been found as 34.4%, 21.1%, 21.6%, and 21.3% respectively. The test power values have been estimated as 50.9%, 14.9%, 15.1%, and 19.8% when both factors have three (3x3) levels, and 57.3%, 11.7%, 11.8%, and 17.2% when both factors have four (4x4) levels. When samples are taken from Chi-Square with 3 d.f. distribution, the test power values have been estimates as 34.5%, 22.7%, 22.6%, and 22.6% for 2x2 design, 49.4%, 15.5%, 15.4%, and 20.4% for 3x3 design, and 54.8%, 11.6%, 11.7%, and 17.5% for 4x4 design respectively. As it can be seen from Table 22, 23, 24, 25, and 26, lower test power values have been obtained when variances are not homogeneous and this case has become more obvious when deviation from homogeneity is increased.

### 3.2. Results of Real Data Set

Results of CAM and WM have been given in Table 12 and Table 16 respectively. When the CAM is used (Table 12), in other way, ignoring of the existence a common control group in the experiment, the main effect of lighting program (P=1.000) and interaction effect (P=0.116) are not found to be statistically significant, whereas the main effect of feeding program is significant (P=0.000). However, as it can be seen from Table 16, when the same data set is analyzed by using WM, the main effects of lighting program (P=1.000) and feeding program (P=0.411) are not found statistically significant whereas the interaction effect is found as statistically significant (P=0.003). As it can be seen from Table 12 and Table 16, different results have been obtained when CAM and WM have been used. Reason for reaching different results is related to considering of existence of a single common control group in the statistical analysis stage or not. Therefore, the use of CAM for analyzing data sets in the experiments where there is a single common control group has caused to mask interaction effect.

## 4. Discussion

In practice, the researchers generally want to know if the treatment of a particular substance has any effect on the experimental units (animals, patients, plants etc). For such cases, the researchers need something or baseline to compare this effect with. For this purpose, the researchers generally put a control group to their experiments in order to determine if the factors or treatments make a significant impact on the response(s). Although it is practically not possible to completely eliminate effect of all variables on the the results of the experiment, but control group allows the researcher to eliminate variables that can't be controlled in an experiment. Therefore, the control group plays an important role in the experimental process. Especially recently factorial experiments with single or common control are widely designed by the researchers wishing to investigate the effect of two or more factors on interested variable(s). Although factorial experiments with a common control group are commonly designed it is noticed that there is a big problem about the statistical analysis of these kinds of experiments. The problem is the usage of the classical statistical analysis of factorial analysis. In other way, the problem stems from the fact that the statistical analysis is performed by conventional methods (Table 3). Preffering this analysis method is not correct and thus it would not be possible to get reliable results. It is because, performing this analysis methods will cause not to retain Type I error rate at the nominal alpha level. It will also cause to negative changes in test power.

Results of this simulation study revealed that doing statistical analysis by ignoring the fact that there is a common control group in the experiment has caused obtaining unreliable results. Type I error rate could not be retained under any conditions even both homogeneity of variances and normality assumptions were met and number of replications were large (n=20). Test power estimates affected negatively when classical computational steps of statistical analysis were performed. Therefore, the usage of the same sub-group at the each level of the row factor (as if the control group is considered as a level of the column factor) (as in Table 9) has caused to obtain significantly higher Type I error rates than 0.05 for the column factor. On the other hand, it has been caused to obtain significantly lower than 0.05 type I error rates for the row factor and interaction. The test power estimates have also been negatively affected by using CAM.

## 5. Conclusion

Results of Monte Carlo Simulation Study showed that the usage of the Winer *et al.* (1991) method (WM) in analyzing data sets of factorial experiments when there was a single common control group enabled the researcher to get more correct and reliable results as long as variances are homogeneous. At the same time, since the WM enables the researchers to compare all sub-groups to the control group like multiple comparison procedure, it will be possible to get more detailed and reliable results in terms of effects of interested factors. As a results, it is possible to strongly suggested to authors and researchers to use Winer et al (1991) Method for analyzing their data sets if they have a common single control group in their experimental design.

## References

1. Bate, S. and Karp, N.A. **A Common Control Group - Optimising the Experiment Design to Maximise Sensitivity.** PLoS One, Vol. 9, No. 12, 2014, pp. e114872.
2. Kinser, P.A. and Robins, J.L. **Control group design: enhancing rigor in research of mind-body therapies for depression.** Evid-Based Complement Alternat Med. 2013 pp. 140467.
3. Kramer, M. and Font, E. **Reducing sample size in experiments with animals: historical controls and related strategies.** Biol Rev Camb Philos Soc. Voll. 92, No. 1, 2015, pp. 431-445.
4. Pithon, M.M. **Importance of the control group in scientific research.** Dental Press J. Orthod. Vol. 18, No. 6, 2013, pp. 13-14.
5. Winer, B.J., Brown, D.R. and Michels, K.M. **Statistical principles in experimental design, 3rd ed.,** New York: McGraw-Hill, 1991

### Apendixes

**Table 17.** Type I error rate estimates when samples are taken from normal distributions

| | | $\sigma_{11}^2:\sigma_{12}^2 \ldots : \sigma_{ij}^2$ | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1:1:…:1 | | | | | | | 1:1:…:10 | | | | | | | 1:1:…:20 | | | | | | |
| rxc | n | A | B | AxB | ***Cont.*** | ***A*** | ***B*** | ***AxB*** | A | B | AxB | ***Cont.*** | ***A*** | ***B*** | ***AxB*** | A | B | AxB | ***Cont.*** | ***A*** | ***B*** | ***AxB*** |
| 2x2 | 3 | 2.5 | 10.3 | 2.7 | ***4.9*** | ***5.0*** | ***5.1*** | ***5.1*** | 6.2 | 11.2 | 7.8 | ***2.6*** | ***9.5*** | ***9.5*** | ***9.5*** | 8.8 | 12.9 | 10.9 | ***2.5*** | ***12.2*** | ***11.9*** | ***12.0*** |
| | 4 | 2.2 | 10.5 | 2.5 | ***5.0*** | ***5.0*** | ***5.1*** | ***5.1*** | 5.7 | 10.6 | 7.4 | ***2.1*** | ***9.1*** | ***9.1*** | ***9.1*** | 7.6 | 11.9 | 10.0 | ***1.8*** | ***10.9*** | ***10.9*** | ***11.1*** |
| | 5 | 2.1 | 10.7 | 2.2 | ***5.1*** | ***5.1*** | ***5.1*** | ***4.9*** | 5.2 | 10.0 | 6.7 | ***1.7*** | ***8.6*** | ***8.7*** | ***8.4*** | 7.1 | 11.3 | 9.5 | ***1.4*** | ***10.5*** | ***10.5*** | ***10.4*** |
| | 10 | 1.8 | 10.7 | 2.0 | ***5.1*** | ***4.9*** | ***5.0*** | ***4.9*** | 4.4 | 9.1 | 6.1 | ***1.2*** | ***7.8*** | ***7.7*** | ***7.8*** | 5.5 | 9.5 | 8.0 | ***0.7*** | ***8.9*** | ***8.9*** | ***8.8*** |
| | 20 | 1.8 | 10.8 | 1.9 | ***4.9*** | ***5.1*** | ***5.2*** | ***5.0*** | 3.9 | 8.5 | 5.6 | ***0.9*** | ***7.4*** | ***7.4*** | ***7.3*** | 4.8 | 8.7 | 7.2 | ***0.4*** | ***8.1*** | ***8.2*** | ***8.0*** |
| 3x3 | 3 | 2.7 | 15.4 | 2.8 | ***5.1*** | ***5.0*** | ***5.0*** | ***5.0*** | 6.1 | 14.4 | 8.8 | ***2.3*** | ***8.9*** | ***8.9*** | ***11.0*** | 9.2 | 15.7 | 13.2 | ***1.6*** | ***12.0*** | ***12.1*** | ***14.8*** |
| | 4 | 2.4 | 15.4 | 2.4 | ***4.9*** | ***5.0*** | ***5.0*** | ***5.0*** | 5.4 | 13.7 | 8.3 | ***1.9*** | ***8.4*** | ***8.4*** | ***10.6*** | 8.1 | 14.2 | 12.6 | ***1.2*** | ***11.1*** | ***10.8*** | ***14.2*** |
| | 5 | 2.3 | 15.5 | 2.3 | ***4.9*** | ***5.1*** | ***5.0*** | ***5.1*** | 5.1 | 13.2 | 8.1 | ***1.7*** | ***8.2*** | ***8.1*** | ***10.3*** | 7.7 | 13.6 | 12.0 | ***1.0*** | ***10.5*** | ***10.4*** | ***13.5*** |
| | 10 | 2.0 | 15.7 | 2.0 | ***5.1*** | ***4.9*** | ***4.9*** | ***5.0*** | 4.7 | 12.4 | 7.7 | ***1.3*** | ***7.5*** | ***7.3*** | ***9.8*** | 6.4 | 12.0 | 10.6 | ***0.6*** | ***9.1*** | ***9.1*** | ***12.0*** |
| | 20 | 1.9 | 15.6 | 1.9 | ***5.0*** | ***5.0*** | ***4.9*** | ***5.2*** | 4.3 | 11.9 | 7.2 | ***1.2*** | ***7.2*** | ***7.1*** | ***9.3*** | 5.9 | 11.1 | 10.3 | ***0.4*** | ***8.4*** | ***8.4*** | ***11.5*** |
| 4x4 | 3 | 2.7 | 19.4 | 2.7 | ***5.1*** | ***4.9*** | ***5.0*** | ***4.9*** | 5.4 | 17.5 | 8.6 | ***2.5*** | ***8.0*** | ***8.0*** | ***11.2*** | 8.5 | 18.1 | 13.7 | ***1.8*** | ***11.2*** | ***11.2*** | ***16.0*** |
| | 4 | 2.5 | 19.5 | 2.5 | ***4.9*** | ***5.0*** | ***5.0*** | ***5.0*** | 5.0 | 17.5 | 8.2 | ***2.3*** | ***7.7*** | ***7.7*** | ***11.0*** | 7.7 | 17.2 | 13.2 | ***1.4*** | ***10.3*** | ***10.5*** | ***15.5*** |
| | 5 | 2.4 | 19.7 | 2.1 | ***4.9*** | ***5.0*** | ***5.0*** | ***5.0*** | 4.8 | 17.1 | 7.9 | ***2.2*** | ***7.4*** | ***7.4*** | ***10.7*** | 7.1 | 16.5 | 12.5 | ***1.3*** | ***9.8*** | ***9.7*** | ***14.8*** |
| | 10 | 2.2 | 19.8 | 2.0 | ***5.1*** | ***5.0*** | ***5.0*** | ***5.0*** | 4.4 | 16.5 | 7.6 | ***1.9*** | ***7.0*** | ***7.0*** | ***10.3*** | 6.5 | 15.0 | 11.7 | ***0.9*** | ***8.9*** | ***8.8*** | ***13.7*** |
| | 20 | 2.1 | 20.2 | 1.8 | ***5.1*** | ***5.0*** | ***4.9*** | ***5.1*** | 4.2 | 16.2 | 7.2 | ***1.9*** | ***6.8*** | ***6.6*** | ***9.9*** | 6.0 | 14.5 | 11.2 | ***0.7*** | ***8.2*** | ***8.2*** | ***13.1*** |
| 2x4 | 3 | 3.2 | 9.4 | 3.2 | ***5.0*** | ***5.0*** | ***5.0*** | ***4.9*** | 5.8 | 12.3 | 8.8 | ***2.3*** | ***7.8*** | ***10.7*** | ***10.7*** | 7.9 | 14.9 | 12.6 | ***1.7*** | ***10.0*** | ***14.2*** | ***14.1*** |
| | 4 | 3.0 | 9.6 | 3.0 | ***5.0*** | ***5.1*** | ***4.9*** | ***5.0*** | 5.3 | 11.4 | 8.2 | ***1.9*** | ***7.2*** | ***9.9*** | ***10.0*** | 7.0 | 13.8 | 11.7 | ***1.2*** | ***9.0*** | ***13.2*** | ***13.1*** |
| | 5 | 3.0 | 9.6 | 2.9 | ***5.0*** | ***4.9*** | ***5.0*** | ***5.0*** | 5.0 | 11.0 | 7.9 | ***1.6*** | ***7.0*** | ***9.6*** | ***9.7*** | 6.4 | 13.1 | 11.1 | ***1.0*** | ***8.4*** | ***12.5*** | ***12.5*** |
| | 10 | 2.9 | 9.7 | 2.7 | ***5.0*** | ***4.9*** | ***5.0*** | ***5.0*** | 4.3 | 10.2 | 7.3 | ***1.3*** | ***6.4*** | ***9.0*** | ***9.0*** | 5.3 | 11.5 | 9.8 | ***0.5*** | ***7.1*** | ***11.2*** | ***11.2*** |
| | 20 | 2.9 | 10.0 | 2.6 | ***5.2*** | ***5.0*** | ***5.0*** | ***4.9*** | 4.2 | 9.6 | 6.9 | ***1.1*** | ***6.1*** | ***8.5*** | ***8.5*** | 4.7 | 11.0 | 9.3 | ***0.4*** | ***6.6*** | ***10.7*** | ***10.6*** |

**Bold:** Results of WM
Regular: Results of CAM

**Table 18.** Type I error rate estimates when samples are taken from Beta(10,10) distributions

| | | $\sigma_{11}^2:\sigma_{12}^2 \dots :\sigma_{ij}^2$ | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1:1:…:1 | | | | | | | 1:1:…:10 | | | | | | | 1:1:…:20 | | | | | | |
| rxc | n | A | B | AxB | ***Cont.*** | ***A*** | ***B*** | ***AxB*** | A | B | AxB | ***Cont.*** | ***A*** | ***B*** | ***AxB*** | A | B | AxB | ***Cont.*** | ***A*** | ***B*** | ***AxB*** |
| 2x2 | 3 | 2.5 | 10.5 | 2.7 | 4.9 | 5.1 | 5.1 | 5.0 | 6.6 | 11.6 | 8.2 | 2.6 | 9.9 | 9.9 | 10.0 | 9.1 | 13.3 | 11.3 | 2.5 | 12.4 | 12.5 | 12.5 |
| | 4 | 2.2 | 10.7 | 2.4 | 5.0 | 5.0 | 5.1 | 5.1 | 5.9 | 10.8 | 7.5 | 2.1 | 9.4 | 9.4 | 9.3 | 8.0 | 12.3 | 10.4 | 1.9 | 11.4 | 11.4 | 11.5 |
| | 5 | 2.2 | 10.7 | 2.3 | 5.0 | 5.2 | 5.1 | 5.0 | 5.4 | 10.2 | 7.1 | 1.8 | 8.8 | 8.9 | 8.8 | 7.1 | 11.2 | 9.5 | 1.4 | 10.4 | 10.4 | 10.6 |
| | 10 | 1.9 | 10.6 | 2.0 | 5.0 | 5.0 | 4.9 | 4.9 | 4.4 | 9.0 | 6.2 | 1.1 | 7.9 | 7.9 | 7.9 | 5.4 | 9.4 | 7.9 | 0.7 | 8.8 | 8.8 | 8.8 |
| | 20 | 1.7 | 10.7 | 1.9 | 5.0 | 5.0 | 5.0 | 4.9 | 4.0 | 8.4 | 5.6 | 0.9 | 7.3 | 7.3 | 7.2 | 4.8 | 8.7 | 7.1 | 0.4 | 8.0 | 8.1 | 8.0 |
| 3x3 | 3 | 2.7 | 15.4 | 2.8 | 5.0 | 5.1 | 5.0 | 5.0 | 6.2 | 14.6 | 9.1 | 2.2 | 9.1 | 9.1 | 11.2 | 9.5 | 16.1 | 13.7 | 1.7 | 12.3 | 12.4 | 15.4 |
| | 4 | 2.4 | 15.8 | 2.5 | 5.0 | 5.1 | 5.2 | 5.1 | 5.7 | 13.9 | 8.7 | 1.9 | 8.6 | 8.6 | 10.9 | 8.3 | 14.6 | 12.6 | 1.3 | 11.1 | 11.2 | 14.2 |
| | 5 | 2.3 | 15.7 | 2.2 | 5.0 | 5.1 | 5.0 | 5.0 | 5.4 | 13.2 | 8.1 | 1.7 | 8.3 | 8.2 | 10.4 | 7.6 | 13.6 | 11.9 | 1.0 | 10.5 | 10.5 | 13.5 |
| | 10 | 2.1 | 15.8 | 2.0 | 5.0 | 5.2 | 5.0 | 4.9 | 4.7 | 12.5 | 7.7 | 1.4 | 7.6 | 7.6 | 9.8 | 6.5 | 11.9 | 10.8 | 0.6 | 9.1 | 9.0 | 12.1 |
| | 20 | 1.9 | 15.9 | 1.8 | 5.1 | 5.0 | 5.1 | 5.1 | 4.2 | 12.0 | 7.1 | 1.2 | 7.1 | 7.1 | 9.2 | 5.8 | 11.1 | 10.2 | 0.4 | 8.4 | 8.4 | 11.4 |
| 4x4 | 3 | 2.7 | 19.8 | 2.8 | 5.0 | 5.0 | 5.0 | 5.2 | 5.6 | 17.6 | 8.9 | 2.4 | 8.2 | 8.2 | 11.5 | 8.8 | 18.3 | 14.0 | 1.7 | 11.6 | 11.5 | 16.3 |
| | 4 | 2.5 | 19.7 | 2.3 | 5.0 | 5.0 | 4.9 | 5.0 | 5.2 | 17.4 | 8.4 | 2.2 | 7.8 | 7.9 | 11.2 | 8.1 | 17.1 | 13.2 | 1.4 | 10.8 | 10.4 | 15.4 |
| | 5 | 2.4 | 20.0 | 2.3 | 5.0 | 5.0 | 5.1 | 5.2 | 4.7 | 17.3 | 8.0 | 2.1 | 7.5 | 7.7 | 10.9 | 7.5 | 16.7 | 12.7 | 1.2 | 10.0 | 10.0 | 14.8 |
| | 10 | 2.2 | 19.6 | 1.8 | 4.8 | 4.9 | 5.0 | 4.9 | 4.3 | 16.5 | 7.7 | 1.9 | 7.0 | 7.1 | 10.4 | 6.6 | 15.2 | 11.9 | 0.9 | 9.0 | 9.0 | 13.8 |
| | 20 | 2.1 | 19.8 | 1.7 | 4.8 | 5.0 | 4.9 | 5.1 | 4.2 | 16.2 | 7.4 | 1.8 | 6.7 | 6.6 | 10.0 | 6.0 | 14.5 | 11.2 | 0.7 | 8.3 | 8.3 | 13.1 |
| 2x4 | 3 | 3.3 | 9.4 | 3.2 | 4.8 | 5.0 | 5.1 | 5.1 | 6.0 | 12.4 | 9.0 | 2.2 | 8.0 | 10.9 | 10.8 | 8.2 | 15.2 | 13.0 | 1.7 | 10.3 | 14.5 | 14.5 |
| | 4 | 3.1 | 9.5 | 3.1 | 4.9 | 5.0 | 5.1 | 5.1 | 5.3 | 11.6 | 8.6 | 1.8 | 7.3 | 10.2 | 10.5 | 7.0 | 14.0 | 11.9 | 1.2 | 9.1 | 13.3 | 13.4 |
| | 5 | 3.0 | 9.7 | 2.9 | 5.0 | 5.1 | 5.1 | 5.0 | 5.0 | 11.3 | 8.2 | 1.6 | 7.0 | 9.9 | 9.9 | 6.5 | 13.4 | 11.3 | 1.0 | 8.6 | 12.8 | 12.7 |
| | 10 | 3.0 | 9.8 | 2.8 | 5.0 | 5.1 | 5.0 | 5.0 | 4.4 | 10.1 | 7.5 | 1.2 | 6.4 | 8.9 | 9.3 | 5.3 | 11.6 | 10.0 | 0.5 | 7.2 | 11.2 | 11.4 |
| | 20 | 2.9 | 9.7 | 2.6 | 4.9 | 5.1 | 5.0 | 4.9 | 4.2 | 9.7 | 6.9 | 1.1 | 6.1 | 8.6 | 8.6 | 4.7 | 10.9 | 9.3 | 0.4 | 6.6 | 10.5 | 10.6 |

**Table 19.** Type I error rate estimates when samples are taken from Beta(10,5) distributions

| | | $\sigma_{11}^2:\sigma_{12}^2 \dots :\sigma_{ij}^2$ | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1:1:…:1 | | | | | | | 1:1:…:10 | | | | | | | 1:1:…:20 | | | | | | |
| rxc | n | A | B | AxB | ***Cont.*** | ***A*** | ***B*** | ***AxB*** | A | B | AxB | ***Cont.*** | ***A*** | ***B*** | ***AxB*** | A | B | AxB | ***Cont.*** | ***A*** | ***B*** | ***AxB*** |
| 2x2 | 3 | 2.5 | 10.7 | 2.7 | 5.1 | 5.1 | 5.0 | 5.1 | 6.7 | 11.7 | 8.4 | 2.6 | 10.2 | 10.1 | 10.1 | 9.3 | 13.8 | 11.6 | 2.7 | 12.7 | 12.7 | 12.8 |
| | 4 | 2.2 | 10.7 | 2.4 | 4.9 | 4.9 | 5.1 | 5.0 | 6.0 | 10.8 | 7.7 | 2.2 | 9.5 | 9.4 | 9.4 | 8.0 | 12.3 | 10.4 | 1.9 | 11.4 | 11.4 | 11.4 |
| | 5 | 2.1 | 10.5 | 2.3 | 4.9 | 5.0 | 5.0 | 5.0 | 5.5 | 10.3 | 7.3 | 1.9 | 9.0 | 8.9 | 9.1 | 7.3 | 11.4 | 9.5 | 1.5 | 10.7 | 10.7 | 10.5 |
| | 10 | 1.9 | 10.6 | 2.0 | 5.0 | 5.1 | 5.0 | 5.0 | 4.6 | 9.2 | 6.3 | 1.3 | 8.1 | 8.0 | 8.0 | 5.7 | 9.6 | 8.1 | 0.8 | 9.0 | 8.9 | 9.1 |
| | 20 | 1.8 | 10.6 | 1.9 | 4.9 | 5.0 | 4.9 | 5.0 | 4.1 | 8.6 | 5.9 | 0.9 | 7.5 | 7.5 | 7.5 | 5.0 | 8.7 | 7.3 | 0.5 | 8.3 | 8.1 | 8.1 |
| 3x3 | 3 | 2.6 | 15.6 | 2.9 | 5.0 | 5.0 | 5.1 | 5.1 | 6.3 | 14.4 | 9.3 | 2.3 | 9.3 | 9.2 | 11.4 | 9.6 | 16.1 | 14.0 | 1.7 | 12.6 | 12.6 | 15.8 |
| | 4 | 2.4 | 15.6 | 2.5 | 5.0 | 5.0 | 5.0 | 5.1 | 5.7 | 13.8 | 8.5 | 1.8 | 8.6 | 8.7 | 10.8 | 8.4 | 14.6 | 12.9 | 1.3 | 11.3 | 11.3 | 14.4 |
| | 5 | 2.3 | 15.6 | 2.3 | 5.0 | 5.1 | 5.0 | 5.0 | 5.4 | 13.6 | 8.3 | 1.7 | 8.3 | 8.3 | 10.5 | 7.8 | 14.0 | 12.3 | 1.1 | 10.7 | 10.8 | 13.9 |
| | 10 | 2.1 | 15.7 | 2.0 | 5.0 | 5.0 | 5.0 | 4.9 | 4.6 | 12.5 | 7.6 | 1.3 | 7.5 | 7.7 | 9.7 | 6.4 | 11.8 | 10.9 | 0.6 | 9.0 | 9.0 | 12.4 |
| | 20 | 2.0 | 15.9 | 1.8 | 5.0 | 4.9 | 5.1 | 4.9 | 4.3 | 12.2 | 7.3 | 1.2 | 7.1 | 7.2 | 9.4 | 6.0 | 11.4 | 10.1 | 0.4 | 8.4 | 8.6 | 11.5 |
| 4x4 | 3 | 2.8 | 19.7 | 2.9 | 5.0 | 5.1 | 5.1 | 5.2 | 5.6 | 17.7 | 8.9 | 2.5 | 8.2 | 8.3 | 11.6 | 9.1 | 18.6 | 14.4 | 1.7 | 11.8 | 11.7 | 16.6 |
| | 4 | 2.5 | 19.7 | 2.4 | 4.8 | 5.0 | 5.0 | 5.0 | 5.2 | 17.4 | 8.4 | 2.2 | 7.9 | 7.8 | 11.2 | 8.1 | 17.3 | 13.6 | 1.4 | 10.8 | 10.8 | 15.8 |
| | 5 | 2.4 | 19.7 | 2.2 | 4.9 | 5.0 | 5.0 | 5.0 | 5.0 | 17.1 | 8.2 | 2.1 | 7.7 | 7.6 | 10.9 | 7.5 | 16.7 | 13.0 | 1.2 | 10.2 | 10.2 | 15.2 |
| | 10 | 2.3 | 19.9 | 1.8 | 5.0 | 5.1 | 5.0 | 5.0 | 4.4 | 16.6 | 7.7 | 1.9 | 7.1 | 7.1 | 10.5 | 6.7 | 15.4 | 12.0 | 0.9 | 9.1 | 8.9 | 14.0 |
| | 20 | 2.2 | 20.0 | 1.7 | 5.0 | 5.1 | 5.1 | 5.0 | 4.2 | 16.4 | 7.4 | 1.8 | 6.8 | 6.8 | 10.1 | 6.1 | 14.4 | 11.3 | 0.7 | 8.4 | 8.3 | 13.2 |
| 2x4 | 3 | 3.2 | 9.5 | 3.2 | 4.9 | 5.0 | 5.0 | 5.0 | 6.0 | 12.3 | 9.1 | 2.2 | 8.0 | 10.8 | 10.9 | 8.2 | 15.5 | 13.3 | 1.7 | 10.4 | 14.8 | 14.9 |
| | 4 | 3.1 | 9.6 | 3.1 | 5.0 | 4.9 | 5.0 | 5.1 | 5.3 | 11.8 | 8.6 | 1.9 | 7.4 | 10.4 | 10.4 | 7.3 | 14.2 | 12.1 | 1.2 | 9.3 | 13.6 | 13.6 |
| | 5 | 3.1 | 9.5 | 2.9 | 5.0 | 5.0 | 4.9 | 5.0 | 5.0 | 11.3 | 8.1 | 1.6 | 7.0 | 10.0 | 9.9 | 6.6 | 13.6 | 11.5 | 1.0 | 8.6 | 13.0 | 12.9 |
| | 10 | 3.0 | 9.6 | 2.8 | 4.9 | 5.1 | 4.9 | 5.1 | 4.4 | 10.3 | 7.4 | 1.3 | 6.3 | 9.1 | 9.1 | 5.6 | 11.7 | 10.1 | 0.5 | 7.4 | 11.4 | 11.4 |
| | 20 | 2.9 | 9.8 | 2.7 | 5.1 | 5.1 | 5.0 | 5.0 | 4.2 | 9.8 | 7.0 | 1.1 | 6.1 | 8.7 | 8.7 | 4.7 | 10.8 | 9.4 | 0.4 | 6.6 | 10.5 | 10.7 |

**Table 20.** Type I error rate estimates when samples are taken from Beta(5,10) distributions

| | | $\sigma_{11}^2:\sigma_{12}^2 \dots :\sigma_{ij}^2$ | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1:1:…:1 | | | | | | | 1:1:…:10 | | | | | | | 1:1:…:20 | | | | | | |
| rxc | n | A | B | AxB | ***Cont.*** | ***A*** | ***B*** | ***AxB*** | A | B | AxB | ***Cont.*** | ***A*** | ***B*** | ***AxB*** | A | B | AxB | ***Cont.*** | ***A*** | ***B*** | ***AxB*** |
| 2x2 | 3 | 2.5 | 10.5 | 2.7 | 4.9 | 5.1 | 5.1 | 5.0 | 6.7 | 11.9 | 8.4 | 2.6 | 10.1 | 10.3 | 10.1 | 9.3 | 13.7 | 11.7 | 2.6 | 12.8 | 12.8 | 12.8 |
| | 4 | 2.3 | 10.6 | 2.4 | 5.0 | 5.1 | 4.9 | 5.0 | 5.9 | 10.9 | 7.7 | 2.2 | 9.3 | 9.4 | 9.4 | 8.2 | 12.3 | 10.5 | 2.0 | 11.7 | 11.4 | 11.6 |
| | 5 | 2.1 | 10.8 | 2.3 | 5.1 | 5.0 | 5.1 | 5.0 | 5.4 | 10.3 | 7.2 | 1.8 | 8.9 | 8.9 | 8.9 | 7.2 | 11.4 | 9.6 | 1.5 | 10.5 | 10.6 | 10.6 |
| | 10 | 1.9 | 10.6 | 2.0 | 5.0 | 5.0 | 4.8 | 5.0 | 4.3 | 9.1 | 6.1 | 1.2 | 7.8 | 7.7 | 7.9 | 5.6 | 9.6 | 8.0 | 0.7 | 8.9 | 8.9 | 9.0 |
| | 20 | 1.6 | 10.6 | 1.9 | 4.8 | 4.9 | 5.1 | 4.9 | 3.9 | 8.5 | 5.8 | 0.9 | 7.3 | 7.5 | 7.3 | 4.9 | 8.7 | 7.3 | 0.5 | 8.2 | 8.2 | 8.2 |
| 3x3 | 3 | 2.6 | 15.4 | 2.7 | 4.9 | 5.0 | 5.0 | 5.0 | 6.4 | 14.4 | 9.3 | 2.3 | 9.3 | 9.2 | 11.6 | 9.5 | 16.2 | 13.7 | 1.8 | 12.5 | 12.5 | 15.4 |
| | 4 | 2.3 | 15.5 | 2.5 | 4.9 | 4.9 | 5.0 | 5.1 | 5.6 | 13.7 | 8.7 | 1.9 | 8.5 | 8.7 | 10.9 | 8.6 | 14.5 | 12.8 | 1.3 | 11.5 | 11.3 | 14.4 |
| | 5 | 2.2 | 15.5 | 2.3 | 4.8 | 4.9 | 5.0 | 4.9 | 5.4 | 13.6 | 8.4 | 1.8 | 8.3 | 8.4 | 10.7 | 8.0 | 13.8 | 12.2 | 1.0 | 10.8 | 10.6 | 13.8 |

| | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 10 | 2.0 | 15.9 | 1.9 | 4.9 | 4.9 | 5.0 | 4.9 | 4.7 | 12.4 | 7.6 | 1.3 | 7.6 | 7.5 | 9.7 | 6.7 | 12.2 | 10.9 | 0.6 | 9.2 | 9.4 | 12.2 |
| | 20 | 2.0 | 15.9 | 1.9 | 5.1 | 5.0 | 4.9 | 5.2 | 4.3 | 11.9 | 7.2 | 1.2 | 6.9 | 7.0 | 9.3 | 6.0 | 11.3 | 10.3 | 0.4 | 8.6 | 8.5 | 11.6 |
| 4x4 | 3 | 2.8 | 19.8 | 2.8 | 4.9 | 5.1 | 5.1 | 5.2 | 5.7 | 17.7 | 8.9 | 2.5 | 8.3 | 8.3 | 11.6 | 8.8 | 18.3 | 14.4 | 1.7 | 11.5 | 11.5 | 16.6 |
| | 4 | 2.6 | 19.8 | 2.5 | 5.0 | 5.0 | 5.0 | 5.1 | 5.1 | 17.2 | 8.5 | 2.3 | 7.9 | 7.9 | 11.3 | 8.0 | 17.4 | 13.3 | 1.4 | 10.7 | 10.7 | 15.6 |
| | 5 | 2.4 | 19.9 | 2.3 | 5.0 | 5.0 | 5.0 | 5.0 | 4.9 | 17.1 | 8.3 | 2.1 | 7.5 | 7.6 | 11.0 | 7.5 | 16.6 | 12.9 | 1.2 | 10.2 | 10.1 | 15.0 |
| | 10 | 2.2 | 20.2 | 1.9 | 5.0 | 5.0 | 5.0 | 5.1 | 4.5 | 16.6 | 7.7 | 2.0 | 7.1 | 7.1 | 10.5 | 6.5 | 15.3 | 11.9 | 0.9 | 8.9 | 9.0 | 13.9 |
| | 20 | 2.2 | 20.1 | 1.7 | 5.1 | 5.1 | 5.0 | 4.9 | 4.1 | 16.2 | 7.3 | 1.8 | 6.6 | 6.7 | 10.0 | 6.1 | 14.5 | 11.3 | 0.8 | 8.4 | 8.3 | 13.2 |
| 2x4 | 3 | 3.2 | 9.5 | 3.2 | 5.1 | 5.1 | 5.0 | 5.0 | 5.8 | 12.5 | 9.1 | 2.2 | 7.8 | 10.9 | 10.9 | 8.3 | 15.5 | 13.2 | 1.8 | 10.4 | 14.8 | 14.8 |
| | 4 | 3.1 | 9.7 | 3.0 | 5.0 | 5.0 | 5.0 | 5.0 | 5.4 | 11.6 | 8.5 | 1.9 | 7.4 | 10.2 | 10.3 | 7.2 | 14.4 | 12.3 | 1.3 | 9.3 | 13.8 | 13.7 |
| | 5 | 3.1 | 9.8 | 2.9 | 4.9 | 5.1 | 5.2 | 5.0 | 5.2 | 11.5 | 8.4 | 1.7 | 7.2 | 10.2 | 10.1 | 6.7 | 13.4 | 11.3 | 1.0 | 8.7 | 12.8 | 12.7 |
| | 10 | 2.9 | 9.7 | 2.8 | 5.0 | 5.0 | 4.9 | 5.0 | 4.4 | 10.2 | 7.5 | 1.2 | 6.4 | 9.1 | 9.2 | 5.4 | 11.7 | 10.0 | 0.5 | 7.3 | 11.4 | 11.3 |
| | 20 | 2.9 | 9.7 | 2.6 | 5.0 | 5.0 | 4.9 | 5.0 | 4.1 | 9.8 | 7.0 | 1.1 | 6.0 | 8.7 | 8.7 | 4.8 | 11.1 | 9.4 | 0.4 | 6.7 | 10.8 | 10.7 |

**Table 21.** Type I error rate estimates when samples are taken from Chi-Sq(3) distributions

| | | $\sigma_{11}^2:\sigma_{12}^2 \ldots:\sigma_{ij}^2$ | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1:1:…:1 | | | | | | | 1:1:…:10 | | | | | | | 1:1:…:20 | | | | | | |
| rxc | n | A | B | AxB | Cont. | A | B | AxB | A | B | AxB | Cont. | A | B | AxB | A | B | AxB | Cont. | A | B | AxB |
| 2x2 | 3 | 2.3 | 9.2 | 2.7 | 4.4 | 4.6 | 4.7 | 4.6 | 7.2 | 12.3 | 8.9 | 3.3 | 10.9 | 10.9 | 10.8 | 10.9 | 15.6 | 13.3 | 3.7 | 14.7 | 14.7 | 14.6 |
| | 4 | 2.1 | 9.3 | 2.3 | 4.5 | 4.6 | 4.6 | 4.6 | 6.5 | 11.6 | 8.4 | 2.9 | 10.4 | 10.3 | 10.4 | 9.8 | 14.4 | 12.4 | 3.1 | 13.6 | 13.5 | 13.6 |
| | 5 | 2.1 | 9.7 | 2.2 | 4.5 | 4.7 | 4.7 | 4.6 | 6.1 | 11.3 | 8.0 | 2.5 | 9.8 | 10.0 | 9.8 | 8.8 | 13.5 | 11.6 | 2.7 | 12.6 | 12.6 | 12.7 |
| | 10 | 1.8 | 10.5 | 2.1 | 4.9 | 4.9 | 4.8 | 4.9 | 5.1 | 10.0 | 6.8 | 2.0 | 8.7 | 8.6 | 8.6 | 6.9 | 11.4 | 9.4 | 1.7 | 10.5 | 10.6 | 10.5 |
| | 20 | 1.8 | 10.4 | 2.0 | 4.9 | 5.0 | 4.9 | 5.0 | 4.4 | 9.1 | 6.3 | 1.4 | 8.0 | 8.0 | 7.9 | 5.7 | 9.9 | 8.2 | 1.0 | 9.1 | 9.2 | 9.1 |
| 3x3 | 3 | 2.5 | 13.6 | 3.1 | 4.5 | 4.6 | 4.6 | 4.8 | 6.0 | 13.8 | 8.9 | 2.9 | 9.0 | 9.0 | 11.1 | 10.1 | 16.6 | 14.9 | 2.5 | 13.3 | 13.4 | 16.8 |
| | 4 | 2.3 | 14.3 | 2.7 | 4.5 | 4.6 | 4.7 | 4.7 | 5.7 | 13.4 | 8.7 | 2.6 | 8.7 | 8.6 | 11.1 | 9.0 | 15.5 | 13.8 | 2.0 | 12.2 | 12.4 | 15.8 |
| | 5 | 2.3 | 14.5 | 2.4 | 4.5 | 4.8 | 4.8 | 4.7 | 5.5 | 13.4 | 8.5 | 2.3 | 8.5 | 8.5 | 10.9 | 8.6 | 15.1 | 13.4 | 1.9 | 11.9 | 11.9 | 15.2 |
| | 10 | 2.1 | 15.5 | 2.2 | 4.7 | 4.9 | 5.0 | 4.9 | 4.8 | 13.0 | 7.8 | 1.9 | 7.7 | 8.0 | 10.1 | 7.2 | 13.1 | 11.9 | 1.2 | 10.1 | 10.2 | 13.5 |
| | 20 | 2.0 | 15.6 | 2.0 | 4.8 | 5.0 | 5.0 | 4.9 | 4.5 | 12.3 | 7.6 | 1.5 | 7.4 | 7.4 | 9.7 | 6.5 | 12.1 | 11.1 | 0.8 | 9.2 | 9.2 | 12.5 |
| 4x4 | 3 | 2.8 | 17.8 | 3.1 | 4.6 | 4.7 | 4.7 | 4.8 | 5.3 | 16.4 | 8.1 | 3.1 | 7.8 | 7.7 | 10.6 | 8.6 | 18.2 | 14.4 | 2.5 | 11.4 | 11.6 | 17.0 |
| | 4 | 2.6 | 18.4 | 2.7 | 4.4 | 4.7 | 4.7 | 4.8 | 5.0 | 16.5 | 8.2 | 2.8 | 7.6 | 7.7 | 10.9 | 8.0 | 17.5 | 13.8 | 2.3 | 10.8 | 10.9 | 16.3 |
| | 5 | 2.5 | 19.0 | 2.5 | 4.5 | 4.7 | 4.8 | 4.8 | 4.8 | 16.5 | 8.0 | 2.7 | 7.4 | 7.4 | 10.9 | 7.6 | 16.9 | 13.5 | 1.9 | 10.4 | 10.4 | 15.9 |
| | 10 | 2.3 | 19.7 | 2.2 | 4.7 | 4.8 | 4.8 | 5.0 | 4.5 | 16.7 | 7.8 | 2.3 | 7.2 | 7.2 | 10.7 | 7.1 | 15.8 | 12.7 | 1.5 | 9.6 | 9.6 | 14.9 |
| | 20 | 2.2 | 20.1 | 1.9 | 4.8 | 5.0 | 5.0 | 4.9 | 4.1 | 16.2 | 7.4 | 2.1 | 6.7 | 6.8 | 10.2 | 6.3 | 15.0 | 11.9 | 1.1 | 8.8 | 8.8 | 14.0 |
| 2x4 | 3 | 3.1 | 8.2 | 3.3 | 4.5 | 4.8 | 4.6 | 4.7 | 5.9 | 12.2 | 8.8 | 2.9 | 7.9 | 10.9 | 10.8 | 8.7 | 16.6 | 14.1 | 2.5 | 11.1 | 15.8 | 15.8 |
| | 4 | 3.0 | 8.5 | 3.0 | 4.6 | 4.8 | 4.7 | 4.7 | 5.4 | 12.1 | 8.7 | 2.6 | 7.5 | 10.7 | 10.7 | 8.0 | 15.8 | 13.3 | 2.2 | 10.2 | 15.1 | 15.0 |
| | 5 | 3.0 | 8.7 | 2.8 | 4.4 | 4.8 | 4.6 | 4.5 | 5.1 | 11.8 | 8.3 | 2.4 | 7.2 | 10.3 | 10.3 | 7.4 | 15.2 | 12.9 | 1.8 | 9.6 | 14.4 | 14.5 |
| | 10 | 3.1 | 9.3 | 2.6 | 4.7 | 5.1 | 4.9 | 4.7 | 4.7 | 10.9 | 7.8 | 1.7 | 6.7 | 9.9 | 9.6 | 6.1 | 13.0 | 11.1 | 1.3 | 8.2 | 12.6 | 12.5 |
| | 20 | 2.9 | 9.5 | 2.7 | 4.8 | 5.0 | 4.9 | 4.9 | 4.4 | 10.2 | 7.4 | 1.5 | 6.3 | 9.1 | 9.1 | 5.3 | 11.8 | 10.1 | 0.7 | 7.1 | 11.4 | 11.4 |

**Table 22.** Test power estimates when samples are taken from Normal distributions

| | | | $\sigma_{11}^2:\sigma_{12}^2 \ldots:\sigma_{ij}^2$ | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 1:1:…:1 | | | | | | | 1:1:…:10 | | | | | | | 1:1:…:20 | | | | | | |
| rxc | δ | n | A | B | AxB | Cont. | A | B | AxB | A | B | AxB | Cont. | A | B | AxB | A | B | AxB | Cont. | A | B | AxB |
| 2x2 | 0.75 | 3 | 4.8 | 22.2 | 5.5 | 12.3 | 9.0 | 9.0 | 9.1 | 7.8 | 16.3 | 9.6 | 5.2 | 11.5 | 11.5 | 11.4 | 9.7 | 16.3 | 11.8 | 4.4 | 13.1 | 13.2 | 13.0 |
| | | 4 | 5.6 | 27.2 | 6.4 | 15.7 | 10.7 | 10.8 | 10.8 | 7.3 | 17.4 | 9.5 | 5.5 | 11.3 | 11.4 | 11.3 | 8.7 | 15.9 | 11.1 | 3.9 | 12.3 | 12.3 | 12.2 |
| | | 5 | 6.3 | 32.2 | 7.6 | 18.8 | 12.5 | 12.4 | 12.6 | 7.3 | 18.7 | 9.8 | 6.0 | 11.5 | 11.5 | 11.5 | 8.5 | 15.9 | 11.1 | 3.8 | 12.2 | 12.2 | 12.2 |
| | | 10 | 11.3 | 53.9 | 14.5 | 34.3 | 21.4 | 21.4 | 21.5 | 8.6 | 27.2 | 11.9 | 9.3 | 13.7 | 13.6 | 13.7 | 8.1 | 18.5 | 11.3 | 4.5 | 12.3 | 12.2 | 12.3 |
| | | 20 | 23.4 | 81.4 | 31.7 | 60.8 | 38.3 | 38.4 | 38.2 | 12.5 | 49.5 | 17.5 | 20.2 | 19.3 | 19.1 | 19.3 | 9.9 | 29.7 | 14.1 | 8.6 | 14.9 | 15.0 | 15.0 |
| | 1.5 | 3 | 13.5 | 53.4 | 16.2 | 34.8 | 21.7 | 21.7 | 21.7 | 11.4 | 32.5 | 14.4 | 14.7 | 16.2 | 16.3 | 16.3 | 12.0 | 26.6 | 14.9 | 10.3 | 16.1 | 16.2 | 15.9 |
| | | 4 | 17.8 | 67.6 | 23.1 | 47.0 | 28.9 | 29.0 | 29.3 | 12.6 | 40.0 | 16.2 | 18.1 | 18.1 | 18.1 | 18.1 | 12.0 | 29.5 | 15.3 | 11.1 | 16.4 | 16.4 | 16.4 |
| | | 5 | 22.6 | 77.7 | 29.9 | 56.8 | 36.1 | 36.2 | 35.6 | 14.0 | 47.9 | 18.3 | 22.2 | 20.0 | 20.1 | 20.2 | 12.3 | 33.8 | 16.2 | 12.9 | 17.1 | 17.1 | 17.2 |
| | | 10 | 47.3 | 97.4 | 63.1 | 87.6 | 64.1 | 64.1 | 63.8 | 22.1 | 81.0 | 29.3 | 45.9 | 30.7 | 30.8 | 31.0 | 16.3 | 58.8 | 21.8 | 24.4 | 22.6 | 22.5 | 22.5 |
| | | 20 | 82.1 | 100.0 | 94.8 | 99.4 | 91.3 | 91.3 | 91.4 | 40.1 | 99.1 | 50.7 | 82.8 | 51.0 | 51.1 | 51.2 | 26.5 | 93.3 | 34.6 | 55.0 | 35.0 | 34.9 | 34.9 |
| 3x3 | 0.75 | 3 | 4.3 | 35.1 | 5.0 | 18.2 | 7.7 | 7.6 | 8.3 | 7.3 | 27.4 | 10.8 | 9.0 | 10.4 | 10.6 | 13.2 | 10.0 | 24.5 | 14.5 | 6.1 | 13.0 | 13.0 | 16.2 |
| | | 4 | 4.5 | 42.2 | 5.5 | 23.1 | 8.5 | 8.4 | 9.7 | 7.1 | 31.2 | 10.9 | 10.9 | 10.6 | 10.6 | 13.4 | 9.3 | 26.2 | 14.2 | 6.5 | 12.5 | 12.4 | 15.7 |
| | | 5 | 4.9 | 48.6 | 6.2 | 28.4 | 9.7 | 9.7 | 11.3 | 7.5 | 35.6 | 11.4 | 13.0 | 10.9 | 10.8 | 13.9 | 8.9 | 28.6 | 13.9 | 7.4 | 12.0 | 12.2 | 15.5 |
| | | 10 | 8.0 | 72.5 | 11.6 | 51.0 | 15.0 | 15.3 | 19.9 | 8.8 | 56.5 | 14.4 | 26.0 | 12.9 | 12.9 | 17.0 | 9.4 | 43.4 | 15.2 | 13.7 | 12.7 | 12.7 | 16.6 |
| | | 20 | 15.7 | 93.3 | 27.2 | 80.5 | 27.0 | 27.2 | 38.5 | 12.9 | 83.9 | 21.3 | 54.4 | 18.2 | 18.2 | 24.2 | 11.6 | 71.6 | 18.7 | 33.4 | 15.3 | 15.2 | 20.2 |
| | 1.5 | 3 | 9.8 | 75.1 | 13.9 | 55.0 | 16.0 | 15.8 | 20.0 | 11.1 | 59.8 | 16.6 | 32.5 | 15.0 | 15.2 | 19.3 | 12.6 | 49.7 | 18.0 | 23.0 | 16.0 | 15.9 | 19.7 |
| | | 4 | 12.8 | 86.1 | 19.4 | 69.1 | 21.0 | 20.9 | 27.8 | 12.4 | 72.0 | 19.0 | 43.0 | 17.0 | 17.2 | 21.9 | 12.9 | 59.5 | 19.0 | 28.8 | 16.3 | 16.5 | 20.7 |
| | | 5 | 15.6 | 92.1 | 25.9 | 78.8 | 25.3 | 25.5 | 35.4 | 13.8 | 81.3 | 21.3 | 53.5 | 18.7 | 18.6 | 24.3 | 13.4 | 68.9 | 19.9 | 36.1 | 17.1 | 17.0 | 21.6 |
| | | 10 | 34.5 | 99.7 | 60.5 | 97.7 | 49.2 | 49.2 | 69.1 | 22.5 | 98.4 | 34.4 | 87.8 | 29.3 | 29.2 | 37.6 | 18.3 | 94.7 | 27.7 | 70.8 | 22.9 | 22.9 | 29.4 |
| | | 20 | 68.6 | 100.0 | 95.1 | 100.0 | 80.9 | 81.1 | 96.0 | 40.1 | 100.0 | 57.2 | 99.7 | 48.7 | 48.5 | 60.0 | 29.6 | 100.0 | 42.3 | 97.6 | 35.6 | 35.4 | 44.0 |
| 4x4 | 0.75 | 3 | 3.7 | 43.5 | 4.3 | 21.1 | 6.6 | 6.7 | 7.5 | 6.5 | 36.8 | 10.4 | 12.5 | 9.4 | 9.4 | 13.3 | 9.4 | 33.2 | 14.9 | 9.0 | 12.1 | 12.1 | 17.3 |

| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 4 | 4.1 | 51.2 | 4.6 | 26.6 | 7.4 | 7.5 | 8.8 | 6.6 | 43.0 | 10.9 | 16.0 | 9.6 | 9.5 | 13.8 | 9.0 | 37.7 | 14.8 | 10.8 | 11.8 | 11.8 | 17.1 |
| | | 5 | 4.4 | 57.6 | 5.2 | 32.3 | 8.2 | 8.2 | 10.1 | 6.7 | 48.6 | 11.5 | 19.7 | 9.9 | 9.7 | 14.7 | 8.8 | 41.7 | 15.0 | 12.8 | 11.6 | 11.6 | 17.2 |
| | | 10 | 6.3 | 80.4 | 9.1 | 57.6 | 11.6 | 11.8 | 17.5 | 8.1 | 72.0 | 14.6 | 39.5 | 11.7 | 11.7 | 18.1 | 9.3 | 63.0 | 16.1 | 26.3 | 12.1 | 12.1 | 18.4 |
| | | 20 | 11.6 | 96.4 | 21.7 | 85.9 | 19.9 | 19.8 | 35.0 | 11.8 | 93.0 | 21.5 | 72.7 | 16.5 | 16.2 | 25.6 | 11.5 | 88.3 | 20.4 | 57.3 | 14.9 | 15.0 | 22.8 |
| | 1.5 | 3 | 7.7 | 83.5 | 11.2 | 63.0 | 12.4 | 12.5 | 17.4 | 10.0 | 75.6 | 16.3 | 47.1 | 13.7 | 13.8 | 19.7 | 12.1 | 67.6 | 18.9 | 35.7 | 15.3 | 15.1 | 21.4 |
| | | 4 | 9.7 | 91.7 | 15.7 | 76.5 | 15.9 | 15.8 | 24.5 | 11.3 | 85.7 | 18.9 | 60.1 | 15.4 | 15.3 | 22.8 | 12.6 | 78.5 | 20.3 | 46.8 | 15.8 | 15.9 | 22.8 |
| | | 5 | 11.7 | 95.9 | 20.9 | 85.1 | 19.2 | 19.3 | 31.9 | 12.3 | 92.1 | 21.5 | 71.5 | 16.7 | 16.9 | 25.4 | 13.1 | 86.4 | 21.5 | 57.2 | 16.5 | 16.4 | 24.1 |
| | | 10 | 24.9 | 99.9 | 53.0 | 99.0 | 36.9 | 36.6 | 66.2 | 20.2 | 99.7 | 34.8 | 96.5 | 26.4 | 26.3 | 39.5 | 17.7 | 99.1 | 29.2 | 90.6 | 21.9 | 22.1 | 31.8 |
| | | 20 | 54.2 | 100.0 | 92.9 | 100.0 | 67.4 | 67.4 | 95.8 | 36.3 | 100.0 | 57.6 | 100.0 | 43.9 | 44.1 | 62.2 | 28.9 | 100.0 | 44.6 | 99.9 | 34.2 | 34.0 | 47.4 |
| 2x4 | 0.75 | 3 | 5.0 | 23.9 | 5.6 | 17.5 | 7.4 | 8.2 | 8.2 | 6.8 | 19.5 | 10.5 | 8.0 | 9.0 | 12.5 | 12.5 | 8.7 | 20.1 | 14.0 | 5.6 | 10.9 | 15.6 | 15.6 |
| | | 4 | 5.4 | 30.4 | 6.6 | 22.5 | 8.2 | 9.6 | 9.8 | 6.5 | 21.8 | 10.7 | 9.7 | 8.8 | 12.6 | 12.6 | 8.0 | 20.2 | 13.3 | 6.0 | 10.3 | 14.8 | 14.7 |
| | | 5 | 5.8 | 36.5 | 7.3 | 27.1 | 8.8 | 11.2 | 11.0 | 6.6 | 24.4 | 11.0 | 11.5 | 8.9 | 13.1 | 13.1 | 7.5 | 20.8 | 13.1 | 6.4 | 9.8 | 14.5 | 14.6 |
| | | 10 | 8.9 | 62.9 | 13.4 | 49.1 | 13.1 | 19.4 | 19.0 | 7.7 | 39.7 | 13.7 | 22.9 | 10.5 | 15.9 | 15.8 | 7.7 | 28.0 | 13.9 | 11.7 | 10.0 | 15.3 | 15.3 |
| | | 20 | 15.7 | 89.8 | 29.1 | 78.6 | 21.8 | 37.0 | 37.0 | 10.9 | 68.6 | 20.0 | 49.4 | 14.5 | 22.5 | 22.5 | 9.5 | 48.9 | 17.7 | 27.9 | 12.1 | 19.3 | 19.3 |
| | 1.5 | 3 | 10.1 | 63.0 | 15.0 | 52.7 | 14.1 | 19.7 | 19.8 | 9.9 | 43.8 | 15.9 | 30.0 | 12.7 | 18.3 | 18.2 | 11.0 | 36.0 | 17.3 | 20.5 | 13.5 | 18.9 | 19.0 |
| | | 4 | 12.7 | 77.6 | 20.8 | 66.3 | 17.7 | 26.8 | 26.9 | 10.9 | 54.9 | 18.2 | 39.1 | 14.0 | 20.6 | 20.6 | 10.8 | 42.1 | 17.8 | 25.6 | 13.5 | 19.5 | 19.5 |
| | | 5 | 15.4 | 87.4 | 27.3 | 76.9 | 21.2 | 34.1 | 34.3 | 11.8 | 64.8 | 20.4 | 48.8 | 15.3 | 23.0 | 23.0 | 11.2 | 49.3 | 18.9 | 31.5 | 13.9 | 20.7 | 20.7 |
| | | 10 | 29.9 | 99.5 | 59.3 | 97.2 | 38.2 | 65.7 | 66.0 | 18.4 | 93.9 | 32.3 | 84.3 | 23.0 | 35.2 | 35.3 | 14.7 | 81.0 | 25.8 | 64.2 | 18.2 | 27.6 | 27.6 |
| | | 20 | 56.4 | 100.0 | 93.0 | 100.0 | 65.2 | 94.1 | 94.2 | 33.0 | 100.0 | 54.5 | 99.4 | 39.3 | 57.3 | 57.4 | 23.5 | 99.3 | 39.6 | 95.9 | 28.3 | 42.0 | 41.7 |

**Table 23.** Test power estimates when samples are taken from Beta (10,10) distributions

| | | | $\sigma_{11}^2:\sigma_{12}^2 \ldots :\sigma_{ij}^2$ | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 1:1:…:1 | | | | | | | 1:1:…:10 | | | | | | | 1:1:…:20 | | | | | | |
| rxc | δ | n | A | B | AxB | **Cont.** | ***A*** | ***B*** | ***AxB*** | A | B | AxB | **Cont.** | ***A*** | ***B*** | ***AxB*** | ***A*** | B | AxB | **Cont.** | ***A*** | ***B*** | ***AxB*** |
| 2x2 | 0.75 | 3 | 4.9 | 22.3 | 5.6 | 12.5 | 9.1 | 9.0 | 9.1 | 7.8 | 16.3 | 9.7 | 5.3 | 11.5 | 11.5 | 11.5 | 10.0 | 16.8 | 12.5 | 4.5 | 13.5 | 13.6 | 13.6 |
| | | 4 | 5.5 | 27.0 | 6.4 | 15.5 | 10.7 | 10.8 | 10.8 | 7.3 | 17.2 | 9.6 | 5.6 | 11.3 | 11.4 | 11.3 | 8.8 | 15.6 | 11.4 | 3.9 | 12.5 | 12.5 | 12.5 |
| | | 5 | 6.4 | 32.0 | 7.4 | 18.7 | 12.5 | 12.3 | 12.5 | 7.4 | 18.3 | 9.9 | 5.8 | 11.6 | 11.6 | 11.7 | 8.4 | 15.8 | 11.1 | 3.8 | 12.2 | 12.2 | 12.1 |
| | | 10 | 11.2 | 54.0 | 14.4 | 34.4 | 21.1 | 21.6 | 21.3 | 8.7 | 27.4 | 11.9 | 9.5 | 13.8 | 13.6 | 13.7 | 8.0 | 18.4 | 11.2 | 4.5 | 12.2 | 12.3 | 12.1 |
| | | 20 | 23.3 | 81.1 | 31.7 | 60.2 | 38.4 | 38.1 | 38.0 | 12.5 | 49.4 | 17.3 | 20.1 | 19.3 | 19.3 | 19.2 | 9.9 | 29.7 | 14.1 | 8.5 | 14.8 | 15.0 | 15.0 |
| | 1.5 | 3 | 13.1 | 53.1 | 16.0 | 34.7 | 21.3 | 21.3 | 21.2 | 11.5 | 32.3 | 14.6 | 14.7 | 16.1 | 16.3 | 16.4 | 12.5 | 26.5 | 15.2 | 10.1 | 16.3 | 16.4 | 16.3 |
| | | 4 | 17.7 | 67.1 | 22.5 | 46.7 | 28.9 | 28.6 | 28.7 | 12.5 | 39.5 | 16.1 | 17.8 | 17.9 | 18.0 | 18.0 | 12.2 | 29.3 | 15.5 | 11.1 | 16.5 | 16.6 | 16.6 |
| | | 5 | 22.3 | 77.8 | 29.3 | 56.9 | 35.6 | 35.5 | 35.4 | 13.9 | 47.1 | 18.1 | 21.7 | 19.9 | 19.8 | 20.0 | 12.3 | 32.9 | 16.0 | 12.4 | 16.9 | 16.9 | 16.9 |
| | | 10 | 47.2 | 97.5 | 63.2 | 87.5 | 64.0 | 63.7 | 64.0 | 22.0 | 80.9 | 29.0 | 45.5 | 30.7 | 30.5 | 30.4 | 16.2 | 58.4 | 21.6 | 23.7 | 22.3 | 22.3 | 22.3 |
| | | 20 | 82.1 | 100.0 | 94.9 | 99.4 | 91.5 | 91.4 | 91.4 | 40.0 | 99.2 | 50.8 | 82.9 | 51.1 | 50.8 | 51.2 | 26.1 | 93.6 | 34.3 | 54.8 | 34.6 | 34.6 | 34.7 |
| 3x3 | 0.75 | 3 | 4.2 | 34.9 | 5.0 | 18.1 | 7.6 | 7.5 | 8.3 | 7.5 | 27.0 | 11.0 | 8.7 | 10.8 | 10.6 | 13.2 | 10.3 | 24.7 | 15.0 | 6.0 | 13.3 | 13.3 | 16.7 |
| | | 4 | 4.5 | 42.0 | 5.4 | 23.4 | 8.6 | 8.5 | 9.7 | 7.4 | 30.9 | 11.2 | 10.5 | 10.6 | 10.7 | 13.6 | 9.4 | 26.2 | 14.2 | 6.5 | 12.4 | 12.6 | 15.8 |
| | | 5 | 4.8 | 48.4 | 6.1 | 28.0 | 9.5 | 9.6 | 11.2 | 7.4 | 35.2 | 11.5 | 12.5 | 10.8 | 10.9 | 13.9 | 9.2 | 28.5 | 14.2 | 7.3 | 12.2 | 12.3 | 15.8 |
| | | 10 | 7.8 | 72.7 | 11.5 | 50.9 | 14.9 | 15.1 | 19.8 | 8.8 | 56.1 | 14.4 | 25.8 | 12.9 | 12.8 | 17.0 | 9.4 | 43.0 | 15.2 | 13.7 | 12.6 | 12.5 | 16.6 |
| | | 20 | 15.7 | 93.3 | 27.3 | 80.1 | 27.0 | 27.0 | 38.7 | 12.8 | 84.1 | 20.9 | 54.2 | 18.0 | 18.0 | 23.8 | 11.7 | 71.6 | 19.0 | 33.2 | 15.4 | 15.4 | 20.5 |
| | 1.5 | 3 | 9.6 | 75.0 | 13.7 | 55.0 | 15.8 | 16.1 | 19.9 | 11.1 | 59.3 | 16.4 | 32.0 | 15.1 | 15.1 | 19.0 | 12.9 | 49.3 | 18.3 | 22.2 | 16.3 | 16.5 | 20.0 |
| | | 4 | 12.5 | 85.9 | 18.9 | 68.3 | 20.5 | 20.6 | 27.2 | 12.4 | 71.6 | 18.8 | 42.9 | 16.9 | 16.9 | 21.6 | 12.9 | 58.9 | 18.9 | 28.3 | 16.4 | 16.6 | 20.7 |
| | | 5 | 15.4 | 92.2 | 25.5 | 78.4 | 25.3 | 25.4 | 35.1 | 13.9 | 81.2 | 21.1 | 53.2 | 18.9 | 18.7 | 24.1 | 13.5 | 68.5 | 20.1 | 35.2 | 17.1 | 17.3 | 21.7 |
| | | 10 | 34.3 | 99.7 | 60.1 | 97.8 | 49.1 | 48.9 | 68.8 | 22.2 | 98.4 | 34.1 | 87.9 | 28.9 | 28.8 | 37.3 | 18.2 | 94.7 | 27.2 | 70.5 | 22.9 | 22.8 | 29.0 |
| | | 20 | 68.7 | 100.0 | 95.2 | 100.0 | 81.0 | 80.9 | 96.0 | 40.0 | 100.0 | 57.0 | 99.7 | 48.3 | 48.6 | 59.8 | 29.2 | 100.0 | 42.1 | 97.8 | 35.2 | 35.1 | 43.9 |
| 4x4 | 0.75 | 3 | 3.8 | 43.2 | 4.3 | 21.1 | 6.8 | 6.8 | 7.6 | 6.8 | 36.8 | 10.6 | 12.5 | 9.7 | 9.6 | 13.5 | 9.8 | 33.1 | 15.4 | 8.7 | 12.6 | 12.4 | 17.8 |
| | | 4 | 4.1 | 50.8 | 4.6 | 26.7 | 7.5 | 7.4 | 8.9 | 6.6 | 42.9 | 10.9 | 16.1 | 9.7 | 9.5 | 14.0 | 9.2 | 37.1 | 14.9 | 10.3 | 12.0 | 11.7 | 17.2 |
| | | 5 | 4.2 | 57.6 | 5.0 | 32.5 | 8.0 | 8.0 | 10.0 | 6.9 | 48.9 | 11.5 | 19.8 | 10.0 | 10.1 | 14.7 | 9.0 | 41.5 | 15.1 | 12.4 | 11.7 | 11.7 | 17.3 |
| | | 10 | 6.3 | 80.3 | 8.9 | 57.3 | 11.7 | 11.8 | 17.2 | 8.0 | 71.9 | 14.3 | 39.4 | 11.7 | 11.5 | 18.0 | 9.2 | 62.7 | 16.1 | 26.3 | 12.0 | 12.0 | 18.2 |
| | | 20 | 11.6 | 96.4 | 21.5 | 86.0 | 19.8 | 19.9 | 34.6 | 11.7 | 93.2 | 21.4 | 72.9 | 16.3 | 16.4 | 25.6 | 11.6 | 88.4 | 20.5 | 57.3 | 15.0 | 15.0 | 22.9 |
| | 1.5 | 3 | 7.7 | 83.3 | 11.3 | 62.9 | 12.5 | 12.5 | 17.5 | 10.0 | 75.3 | 16.4 | 46.6 | 13.7 | 13.7 | 19.7 | 12.4 | 67.3 | 19.2 | 35.6 | 15.5 | 15.3 | 21.6 |
| | | 4 | 9.5 | 91.8 | 15.4 | 76.2 | 15.8 | 15.7 | 24.1 | 11.2 | 85.7 | 18.7 | 59.9 | 15.3 | 15.1 | 22.4 | 12.6 | 78.1 | 20.0 | 46.1 | 15.8 | 15.6 | 22.4 |
| | | 5 | 11.6 | 96.0 | 20.5 | 85.3 | 19.1 | 19.1 | 31.3 | 12.3 | 92.2 | 21.3 | 71.6 | 16.7 | 16.8 | 25.4 | 13.0 | 86.3 | 21.4 | 57.1 | 16.4 | 16.4 | 23.9 |
| | | 10 | 25.0 | 99.9 | 52.7 | 99.0 | 36.9 | 36.7 | 66.0 | 20.0 | 99.7 | 34.5 | 96.6 | 25.9 | 26.2 | 39.4 | 17.7 | 99.1 | 29.0 | 90.9 | 21.8 | 21.8 | 31.6 |
| | | 20 | 54.0 | 100.0 | 93.1 | 100.0 | 67.2 | 67.5 | 96.0 | 36.4 | 100.0 | 57.9 | 100.0 | 44.2 | 44.1 | 62.6 | 28.8 | 100.0 | 44.4 | 99.9 | 34.0 | 33.9 | 47.2 |
| 2x4 | 0.75 | 3 | 4.9 | 23.9 | 5.5 | 17.6 | 7.3 | 8.0 | 8.1 | 6.8 | 19.7 | 10.8 | 8.0 | 9.1 | 12.8 | 12.7 | 8.8 | 20.0 | 14.0 | 5.6 | 11.0 | 15.6 | 15.6 |
| | | 4 | 5.3 | 30.2 | 6.2 | 22.1 | 8.0 | 9.7 | 9.4 | 6.7 | 22.0 | 10.9 | 9.6 | 9.1 | 13.0 | 12.9 | 8.1 | 20.1 | 13.4 | 5.9 | 10.2 | 14.8 | 14.9 |
| | | 5 | 5.9 | 36.4 | 7.4 | 27.1 | 8.9 | 11.3 | 11.2 | 6.7 | 24.4 | 11.2 | 11.3 | 9.1 | 13.3 | 13.3 | 7.6 | 21.1 | 13.4 | 6.3 | 9.8 | 14.9 | 14.9 |
| | | 10 | 9.1 | 62.4 | 13.4 | 48.8 | 13.2 | 19.3 | 19.1 | 7.9 | 39.6 | 13.7 | 22.8 | 10.7 | 16.1 | 16.0 | 7.5 | 27.4 | 13.6 | 11.1 | 9.8 | 15.2 | 15.1 |
| | | 20 | 15.7 | 89.9 | 29.3 | 78.7 | 21.9 | 36.7 | 36.9 | 10.7 | 68.6 | 19.8 | 49.7 | 14.2 | 22.3 | 22.4 | 9.3 | 48.4 | 17.4 | 27.7 | 12.1 | 18.9 | 19.0 |
| | 1.5 | 3 | 10.1 | 62.4 | 14.8 | 52.4 | 13.9 | 19.5 | 19.7 | 10.0 | 42.8 | 15.6 | 29.1 | 12.7 | 18.0 | 17.9 | 11.1 | 35.5 | 17.3 | 19.9 | 13.6 | 19.0 | 19.1 |
| | | 4 | 12.7 | 77.5 | 20.5 | 66.0 | 17.5 | 26.6 | 26.5 | 10.7 | 54.2 | 18.1 | 39.0 | 13.9 | 20.5 | 20.6 | 10.9 | 41.4 | 17.8 | 25.0 | 13.4 | 19.5 | 19.6 |
| | | 5 | 15.2 | 87.1 | 26.6 | 76.6 | 20.8 | 33.7 | 33.5 | 11.9 | 64.7 | 20.3 | 48.5 | 15.4 | 22.9 | 22.8 | 11.0 | 48.4 | 18.8 | 31.0 | 13.7 | 20.6 | 20.5 |
| | | 10 | 29.6 | 99.5 | 59.3 | 97.1 | 37.8 | 65.5 | 65.7 | 18.5 | 94.0 | 32.3 | 84.3 | 23.1 | 35.4 | 35.2 | 14.7 | 80.8 | 25.4 | 63.8 | 18.1 | 27.2 | 27.3 |
| | | 20 | 56.8 | 100.0 | 92.9 | 100.0 | 65.5 | 94.2 | 94.1 | 32.7 | 100.0 | 54.3 | 99.4 | 38.9 | 57.3 | 57.2 | 23.4 | 99.4 | 39.6 | 96.0 | 28.1 | 41.7 | 41.7 |

**Table 24.** Test power estimates when samples are taken from Beta (10,5) distributions

| | | | $\sigma_{11}^2:\sigma_{12}^2 \dots :\sigma_{ij}^2$ | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 1:1:...:1 | | | | | | | | 1:1:...:10 | | | | | | | | 1:1:...:20 | | | | | | | |
| rxc | δ | n | A | B | AxB | Cont. | A | B | AxB | A | B | AxB | Cont. | A | B | AxB | A | B | AxB | Cont. | A | B | AxB |
| 2x2 | 0.75 | 3 | 4.9 | 21.6 | 5.6 | 12.4 | 9.0 | 9.0 | 9.0 | 7.2 | 16.4 | 9.1 | 6.1 | 10.8 | 10.8 | 10.8 | 9.2 | 16.6 | 11.7 | 5.2 | 12.8 | 12.8 | 12.9 |
| | | 4 | 5.5 | 26.5 | 6.5 | 15.4 | 10.9 | 10.7 | 10.8 | 6.7 | 17.2 | 8.7 | 6.4 | 10.5 | 10.5 | 10.5 | 8.3 | 16.2 | 10.8 | 4.9 | 11.9 | 11.8 | 11.9 |
| | | 5 | 6.3 | 31.5 | 7.5 | 18.5 | 12.6 | 12.5 | 12.6 | 6.7 | 18.3 | 8.9 | 6.7 | 10.8 | 10.6 | 10.7 | 7.4 | 15.7 | 10.0 | 4.7 | 11.1 | 11.2 | 11.1 |
| | | 10 | 11.2 | 53.2 | 14.4 | 33.8 | 21.3 | 21.0 | 21.2 | 7.7 | 27.0 | 10.7 | 10.4 | 12.6 | 12.5 | 12.5 | 7.1 | 18.2 | 10.3 | 5.4 | 11.2 | 11.1 | 11.2 |
| | | 20 | 23.4 | 81.5 | 32.0 | 60.0 | 38.4 | 38.2 | 38.5 | 11.6 | 49.1 | 16.3 | 20.7 | 18.3 | 18.3 | 18.2 | 9.0 | 29.4 | 12.9 | 9.7 | 14.0 | 13.8 | 13.7 |
| | 1.5 | 3 | 13.3 | 52.7 | 16.2 | 34.1 | 21.6 | 21.5 | 21.4 | 10.3 | 31.8 | 12.9 | 15.7 | 14.7 | 14.7 | 14.7 | 11.2 | 26.3 | 13.8 | 11.6 | 14.7 | 14.8 | 14.9 |
| | | 4 | 17.7 | 67.4 | 22.7 | 46.2 | 28.9 | 28.6 | 28.9 | 11.2 | 39.3 | 14.3 | 19.3 | 16.3 | 16.3 | 16.2 | 10.7 | 29.2 | 13.7 | 12.5 | 14.8 | 15.0 | 14.8 |
| | | 5 | 22.4 | 78.0 | 29.5 | 56.5 | 35.6 | 35.6 | 35.6 | 12.3 | 47.1 | 16.2 | 23.4 | 18.1 | 17.9 | 18.1 | 10.7 | 32.8 | 14.0 | 14.2 | 15.1 | 15.1 | 15.1 |
| | | 10 | 47.3 | 97.7 | 63.4 | 87.7 | 63.9 | 64.1 | 64.1 | 20.7 | 81.0 | 27.7 | 45.9 | 29.5 | 29.2 | 29.4 | 14.3 | 58.1 | 19.5 | 25.6 | 20.5 | 20.4 | 20.4 |
| | | 20 | 82.1 | 100.0 | 95.0 | 99.5 | 91.3 | 91.2 | 91.4 | 39.2 | 99.3 | 50.1 | 82.0 | 50.7 | 50.9 | 50.8 | 25.1 | 93.7 | 33.3 | 54.7 | 33.9 | 33.8 | 33.7 |
| 3x3 | 0.75 | 3 | 4.2 | 33.5 | 4.7 | 17.8 | 7.6 | 7.5 | 8.2 | 7.0 | 26.4 | 10.2 | 9.4 | 10.0 | 10.1 | 12.4 | 9.7 | 24.4 | 14.2 | 6.7 | 12.6 | 12.7 | 15.8 |
| | | 4 | 4.5 | 41.0 | 5.5 | 22.9 | 8.5 | 8.5 | 9.7 | 6.9 | 30.4 | 10.2 | 11.3 | 10.1 | 10.0 | 12.6 | 8.7 | 25.8 | 13.4 | 7.4 | 11.7 | 11.6 | 15.1 |
| | | 5 | 5.0 | 47.7 | 6.2 | 27.8 | 9.6 | 9.5 | 11.4 | 6.7 | 34.8 | 10.6 | 13.5 | 10.0 | 10.0 | 12.9 | 8.6 | 27.9 | 13.3 | 8.1 | 11.4 | 11.4 | 14.8 |
| | | 10 | 7.9 | 72.6 | 11.4 | 50.3 | 15.0 | 15.1 | 19.7 | 8.2 | 55.9 | 13.4 | 26.1 | 12.2 | 12.0 | 15.8 | 8.5 | 42.6 | 14.2 | 14.4 | 11.7 | 11.8 | 15.7 |
| | | 20 | 15.6 | 93.8 | 27.2 | 80.2 | 26.9 | 27.0 | 38.6 | 12.3 | 84.2 | 20.2 | 54.1 | 17.4 | 17.4 | 23.0 | 10.8 | 71.3 | 17.7 | 33.3 | 14.4 | 14.4 | 19.2 |
| | 1.5 | 3 | 9.7 | 74.9 | 13.7 | 53.7 | 15.9 | 15.9 | 19.8 | 10.1 | 58.9 | 14.8 | 32.6 | 13.9 | 13.9 | 17.3 | 11.6 | 48.8 | 16.7 | 23.2 | 14.8 | 14.8 | 18.5 |
| | | 4 | 12.3 | 86.7 | 19.1 | 68.0 | 20.5 | 20.6 | 27.5 | 11.4 | 71.6 | 16.9 | 42.8 | 15.6 | 15.4 | 19.5 | 11.7 | 58.8 | 17.2 | 29.0 | 15.1 | 14.9 | 18.9 |
| | | 5 | 15.6 | 93.1 | 25.4 | 78.5 | 25.5 | 25.4 | 35.0 | 12.7 | 81.4 | 19.6 | 52.8 | 17.4 | 17.3 | 22.5 | 12.1 | 68.1 | 18.4 | 35.8 | 15.7 | 15.7 | 20.0 |
| | | 10 | 34.2 | 99.8 | 60.4 | 98.0 | 49.1 | 48.7 | 69.2 | 20.9 | 98.7 | 32.8 | 87.4 | 27.7 | 27.7 | 36.1 | 16.9 | 94.9 | 25.9 | 69.5 | 21.3 | 21.3 | 27.6 |
| | | 20 | 68.8 | 100.0 | 95.2 | 100.0 | 81.2 | 80.9 | 96.0 | 39.5 | 100.0 | 57.0 | 99.7 | 48.1 | 48.3 | 59.9 | 28.4 | 100.0 | 41.3 | 97.4 | 34.4 | 34.4 | 43.0 |
| 4x4 | 0.75 | 3 | 3.7 | 41.9 | 4.2 | 20.6 | 6.6 | 6.7 | 7.5 | 6.4 | 36.2 | 10.1 | 13.2 | 9.2 | 9.3 | 12.8 | 9.2 | 32.6 | 14.7 | 9.5 | 12.0 | 12.0 | 16.9 |
| | | 4 | 4.1 | 49.9 | 4.5 | 26.2 | 7.4 | 7.4 | 8.6 | 6.3 | 41.8 | 10.4 | 16.2 | 9.3 | 9.4 | 13.3 | 8.6 | 36.6 | 14.1 | 11.0 | 11.3 | 11.4 | 16.4 |
| | | 5 | 4.4 | 57.2 | 5.0 | 32.0 | 8.2 | 8.1 | 9.8 | 6.1 | 47.6 | 10.5 | 19.7 | 9.2 | 9.1 | 13.4 | 8.3 | 40.9 | 13.9 | 13.1 | 11.0 | 10.8 | 16.2 |
| | | 10 | 6.2 | 80.6 | 8.9 | 56.7 | 11.8 | 11.6 | 17.4 | 7.5 | 71.7 | 13.5 | 39.1 | 11.1 | 11.1 | 17.0 | 8.5 | 62.6 | 15.4 | 26.4 | 11.4 | 11.4 | 17.6 |
| | | 20 | 11.4 | 96.7 | 21.5 | 86.1 | 19.7 | 19.6 | 34.7 | 11.2 | 93.6 | 20.7 | 72.4 | 15.7 | 15.9 | 24.8 | 11.1 | 88.3 | 19.5 | 57.0 | 14.3 | 14.2 | 21.9 |
| | 1.5 | 3 | 7.8 | 84.1 | 11.2 | 62.1 | 12.5 | 12.4 | 17.3 | 9.2 | 75.3 | 14.8 | 45.8 | 12.7 | 12.6 | 17.8 | 11.2 | 66.9 | 17.5 | 35.5 | 14.2 | 14.1 | 19.9 |
| | | 4 | 9.6 | 92.8 | 15.5 | 75.9 | 15.7 | 15.6 | 24.3 | 10.1 | 86.2 | 17.0 | 59.4 | 14.0 | 14.1 | 20.6 | 11.4 | 78.3 | 18.4 | 46.1 | 14.5 | 14.4 | 20.8 |
| | | 5 | 11.8 | 96.6 | 20.6 | 85.2 | 19.1 | 19.1 | 31.5 | 11.6 | 92.6 | 19.6 | 70.9 | 15.8 | 15.7 | 23.5 | 11.9 | 86.7 | 19.7 | 56.3 | 15.1 | 15.1 | 22.2 |
| | | 10 | 24.9 | 100.0 | 52.9 | 99.2 | 36.8 | 36.9 | 66.0 | 19.0 | 99.8 | 33.3 | 96.7 | 25.0 | 25.2 | 38.2 | 16.5 | 99.4 | 27.6 | 90.6 | 20.5 | 20.5 | 30.2 |
| | | 20 | 54.1 | 100.0 | 93.1 | 100.0 | 67.3 | 67.4 | 96.1 | 36.0 | 100.0 | 57.7 | 100.0 | 43.8 | 43.7 | 62.3 | 28.0 | 100.0 | 43.9 | 99.9 | 33.2 | 33.3 | 46.7 |
| 2x4 | 0.75 | 3 | 4.8 | 22.9 | 5.7 | 17.1 | 7.1 | 8.2 | 8.4 | 6.6 | 19.0 | 9.9 | 8.8 | 8.7 | 11.7 | 11.8 | 8.5 | 19.5 | 13.2 | 6.4 | 10.6 | 14.6 | 14.7 |
| | | 4 | 5.3 | 29.6 | 6.6 | 21.9 | 8.1 | 9.7 | 9.9 | 6.2 | 21.2 | 9.9 | 10.2 | 8.5 | 12.0 | 11.9 | 7.5 | 19.8 | 12.6 | 6.6 | 9.6 | 14.2 | 14.0 |
| | | 5 | 5.9 | 36.1 | 7.5 | 26.7 | 8.9 | 11.3 | 11.3 | 6.2 | 23.6 | 10.3 | 12.1 | 8.6 | 12.1 | 12.2 | 7.2 | 20.5 | 12.4 | 7.2 | 9.2 | 13.9 | 13.9 |
| | | 10 | 9.0 | 62.3 | 13.6 | 48.4 | 13.3 | 19.2 | 19.6 | 7.2 | 39.0 | 12.8 | 23.4 | 9.9 | 15.0 | 15.0 | 6.9 | 27.4 | 13.1 | 12.4 | 9.1 | 14.5 | 14.6 |
| | | 20 | 15.8 | 90.0 | 29.3 | 78.3 | 22.0 | 36.8 | 36.9 | 10.4 | 69.0 | 19.0 | 49.1 | 13.8 | 21.7 | 21.6 | 8.6 | 48.8 | 16.4 | 28.8 | 11.2 | 18.2 | 17.9 |
| | 1.5 | 3 | 10.0 | 62.4 | 14.7 | 51.8 | 13.9 | 19.4 | 19.5 | 9.2 | 42.3 | 14.5 | 29.8 | 11.7 | 16.5 | 16.5 | 10.2 | 34.9 | 16.0 | 21.0 | 12.6 | 17.6 | 17.6 |
| | | 4 | 12.6 | 77.7 | 20.9 | 65.6 | 17.5 | 26.7 | 27.1 | 9.8 | 53.7 | 16.4 | 38.8 | 12.8 | 18.7 | 18.7 | 9.8 | 40.9 | 16.2 | 26.2 | 12.3 | 17.9 | 17.8 |
| | | 5 | 15.3 | 87.7 | 26.9 | 76.4 | 21.1 | 33.7 | 33.9 | 10.9 | 64.7 | 18.5 | 48.4 | 14.2 | 21.0 | 21.0 | 9.9 | 47.9 | 17.1 | 31.8 | 12.6 | 18.6 | 18.6 |
| | | 10 | 29.9 | 99.6 | 59.0 | 97.5 | 38.1 | 65.7 | 65.5 | 17.2 | 94.5 | 30.9 | 83.6 | 21.8 | 34.1 | 33.9 | 13.4 | 81.8 | 24.0 | 63.6 | 16.8 | 26.0 | 25.9 |
| | | 20 | 56.6 | 100.0 | 93.1 | 100.0 | 65.3 | 94.3 | 94.1 | 32.1 | 100.0 | 54.3 | 99.4 | 38.6 | 57.2 | 57.3 | 22.4 | 99.6 | 38.7 | 95.4 | 27.2 | 40.9 | 40.8 |

**Table 25.** Test power estimates when samples are taken from Beta (5,10) distributions

| | | | $\sigma_{11}^2:\sigma_{12}^2 \dots :\sigma_{ij}^2$ | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 1:1:...:1 | | | | | | | | 1:1:...:10 | | | | | | | | 1:1:...:20 | | | | | | | |
| rxc | δ | n | A | B | AxB | Cont. | A | B | AxB | A | B | AxB | Cont. | A | B | AxB | A | B | AxB | Cont. | A | B | AxB |
| 2x2 | 0.75 | 3 | 4.9 | 22.8 | 5.6 | 12.4 | 9.0 | 9.1 | 9.1 | 8.8 | 16.7 | 10.9 | 4.8 | 12.8 | 12.7 | 12.7 | 10.9 | 16.9 | 13.6 | 3.9 | 14.7 | 14.7 | 14.7 |
| | | 4 | 5.5 | 28.1 | 6.4 | 15.8 | 10.7 | 10.8 | 10.8 | 8.3 | 17.4 | 10.8 | 4.6 | 12.5 | 12.6 | 12.5 | 10.0 | 16.2 | 12.8 | 3.3 | 13.8 | 13.7 | 13.8 |
| | | 5 | 6.4 | 32.7 | 7.5 | 18.9 | 12.5 | 12.4 | 12.5 | 8.4 | 18.9 | 11.1 | 5.2 | 12.7 | 12.9 | 12.8 | 9.5 | 16.0 | 12.3 | 2.8 | 13.3 | 13.3 | 13.3 |
| | | 10 | 11.1 | 53.9 | 14.6 | 34.6 | 21.2 | 21.2 | 21.5 | 9.7 | 27.7 | 13.1 | 8.4 | 14.8 | 15.0 | 14.7 | 9.1 | 18.5 | 12.4 | 3.5 | 13.4 | 13.3 | 13.4 |
| | | 20 | 23.4 | 81.1 | 31.7 | 60.9 | 38.3 | 38.2 | 38.3 | 13.4 | 49.8 | 18.4 | 19.2 | 20.2 | 20.2 | 20.2 | 10.8 | 29.8 | 15.2 | 7.3 | 15.9 | 16.1 | 16.0 |
| | 1.5 | 3 | 13.2 | 53.9 | 16.2 | 35.4 | 21.5 | 21.6 | 21.2 | 13.2 | 32.5 | 16.4 | 13.2 | 18.1 | 18.2 | 18.2 | 13.9 | 26.8 | 17.0 | 8.9 | 18.0 | 18.1 | 18.0 |
| | | 4 | 17.6 | 67.4 | 22.7 | 47.4 | 28.8 | 28.8 | 29.0 | 14.2 | 40.0 | 18.0 | 16.4 | 19.8 | 19.9 | 19.7 | 13.8 | 29.4 | 17.2 | 9.4 | 18.4 | 18.0 | 18.0 |
| | | 5 | 22.4 | 77.5 | 29.5 | 57.7 | 35.6 | 35.8 | 35.6 | 15.6 | 47.8 | 20.0 | 20.4 | 21.9 | 21.7 | 21.7 | 14.1 | 33.4 | 18.2 | 10.5 | 18.9 | 18.9 | 19.0 |
| | | 10 | 47.0 | 97.2 | 63.3 | 87.4 | 63.8 | 64.1 | 64.2 | 23.7 | 80.8 | 30.9 | 45.3 | 32.2 | 32.0 | 32.2 | 17.8 | 58.3 | 23.2 | 21.9 | 23.8 | 23.9 | 23.8 |
| | | 20 | 82.2 | 100.0 | 94.9 | 99.3 | 91.4 | 91.4 | 91.3 | 40.8 | 99.1 | 50.9 | 83.9 | 51.3 | 51.3 | 51.3 | 27.7 | 93.5 | 35.4 | 55.0 | 35.8 | 36.0 | 35.7 |
| 3x3 | 0.75 | 3 | 4.1 | 36.1 | 5.0 | 18.3 | 7.6 | 7.6 | 8.3 | 8.0 | 28.0 | 11.9 | 8.2 | 11.4 | 11.5 | 14.4 | 11.2 | 25.3 | 15.9 | 5.4 | 14.2 | 14.3 | 17.7 |
| | | 4 | 4.5 | 42.8 | 5.4 | 23.6 | 8.7 | 8.6 | 9.6 | 7.9 | 31.7 | 12.0 | 10.0 | 11.5 | 11.6 | 14.5 | 10.5 | 26.8 | 15.4 | 5.7 | 13.6 | 13.6 | 17.1 |
| | | 5 | 5.0 | 48.9 | 6.0 | 28.5 | 9.8 | 9.5 | 11.1 | 8.1 | 35.9 | 12.8 | 12.1 | 11.8 | 11.7 | 15.3 | 10.2 | 29.3 | 15.4 | 6.5 | 13.3 | 13.5 | 16.8 |
| | | 10 | 7.9 | 72.5 | 11.6 | 51.4 | 15.0 | 15.1 | 19.9 | 9.7 | 56.8 | 15.4 | 25.5 | 14.0 | 13.9 | 18.1 | 10.1 | 43.7 | 16.2 | 12.6 | 13.3 | 13.6 | 17.7 |
| | | 20 | 15.9 | 93.0 | 27.2 | 80.2 | 27.2 | 26.9 | 38.7 | 13.6 | 84.0 | 22.0 | 55.2 | 18.9 | 19.0 | 25.0 | 12.4 | 71.6 | 20.0 | 32.9 | 16.2 | 16.4 | 21.5 |

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1.5 | 3 | 9.7 | 74.7 | 13.8 | 55.5 | 16.0 | 15.9 | 19.9 | 12.3 | 59.9 | 18.3 | 32.2 | 16.5 | 16.7 | 21.0 | 14.3 | 49.8 | 20.3 | 21.3 | 18.0 | 18.0 | 22.1 |
| | | 4 | 12.6 | 85.3 | 19.1 | 69.0 | 20.7 | 20.7 | 27.6 | 13.6 | 71.7 | 20.3 | 43.3 | 18.1 | 18.3 | 23.2 | 14.4 | 59.4 | 20.8 | 27.6 | 18.1 | 18.0 | 22.5 |
| | | 5 | 15.8 | 91.8 | 25.7 | 79.1 | 25.6 | 25.8 | 35.5 | 15.0 | 81.0 | 23.2 | 53.8 | 20.2 | 20.4 | 26.1 | 15.2 | 68.9 | 22.1 | 34.7 | 19.0 | 19.0 | 23.8 |
| | | 10 | 34.5 | 99.6 | 60.1 | 97.5 | 49.1 | 48.9 | 68.9 | 23.3 | 98.1 | 35.5 | 88.0 | 30.1 | 30.1 | 38.6 | 19.5 | 94.4 | 28.6 | 71.5 | 24.0 | 24.2 | 30.3 |
| | | 20 | 68.6 | 100.0 | 95.0 | 100.0 | 80.8 | 80.7 | 95.9 | 40.5 | 100.0 | 57.2 | 99.7 | 48.7 | 48.7 | 59.9 | 30.3 | 100.0 | 42.7 | 98.0 | 36.1 | 36.2 | 44.3 |
| 4x4 | 0.75 | 3 | 3.7 | 44.5 | 4.3 | 21.4 | 6.7 | 6.7 | 7.5 | 7.0 | 37.5 | 11.6 | 12.0 | 10.1 | 10.2 | 14.6 | 10.3 | 33.9 | 16.3 | 8.2 | 13.2 | 13.3 | 18.7 |
| | | 4 | 4.0 | 51.9 | 4.6 | 27.5 | 7.4 | 7.4 | 8.7 | 7.0 | 43.8 | 11.9 | 15.7 | 10.2 | 10.2 | 15.1 | 9.7 | 38.0 | 15.9 | 9.8 | 12.6 | 12.8 | 18.1 |
| | | 5 | 4.2 | 58.2 | 5.1 | 33.3 | 8.1 | 7.9 | 10.1 | 7.2 | 49.2 | 12.4 | 19.2 | 10.5 | 10.6 | 15.7 | 9.6 | 42.7 | 16.1 | 11.9 | 12.5 | 12.6 | 18.3 |
| | | 10 | 6.2 | 79.8 | 9.0 | 57.7 | 11.8 | 11.7 | 17.3 | 8.5 | 72.0 | 15.3 | 40.2 | 12.4 | 12.2 | 18.9 | 10.0 | 63.4 | 17.2 | 26.1 | 13.0 | 13.0 | 19.4 |
| | | 20 | 11.6 | 96.1 | 21.6 | 86.1 | 20.0 | 20.1 | 34.7 | 12.2 | 92.9 | 22.2 | 73.2 | 16.8 | 16.9 | 26.4 | 12.5 | 88.2 | 21.2 | 57.8 | 15.9 | 15.9 | 23.6 |
| | 1.5 | 3 | 7.7 | 82.8 | 11.3 | 63.8 | 12.6 | 12.5 | 17.6 | 10.7 | 75.3 | 17.7 | 47.2 | 14.5 | 14.6 | 21.1 | 13.5 | 67.8 | 20.9 | 35.9 | 16.8 | 16.9 | 23.3 |
| | | 4 | 9.6 | 90.9 | 15.4 | 76.4 | 15.8 | 15.8 | 24.3 | 12.0 | 85.2 | 20.3 | 61.0 | 16.3 | 16.2 | 23.9 | 13.9 | 78.1 | 22.0 | 46.6 | 17.4 | 17.3 | 24.6 |
| | | 5 | 11.6 | 95.4 | 20.7 | 85.4 | 19.1 | 19.4 | 31.5 | 13.4 | 91.5 | 22.7 | 71.9 | 17.9 | 17.9 | 26.7 | 14.5 | 86.2 | 23.2 | 57.8 | 17.9 | 17.9 | 25.6 |
| | | 10 | 25.0 | 99.9 | 53.1 | 98.9 | 37.1 | 36.6 | 66.2 | 21.1 | 99.6 | 35.8 | 96.4 | 27.1 | 27.1 | 40.3 | 18.8 | 98.9 | 30.5 | 91.1 | 23.0 | 23.1 | 33.1 |
| | | 20 | 54.1 | 100.0 | 92.8 | 100.0 | 67.3 | 67.5 | 95.8 | 37.1 | 100.0 | 57.5 | 100.0 | 44.6 | 44.3 | 61.8 | 29.6 | 100.0 | 44.9 | 99.9 | 34.6 | 34.8 | 47.6 |
| 2x4 | 0.75 | 3 | 4.8 | 24.4 | 5.5 | 17.6 | 7.2 | 8.2 | 8.1 | 7.4 | 20.4 | 11.7 | 7.4 | 9.8 | 13.8 | 13.8 | 9.6 | 20.9 | 15.4 | 4.9 | 12.0 | 16.9 | 17.0 |
| | | 4 | 5.3 | 30.8 | 6.5 | 22.6 | 8.0 | 9.6 | 9.7 | 7.4 | 22.8 | 11.9 | 8.9 | 9.8 | 14.1 | 14.1 | 8.8 | 20.9 | 14.5 | 5.0 | 11.1 | 16.0 | 16.1 |
| | | 5 | 5.8 | 36.9 | 7.5 | 27.3 | 8.9 | 11.1 | 11.2 | 7.3 | 25.0 | 12.2 | 10.6 | 9.8 | 14.3 | 14.2 | 8.5 | 21.6 | 14.4 | 5.5 | 10.7 | 15.9 | 15.8 |
| | | 10 | 9.0 | 62.5 | 13.7 | 49.3 | 13.3 | 19.3 | 19.4 | 8.4 | 40.0 | 14.8 | 22.7 | 11.2 | 17.0 | 17.1 | 8.3 | 28.2 | 15.0 | 10.6 | 10.7 | 16.3 | 16.4 |
| | | 20 | 15.8 | 89.6 | 29.1 | 78.8 | 21.9 | 37.0 | 36.9 | 11.7 | 68.5 | 20.9 | 49.7 | 15.2 | 23.4 | 23.5 | 10.0 | 48.7 | 18.1 | 27.4 | 12.7 | 19.9 | 19.7 |
| | 1.5 | 3 | 10.1 | 62.8 | 14.8 | 53.3 | 14.0 | 19.8 | 19.5 | 11.0 | 43.9 | 17.8 | 28.9 | 13.9 | 20.1 | 20.1 | 12.2 | 36.2 | 19.0 | 18.6 | 14.8 | 20.7 | 20.8 |
| | | 4 | 12.6 | 77.3 | 20.6 | 66.8 | 17.5 | 26.9 | 26.7 | 12.1 | 54.6 | 19.9 | 38.8 | 15.3 | 22.4 | 22.3 | 12.3 | 42.1 | 19.7 | 23.9 | 15.1 | 21.5 | 21.4 |
| | | 5 | 15.4 | 86.7 | 27.0 | 76.9 | 21.0 | 34.3 | 34.1 | 12.9 | 64.5 | 21.9 | 48.9 | 16.4 | 24.4 | 24.4 | 12.5 | 48.9 | 20.7 | 30.2 | 15.3 | 22.5 | 22.5 |
| | | 10 | 29.7 | 99.4 | 59.2 | 97.0 | 38.0 | 65.6 | 65.7 | 19.5 | 93.5 | 33.5 | 84.6 | 24.1 | 36.5 | 36.4 | 16.1 | 80.4 | 27.0 | 64.8 | 19.5 | 28.9 | 28.9 |
| | | 20 | 56.7 | 100.0 | 93.1 | 100.0 | 65.6 | 93.9 | 94.3 | 33.6 | 99.9 | 54.7 | 99.4 | 39.7 | 57.4 | 57.4 | 24.6 | 99.2 | 40.3 | 96.4 | 29.3 | 42.2 | 42.2 |

**Table 26.** Test power estimates when samples are taken from Chi-Sq (3) distributions

| | | | $\sigma_{11}^2 : \sigma_{12}^2 \ldots : \sigma_{ij}^2$ | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 1:1:…:1 | | | | | | | 1:1:…:10 | | | | | | | 1:1:…:20 | | | | | | |
| rxc | δ | n | A | B | AxB | Cont. | A | B | AxB | A | B | AxB | Cont. | A | B | AxB | A | B | AxB | Cont. | A | B | AxB |
| 2x2 | 0.75 | 3 | 5.5 | 21.8 | 6.3 | 13.9 | 9.8 | 9.6 | 9.8 | 5.4 | 17.5 | 6.5 | 9.8 | 8.4 | 8.3 | 8.3 | 8.1 | 19.1 | 10.0 | 9.5 | 11.4 | 11.4 | 11.4 |
| | | 4 | 6.2 | 27.7 | 7.6 | 17.4 | 11.6 | 11.8 | 11.9 | 4.8 | 18.8 | 6.2 | 10.8 | 8.1 | 8.0 | 8.1 | 7.0 | 19.0 | 9.1 | 9.8 | 10.4 | 10.3 | 10.3 |
| | | 5 | 7.2 | 33.0 | 8.8 | 20.4 | 13.6 | 13.7 | 13.7 | 4.6 | 20.4 | 6.3 | 11.6 | 8.0 | 8.1 | 8.2 | 6.3 | 18.9 | 8.6 | 9.7 | 9.8 | 9.8 | 9.8 |
| | | 10 | 12.4 | 54.8 | 16.0 | 34.5 | 22.7 | 22.6 | 22.6 | 5.4 | 29.1 | 7.8 | 16.0 | 9.9 | 9.8 | 9.8 | 5.3 | 21.2 | 7.9 | 11.2 | 9.1 | 9.1 | 9.1 |
| | | 20 | 24.8 | 82.3 | 33.6 | 60.2 | 39.6 | 39.1 | 39.6 | 9.2 | 50.7 | 13.4 | 26.0 | 15.8 | 15.6 | 15.6 | 6.5 | 31.7 | 10.2 | 15.7 | 11.4 | 11.4 | 11.4 |
| | 1.5 | 3 | 16.7 | 57.5 | 20.6 | 38.3 | 25.7 | 25.7 | 25.7 | 6.8 | 36.1 | 8.6 | 23.6 | 10.3 | 10.3 | 10.4 | 7.7 | 30.7 | 9.6 | 19.7 | 10.8 | 10.8 | 10.9 |
| | | 4 | 21.6 | 71.1 | 27.4 | 49.5 | 32.9 | 33.2 | 32.9 | 7.5 | 44.2 | 9.9 | 28.0 | 12.0 | 11.7 | 11.9 | 7.0 | 34.2 | 9.3 | 22.1 | 10.7 | 10.5 | 10.6 |
| | | 5 | 26.3 | 80.0 | 34.3 | 58.4 | 39.8 | 39.4 | 39.4 | 8.7 | 51.7 | 11.8 | 32.0 | 14.1 | 14.1 | 14.0 | 7.0 | 37.9 | 9.7 | 23.7 | 11.0 | 10.9 | 11.0 |
| | | 10 | 49.8 | 97.6 | 65.2 | 87.6 | 65.6 | 66.1 | 65.5 | 17.2 | 81.6 | 24.1 | 50.8 | 26.7 | 26.7 | 26.6 | 10.3 | 61.3 | 15.2 | 34.7 | 16.5 | 16.5 | 16.4 |
| | | 20 | 81.9 | 100.0 | 94.1 | 99.5 | 90.8 | 90.7 | 91.0 | 38.2 | 98.9 | 50.5 | 79.5 | 51.2 | 51.5 | 51.4 | 21.4 | 92.7 | 30.4 | 56.9 | 31.2 | 31.3 | 31.3 |
| 3x3 | 0.75 | 3 | 4.4 | 32.8 | 5.3 | 18.7 | 7.4 | 7.4 | 8.1 | 5.1 | 26.3 | 7.4 | 12.6 | 7.5 | 7.6 | 9.1 | 7.8 | 25.5 | 11.6 | 10.7 | 10.5 | 10.6 | 13.3 |
| | | 4 | 4.8 | 41.2 | 6.0 | 23.5 | 8.8 | 8.6 | 9.7 | 4.9 | 30.7 | 7.4 | 14.8 | 7.5 | 7.6 | 9.4 | 7.0 | 27.7 | 10.9 | 11.7 | 9.8 | 9.8 | 12.6 |
| | | 5 | 5.3 | 47.7 | 6.9 | 27.7 | 9.8 | 9.8 | 11.4 | 5.1 | 35.4 | 7.9 | 17.0 | 7.9 | 7.8 | 9.9 | 6.6 | 30.1 | 10.8 | 13.1 | 9.4 | 9.4 | 12.4 |
| | | 10 | 8.5 | 74.1 | 12.6 | 49.4 | 15.5 | 15.4 | 20.4 | 6.3 | 56.1 | 10.3 | 28.8 | 9.8 | 9.7 | 12.7 | 6.4 | 43.9 | 11.4 | 20.0 | 9.3 | 9.2 | 12.9 |
| | | 20 | 16.0 | 95.0 | 28.0 | 80.2 | 27.1 | 27.1 | 39.1 | 10.1 | 84.8 | 17.2 | 54.3 | 15.0 | 15.1 | 20.4 | 8.5 | 71.5 | 14.9 | 37.1 | 12.0 | 11.9 | 16.3 |
| | 1.5 | 3 | 10.9 | 78.2 | 16.1 | 54.2 | 16.8 | 17.0 | 21.7 | 7.2 | 62.1 | 10.4 | 37.5 | 10.1 | 10.1 | 12.4 | 8.0 | 52.6 | 11.6 | 30.7 | 10.6 | 10.6 | 13.2 |
| | | 4 | 13.8 | 89.0 | 21.9 | 68.1 | 21.9 | 21.9 | 29.6 | 8.1 | 74.3 | 12.5 | 46.9 | 11.7 | 11.7 | 14.8 | 7.9 | 62.3 | 12.3 | 37.1 | 10.8 | 10.7 | 13.9 |
| | | 5 | 17.1 | 94.7 | 28.5 | 78.6 | 26.7 | 27.2 | 37.5 | 9.5 | 83.4 | 15.3 | 55.9 | 13.8 | 13.9 | 18.1 | 8.2 | 70.9 | 13.6 | 43.0 | 11.4 | 11.5 | 15.3 |
| | | 10 | 35.4 | 99.9 | 61.5 | 98.3 | 49.9 | 49.6 | 69.9 | 18.0 | 98.9 | 29.9 | 85.6 | 25.0 | 24.9 | 33.8 | 12.9 | 95.0 | 21.6 | 69.8 | 17.4 | 17.3 | 23.5 |
| | | 20 | 69.2 | 100.0 | 94.8 | 100.0 | 81.5 | 81.4 | 95.9 | 38.2 | 100.0 | 57.4 | 99.4 | 48.0 | 47.8 | 61.1 | 25.4 | 100.0 | 39.5 | 95.1 | 31.9 | 31.8 | 41.7 |
| 4x4 | 0.75 | 3 | 4.0 | 40.7 | 4.9 | 20.5 | 6.5 | 6.6 | 7.5 | 5.0 | 34.6 | 7.6 | 15.2 | 7.1 | 7.1 | 9.6 | 7.3 | 32.3 | 11.6 | 12.5 | 9.8 | 9.6 | 13.8 |
| | | 4 | 4.3 | 49.2 | 5.3 | 25.7 | 7.4 | 7.4 | 8.8 | 4.7 | 40.8 | 7.7 | 18.0 | 7.1 | 7.2 | 10.0 | 6.6 | 36.5 | 11.4 | 14.4 | 9.0 | 9.1 | 13.7 |
| | | 5 | 4.5 | 56.8 | 5.9 | 30.7 | 8.0 | 8.0 | 10.1 | 5.0 | 47.4 | 8.1 | 21.3 | 7.5 | 7.5 | 10.6 | 6.4 | 41.5 | 11.5 | 16.9 | 8.9 | 8.9 | 13.7 |
| | | 10 | 6.5 | 81.9 | 9.9 | 54.8 | 11.6 | 11.7 | 17.5 | 6.1 | 72.2 | 11.1 | 39.2 | 9.3 | 9.2 | 14.2 | 6.6 | 62.9 | 12.5 | 29.3 | 9.2 | 9.2 | 14.7 |
| | | 20 | 11.7 | 97.7 | 22.4 | 86.4 | 19.9 | 19.9 | 35.1 | 9.5 | 94.7 | 18.1 | 71.6 | 13.8 | 13.9 | 22.1 | 8.9 | 89.5 | 17.0 | 56.9 | 12.0 | 12.0 | 19.4 |
| | 1.5 | 3 | 8.4 | 87.3 | 12.7 | 61.0 | 12.8 | 12.7 | 18.1 | 7.0 | 78.0 | 11.0 | 47.4 | 9.6 | 9.5 | 13.3 | 7.7 | 69.7 | 12.0 | 39.6 | 10.0 | 9.9 | 14.0 |
| | | 4 | 10.4 | 94.9 | 17.4 | 75.5 | 16.2 | 16.0 | 25.3 | 7.9 | 88.4 | 13.3 | 60.0 | 11.0 | 11.1 | 16.3 | 7.9 | 80.3 | 13.3 | 49.6 | 10.5 | 10.5 | 15.4 |
| | | 5 | 12.6 | 98.2 | 22.8 | 85.8 | 19.5 | 19.8 | 32.7 | 9.2 | 94.4 | 16.3 | 70.5 | 13.0 | 12.9 | 19.8 | 8.4 | 88.3 | 14.9 | 58.9 | 11.3 | 11.1 | 17.2 |
| | | 10 | 25.5 | 100.0 | 53.7 | 99.6 | 37.1 | 37.2 | 66.3 | 16.9 | 99.9 | 31.1 | 96.2 | 22.9 | 23.0 | 36.5 | 13.2 | 99.5 | 24.0 | 88.8 | 17.2 | 17.4 | 26.8 |
| | | 20 | 54.3 | 100.0 | 92.8 | 100.0 | 67.5 | 67.7 | 96.0 | 34.5 | 100.0 | 57.8 | 100.0 | 42.9 | 42.9 | 63.5 | 25.4 | 100.0 | 42.2 | 99.8 | 31.0 | 30.8 | 45.7 |
| 2x4 | 0.75 | 3 | 5.1 | 22.9 | 5.8 | 18.2 | 7.4 | 8.0 | 8.2 | 4.9 | 18.2 | 7.2 | 12.0 | 6.8 | 8.9 | 8.8 | 7.0 | 20.1 | 11.0 | 10.5 | 8.9 | 12.7 | 12.6 |
| | | 4 | 5.6 | 30.0 | 6.7 | 22.7 | 8.3 | 9.9 | 9.8 | 4.8 | 21.0 | 7.2 | 14.0 | 6.7 | 8.9 | 9.0 | 6.0 | 20.6 | 10.4 | 11.5 | 8.1 | 11.9 | 11.9 |
| | | 5 | 6.2 | 36.3 | 7.9 | 27.0 | 9.2 | 11.4 | 11.6 | 4.7 | 24.1 | 7.4 | 16.3 | 6.7 | 9.2 | 9.2 | 5.7 | 21.4 | 10.0 | 12.3 | 7.7 | 11.6 | 11.5 |
| | | 10 | 9.3 | 63.2 | 14.4 | 47.6 | 13.5 | 19.7 | 20.0 | 5.6 | 39.4 | 10.0 | 26.7 | 8.1 | 12.0 | 12.1 | 5.3 | 28.7 | 10.3 | 18.2 | 7.4 | 12.0 | 11.8 |

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 20 | 16.2 | 90.7 | 30.0 | ***78.4*** | ***22.5*** | ***37.3*** | ***37.7*** | 8.6 | 70.1 | 16.3 | ***50.2*** | ***12.0*** | ***18.8*** | ***19.0*** | 6.5 | 50.0 | 13.4 | ***33.1*** | ***9.1*** | ***15.2*** | ***15.1*** |
| | 1.5 | 3 | 11.3 | 65.5 | 17.0 | ***52.7*** | ***15.3*** | ***21.1*** | ***21.7*** | 6.7 | 44.4 | 9.9 | ***35.6*** | ***8.9*** | ***11.7*** | ***11.7*** | 6.9 | 36.9 | 10.9 | ***29.0*** | ***8.8*** | ***12.4*** | ***12.4*** |
| | | 4 | 14.4 | 80.0 | 23.6 | ***66.0*** | ***19.2*** | ***28.8*** | ***29.4*** | 7.3 | 57.1 | 11.8 | ***44.2*** | ***9.9*** | ***13.8*** | ***14.0*** | 6.7 | 44.3 | 11.3 | ***34.4*** | ***8.8*** | ***12.8*** | ***12.9*** |
| | | 5 | 16.8 | 88.7 | 30.0 | ***76.4*** | ***22.6*** | ***35.9*** | ***36.7*** | 8.3 | 68.1 | 14.2 | ***52.1*** | ***11.4*** | ***16.7*** | ***16.8*** | 6.8 | 52.0 | 12.1 | ***39.8*** | ***9.2*** | ***13.8*** | ***13.8*** |
| | | 10 | 31.3 | 99.6 | 60.4 | ***97.7*** | ***39.4*** | ***66.6*** | ***66.6*** | 15.3 | 95.6 | 28.6 | ***81.8*** | ***20.1*** | ***31.8*** | ***32.1*** | 9.8 | 84.4 | 19.3 | ***65.0*** | ***13.1*** | ***21.6*** | ***21.4*** |
| | | 20 | 57.3 | 100.0 | 92.5 | ***100.0*** | ***65.8*** | ***94.3*** | ***93.8*** | 30.8 | 100.0 | 54.8 | ***98.8*** | ***37.8*** | ***58.0*** | ***58.4*** | 19.3 | 99.6 | 36.5 | ***92.5*** | ***24.5*** | ***38.6*** | ***39.1*** |