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BOOTSTRAP AND JACKKNIFE RESAMPLING ALGORITHMS FOR ESTIMATION OF REGRESSION PARAMETERS

Suat SAHINLER[1]
Biometry and Genetics Unit, Department of Animal Science,
Agriculture Faculty, University of Mustafa Kemal, 31100, Hatay, Turkey

E-mail: sahinler@mku.edu.tr

Dervis TOPUZ
Vocational School of Ulukisla, University of Nigde, Nigde, Turkey

E-mail: dervis_topuz@hotmail.com

Abstract: *In this paper, the hierarchical ways for building a regression model by using bootstrap and jackknife resampling methods were presented. Bootstrap approaches based on the observations and errors resampling, and jackknife approaches based on the delete-one and delete-d observations were considered. And also we consider estimating bootstrap and jackknife bias, standard errors and confidence intervals of the regression coefficients, and comparing with the concerning estimates of ordinary least squares. Obtaining of the estimates was presented with an illustrative real numerical example. The jackknife bias, the standard errors and confidence intervals of regression coefficients are substantially larger than the bootstrap and estimated asymptotic OLS standard errors. The jackknife percentile intervals also are larger than to the bootstrap percentile intervals of the regression coefficients.*

Key words: *bootstrap; jackknife; resampling; regression*

Introduction

Regression analysis is a statistical analysis technique that characterizes the relationship between two or more variables for prediction and estimation by a mathematical model called regression model. Finding estimates of bias and variance of the estimator $\hat{\beta}$ in estimation β and constructing confidence intervals for β and prediction intervals for a future observation with explanatory variables x_j are also interested in. Let the linear regression model be $y = X\beta + \varepsilon$ with the variance $\mathrm{var}(y) = \sigma^2$, where $y=(y_1,y_2,...y_n)`$ denotes the nx1 vector of the response, $X=(x_1,x_2,...x_n)`$ is matrix of regressors with nxp dimension including intercept, p is the number of parameters, ε_i is an nx1 vector of uncorrelated error terms of zero mean and identical variance σ^2 (Fox,1997; Sahinler and Bek, 2006). Then the least squares estimator $\hat{\beta} = (X^{/} X)^{-1} X^{/} Y$ has variance-covariance matrix $Var(\hat{\beta}) = \sigma^2 (X^{/} X)^{-1}$ and 100(1-α) % confidence intervals $\hat{\beta}_j \pm t_{n-p,\alpha/2} * S_e(\hat{\beta}_j)$. Traditional approaches, like ordinary least squares, rely on some major modelling assumptions strongly. Although they are provided, the conclusions are based on asymtotical or approximate properties frequently. The reliability ot the satatistical analysis depends therefore on the validity of these assumptions and on the sample size. There are several useful methods for diagnosing and treating violations of the regression assumptions. Robust estimation strategies and residual diagnostics have improved the usefulness of these tecniques (Sahinler, 2000). However, they may not be provided these assumptions by using these methods.

The observed data was considered as a representative picture of the entire population in resampling methods. Hence, the main idea to make statistical inference based on an artificial resample, which is drawn from the full sample (Friedl and Stampfer, 2002b). The ordinary sampling techniques use some assumptions related to the form of the estimator distribution, but resampling methods do not need these assumptions because the sample is thought as population. The bootstrap and jackknife are nonparametric and specific resampling techniques that purpose of deriving estimates of standard errors and confidence intervals of a population parameter like a mean, median, proportion, odds ratio, correlation coefficient or regression coefficient calculations without making distributional assumptions when those assumptions are in doubt, or where parametric inference is impossible or requires very complicated formulas for the calculation of standard errors (Efron, 1982).

This study focuses on illustration and aplication of resampling techniques in regression analysis. Some hyerarchical algoritms of concerning techniques in regression analysis are demonstrated. The basics of the bootstrap and jackknife resampling techniques and their applications to the real numerical example that can be described by linear regression model were discussed and compared the results with ordinary least squares regression results.

Materials and Methods

Material. The aim of the following study is to illustrate the bootstrap and jackknife regression parameter estimation as the methodology in method. The real data produced in the fisheries study in Mustafa Kemal University (Turkey) was used as material. Amongst others, the Total Length of fish (TL) and Otolith Length (OL) were considered as independent variables in order to explain the variation in Fish Age (FA) of n=100 fish related to a fish species (Can and Sahinler, 2005). The statistical packages S-PLUS FOR WINDOWS was used for the statistical analysis of these data.

Method. To describe the resampling methods we start with an n sized sample $w_i = (Y_i, X_{ji})'$ and assume that w_is are drawn independently and identically from a distribution of F, where $Y_i = (y_1, y_2, ..., y_n)'$ contains the responses, $X_{ji} = (x_{j1}, x_{j2}, x_{j3} ..., x_{jn})'$ is a matrix of dimension n x k, where j=1,2,...k, i=1,2,3,...,n.

Bootstrapping Regression Algoritm. Here, two approachs for bootstrapping regression methods were given. The coise of either methods depends upon the regressors are fixed or random. If the regressors are fixed, the bootstrap uses resampling of the error term. If the regressors are random, the bootstrap uses resampling of observation sets w_i (Stine, 1990; Shao, 1996).

Bootstrap Based On The Resampling Observations. This approach is usually applied when the regression models built from data have regressors that are as random as the response. Let the (k+1)x1 vector $w_i = (y_i, x_{ji}')'$ denote the values associated with *i*th observation. In this case, the set of observations are the vectors ($w_1, w_2, ..., w_n$). The bootstrap procedure based on the resampling observations is as follows.

1$^{(o)}$. Draw a n sized bootstrap sample ($w_1^{(b)}, w_2^{(b)}, ..., w_n^{(b)}$) with replacement from the observations giving 1/n probability each w_i values and label the elements of each vector $w_i^{(b)} = (y_i^{(b)}, x_{ji}^{(b)})'$, where j=1,2,...k, i=1,2,...n. From these form the vector $Y_i^{(b)} = (y_1^{(b)}, y_2^{(b)}, ..., y_n^{(b)})'$ and the matrix $X_{ji}^{(b)} = (x_{j1}^{(b)}, x_{j2}^{(b)}, ..., x_{jn}^{(b)})'$

2$^{(o)}$. Calculate the OLS coefficients from the bootstrap sample:

$$\hat{\beta}^{(b1)} = (X^{(b)'} X^{(b)})^{-1} X^{(b)'} Y^{(b)} \quad (1)$$

3$^{(o)}$. Repeat steps 1 and 2 for r=1,2,...,B, where B is the number of repetition.

4$^{(o)}$. Obtain the probability distribution ($F_{(\hat{\beta}^{(b)})}$) of bootstrap estimates $\hat{\beta}^{(b1)}, \hat{\beta}^{(b2)}, ..., \hat{\beta}^{(bB)}$ and use the ($F_{(\hat{\beta}^{(b)})}$) to estimate regression coefficients, variances and confidence intervals as follows. The bootstrap estimate of regression coefficient is the mean of the distribution $F_{(\hat{\beta}^{(b)})}$ (Fox,1997),

$$\hat{\beta}^{(b)} = \sum_{b=1}^{B} \hat{\beta}^{(br)} / B = \overline{\hat{\beta}}^{(br)} \quad (2)$$

5$^{(o)}$. Thus, the bootstrap regression equation is

$$\hat{Y} = X\hat{\beta}^{(b)} + \varepsilon \quad (3)$$

where $\hat{\beta}^{(b)}$ is unbiased estimator of β (Shao,1995).

An illustrative example that presents how the regression parameters are estimated from the bootstrap based on the the resampling observations was given in Table 1.

Bootstrap Based On The Resampling Errors. If the regressors are fixed, as in desing experiment, then the bootstrap resampling must preserve that structure. The bootstrap procedure based on the resampling errors as follows.

1$^{(e)}$. Fit the least squares regression equation for full sample.

2$^{(e)}$. Calculate the e_i values ($e_i = Y_i - \hat{Y}_i$).

3$^{(e)}$. Draw a n sized bootstrap random sample with replacement ($e_1^{(b)}, e_2^{(b)}, ..., e_n^{(b)}$) from the e_i values calculated in step 2$^{(e)}$ giving 1/n probability each e_i values(Stine, 1985; 1990; Wu,1986)

4(e). Compute the bootstrap Y values by adding resampled residuals onto the ordinary least squares regression fit, holding the regression desing fixed(Liu,1988; Leger et al,1992):

$$Y^{(b)} = X\hat{\beta} + e^{(b)} \quad (4)$$

5(e). Obtain least squares estimates from the 1th bootstrap sample:

$$\hat{\beta}^{(b1)} = (X^{/}\ X)^{-1}\ X^{/}\ Y^{(b)} \quad \text{(we need Y*) or} \quad (5)$$

$$= \hat{\beta} + (X^{/}\ X)^{-1}\ X^{/} e^{(b)} \quad \text{(we don not need Y*)} \quad (6)$$

6(e). Repeat steps 3(e),4(e) and 5(e) for r=1,2,...,B, and proceed as in resampling with random regressors 4(o) and 5(o).

The bootstrap bias, variance, confidence and percentile interval. The bootstrap bias equals,

$$bi\hat{a}s_b = \hat{\beta}^{(b)} - \hat{\beta} \quad (7)$$

(Further discussion are described in Efron and Tibshirani, 1993). The bootstrap variance from the distribution F$(\hat{\beta}^{(b)})$ are calculated by (Liu, 1988; Stine 1990)

$$\operatorname{var}(\hat{\beta}^{(b)}) = \sum_{b=1}^{B}\left[\left(\hat{\beta}^{(br)} - \hat{\beta}^{(b)}\right)\left(\hat{\beta}^{(br)} - \hat{\beta}^{(b)}\right)'\right]/(B-1), \quad r=1,2,...,B \quad (8)$$

The bootstrap confidence interval by normal approach is obtained by

$$\hat{\beta}^{(b)} - t_{n-p,\alpha/2} * S_e(\hat{\beta}^{(b)}) < \beta < \hat{\beta}^{(b)} + t_{n-p,\alpha/2} * S_e(\hat{\beta}^{(b)}) \quad (9)$$

where $t_{n-p,\ /2}$ is the critical value of t with probability α/2 the right for n-p degrees of freedom; and Se($\hat{\beta}^{(b)}$) is the standard error of the $\hat{\beta}^{(b)}$. If sample size is n ≥30, then Z-distribution values are used instead of t in estimation of confidence intervals (Diciccio and Tibshirani, 1987).

A nonparametric confidence interval named percentile Interval can be constructed from the quantiles of the bootstrap sampling distribution of $\hat{\beta}^{(b)}$. The (α/2)% and (1-α/2)% percentile interval is

$$\hat{\beta}^{(br)}{}_{(lower)} < \beta < \hat{\beta}^{(br)}{}_{(upper)} \quad (10)$$

where $\hat{\beta}^{(br)}$ is the ordered bootstrap estimates of regression coefficient from Equation 2 or 5, lower=(α/2)B, and upper = (1-α/2)B.

Jackknifing Regression Algoritm. Here, two algoritm for Jackknifing regression models based on the resampling observations were given. These approachs are usually applied when the regression models built from data have fixed explanatory variables. There are two cases of jackknife resampling. First of them is based on the deleting single case from the original sample (delete one jackknife), and second is based on the deleting multiple case from the original sample (delete d jackknife) sequentially (Efron and Gong, 1983; Wu, 1986; Shao and Tu, 1995). Let the px1 vector $w_i = (y_i, x_{ji}')'$, (i=1,2,...n) denote the values associated with *i*th observation. In this case, the set of observations are the vectors ($w_1, w_2, ..., w_n$).

Steps of The Algoritms for Delete-One Jackknife Regression. The jackknife procedure based on delete-one (do) is as follows.

$1^{(do)}$. Draw n sized sample from population randomly and label the elements of the vector $w_i = (Y_i\ , X_{ji})'$ as the vector $Y_i = (y_1, y_2, ..., y_n)'$ and the matrix $X_{ji} = (x_{j1}, x_{j2}, x_{j3} ..., x_{jn})'$ where j=1,2,...k, i=1,2,3,...,n.

$2^{(do)}$. Omit first row of the vector $w_i = (Y_i\ , X_{ji})'$ and label remaining n-1 sized observation sets $Y_i^{(J)} = (y_2^{(J)}, ..., y_n^{(J)})'$ and $X_{ji}^{(J)} = (x_{j2}^{(J)}, x_{j3}^{(J)} ..., x_{jn}^{(J)})'$ as delete-*one* Jackknife sample ($w_1^{(J)}$) and estimate the OLS regression coefficients $\hat{\beta}^{(J_1)}$ from ($w_1^{(J)}$). Then, omit second row of the vector $w_i = (Y_i\ , X_{ji})'$ and label remaining n-1 sized observation sets $Y_i^{(J)} = (y_1^{(J)}, y_3^{(J)} ..., y_n^{(J)})'$ and $X_{ji}^{(J)} = (x_{j1}^{(J)}, x_{j3}^{(J)}, ..., x_{jn}^{(J)})'$ as $w_2^{(J)}$ and estimate the OLS regression coefficients $\hat{\beta}^{(J_2)}$. Similarly, omit each one of the n observation sets and estimate the regression coefficients as $\hat{\beta}^{(J_i)}$ alternately, where $\hat{\beta}^{(J_i)}$ is Jackknife regression coefficient vector estimated after deleting of i^th observation set from w_i.

$3^{(do)}$. Obtain the probability distribution $F_{(\hat{\beta}^{(J)})}$ of Jackknife estimates $\hat{\beta}^{(J_1)}, \hat{\beta}^{(J_2)}, ..., \hat{\beta}^{(J_n)}$

$4^{(do)}$. Calculate the jackknife regression coefficient estimate which is the mean of the $F_{(\hat{\beta}^{(J)})}$ distribution (Fox,1997) as;

$$\hat{\beta}^{(J)} = \sum_{i=1}^{n} \hat{\beta}^{(J_i)} / n = \overline{\hat{\beta}}^{(J_i)} \tag{11}$$

$5^{(do)}$. Thus, the delete-*one* Jackknife regression equation is

$$\hat{Y} = X\hat{\beta}^{(J)} + \varepsilon \tag{12}$$

An illustrative study which shows how the delete-*one* jackknife regression parameters are estimated was given in Table 2.

Steps of The Algoritms for Delete-*d* Jackknife Regression. The jackknife procedure based on delete-*d* (dd) is as follows.

$1^{(dd)}$. Draw n sized sample ($w_1, w_2, ..., w_n$) from population randomly and devide the sample into s independent groups of which size is d.

$2^{(dd)}$. Omit first d observation set from full sample at a time and estimate the OLS coefficients $\hat{\beta}^{(J_1)}$ from (n-d) sized remaining observation set called delete-*d* jackknife sample (Wu, 1986).

$3^{(dd)}$. Omit second d observation set from full sample at a time and estimate the OLS coefficients $\hat{\beta}^{(J_2)}$ from (n-d) sized remaining observation set.

$4^{(dd)}$. Omit each d of the n observation sets and estimate the regression coefficients as $\hat{\beta}^{(J_k)}$ alternately, where $\hat{\beta}^{(J_k)}$ is jackknife regression coefficient vector estimated after deleting of k^th d observation set from full sample. Thus, $s = \binom{n}{d}$ delete-*d* jackknife samples are obtained, k=1,2,…,s.

6(dd). Obtain the probability distribution F($\hat{\beta}^{(J)}$) of delete-*d* jackknife estimates $\hat{\beta}^{(J_1)}, \hat{\beta}^{(J_2)}, ..., \hat{\beta}^{(J_s)}$

7(dd). Calculate the jackknife regression coefficient estimate which is the mean of the F($\hat{\beta}^{(J)}$) distribution as;

$$\hat{\beta}^{(J)} = \sum_{k=1}^{s} \hat{\beta}^{(J_k)} / s = \overline{\hat{\beta}}^{(J_k)} \quad (13)$$

8(dd). Thus, the delete-*d* Jackknife regression equation is

$$\hat{Y} = X\hat{\beta}^{(J)} + \varepsilon \quad (14)$$

Jackknife bias, variance, confidence and percentile interval. The jackknife bias, variance and confidence intervals are estimated by using the following equations from F($\hat{\beta}^{(J)}$) distribution (Miller, 1974).

The jackknife bias equals,

$$bi\hat{a}s_J(\hat{\beta}) = (n-1)(\hat{\beta}^{(J)} - \hat{\beta}) \quad (15)$$

The jackknife variance equals,

$$\text{var}(\hat{\beta}^{(J)}) = \frac{(n-1)}{n} \sum_{i=1}^{n} \left(\hat{\beta}^{(Ji)} - \hat{\beta}^{(J)}\right)\left(\hat{\beta}^{(Ji)} - \hat{\beta}^{(J)}\right)' \quad (16)$$

where $\hat{\beta}_j^{(Ji)}$ is the estimate produced from the replicate with i^{th} observation set or j^{th} group deleted (Friedl and Stampfer, 2002a).

Jackknife (1-α) 100 % confidence interval equals (Efron and Tibshirani, 1993).

$$\hat{\beta}^{(J)} - t_{n-p,\alpha/2} * S_e(\hat{\beta}^{(J)}) < \beta < \hat{\beta}^{(J)} + t_{n-p,\alpha/2} * S_e(\hat{\beta}^{(J)}) \quad (17)$$

where $t_{n-p,\ \alpha/2}$ is the critical value of t with probability $\alpha/2$ the right for n-p degrees of freedom; and Se($\hat{\beta}^{(J)}$) is the standard error of the $\hat{\beta}^{(J)}$.

The jackknife percentile Interval can be constructed from the quantiles of the jackknife sampling distribution of $\hat{\beta}^{(J)}$. The ($\alpha/2$)% and (1-$\alpha/2$)% percentile interval is

$$\hat{\beta}^{(J)}_{(lower)} < \beta < \hat{\beta}^{(J)}_{(upper)} \quad (18)$$

where $\hat{\beta}^{(J)}$ is the ordered jackknife estimates of regression coefficient from Equation 11 or 13, lower=($\alpha/2$)n, and upper = (1-$\alpha/2$)n.

Results

First, the ordinary least squares regression model was fitted to data given in Figure 1 and the results of the ordinary least squares regression was summarized in Table 1. The regression of FA on TL and OL is significant as result of variance analysis ($P<0.01$**). According to the t-tests for significance of regression coefficients, all of the regression coefficients are significant ($P<0.01$).

Table 1. The summary statistics of regression coefficients for OLS regression

Variables	$\hat{\beta}$	S.E.($\hat{\beta}$)	t	Sig.	95 % Confidence Interval
Constant	-2.16133	0.178	-12.126	0.000	-2.4538, -1.8682
TL	0.08336	0.034	2.421	0.017	0.0271, 0.1389
OL	0.49573	0.084	5.913	0.000	0.3578, 0.6342
R^2=0.867, N=100, s^2=0.233, SSE=31.491, F=442.3**					

The data and fitted line was given in Figure 1.

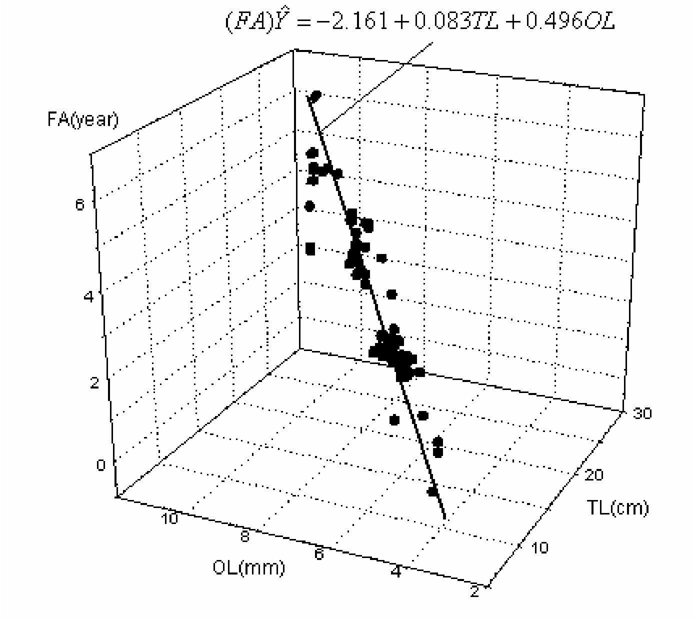


Figure 1. The data and fitted OLS regression line

The illustration of the bootstrap (B=10000 bootstrap samples, each of size n=100) and the jackknife (jackknife samples, each of size n-1=100-1=99) regression procedure, from the data given in Figure 1, calculating the bootstrap and jackknife estimates of the regression parameters for each sample are shown in Table 2 and 3.

Table 2. The illustration of the bootstrap (B=10000 bootstrap samples, each of size n=100) regression procedure from the data given in Figure 1, calculating the bootstrap estimates of the regression parameters for each sample for fish age model

r	Variables	$w_1^{(b)}$,	$w_2^{(b)}$,	$w_3^{(b)}$,	...	$W_{100}^{(b)}$,	$\hat{\beta}_o^{(b)}$	$\hat{\beta}_1^{(b)}$	$\hat{\beta}_2^{(b)}$
1	FA(year)(Y)	1.16	1.84	0.92	...	3.41	-2.183	0.083	0.487
	TL(cm) (X_1)	10.00	13.90	10.00	...	19.7			
	OL(mm) (X_2)	4.10	5.70	4.10	...	8.10			
2	FA(year)(Y)	5.08	0.92	2.25	...	5.08	-2.179	0.081	0.495
	TL(cm) (X_1)	22.10	10.00	15.90	...	22.10			
	OL(mm) (X_2)	9.10	4.10	6.50	...	9.10			
3	FA(year)(Y)	3.16	2.08	0.08	...	4.25	-2.191	0.080	0.491
	TL(cm) (X_1)	20.70	13.00	9.30	...	25.90			

	OL(mm) (X_2)	8.50	5.40	4.10	...	10.90			
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
10000	FA(year)(Y) TL(cm) (X_1) OL(mm) (X_2)	0.08 9.30 4.10	4.16 21.20 8.50	5.08 22.10 9.10	... ...	0.92 10.00 4.10	-2.162	0.084	0.498
$\hat{\beta}^{(b)} = \sum_{b=1}^{B} \hat{\beta}^{(br)} / B = \overline{\hat{\beta}}^{(br)}$							-2.1589	0.0834	0.4954

Table 3. The illustration of the jackknife (jackknife samples, each of size n-1=100-1=99) regression procedure from the data given in Figure 1, calculating the jackknife estimates of the regression parameters for each sample for fish age model

$w_i^{(J)}$	Variables	Observation sets					$\hat{\beta}_o^{(J)}$	$\hat{\beta}_1^{(J)}$	$\hat{\beta}_2^{(J)}$
		1	2	3	...	100			
1	FA(year)(Y) TL(cm) (X_1) OL(mm) (X_2)	omitted	0.92 10.00 4.10	0.08 9.30 4.10	... ...	5.08 22.10 9.10	-2.192	0.084	0.497
2	FA(year)(Y) TL(cm) (X_1) OL(mm) (X_2)	1.16 10.00 4.10	omitted	0.08 9.30 4.10	... ...	5.08 22.10 9.10	-2.176	0.084	0.497
3	FA(year)(Y) TL(cm) (X_1) OL(mm) (X_2)	1.16 10.00 4.10	0.92 10.00 4.10	omitted	... ...	5.08 22.10 9.10	-2.122	0.080	0.498
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
100	FA(year)(Y) TL(cm) (X_1) OL(mm) (X_2)	1.16 10.00 4.10	0.92 10.00 4.10	0.08 9.30 4.10	... ...	omitted	-2.141	0.083	0.493
$\hat{\beta}^{(J_o)} = \sum_{i=1}^{n} \hat{\beta}^{(J_i)} / 100$							-2.1613	0.0834	0.4957

The summaries of the some bootstrap and jackknife values of regression coefficients are presented in Table 4.

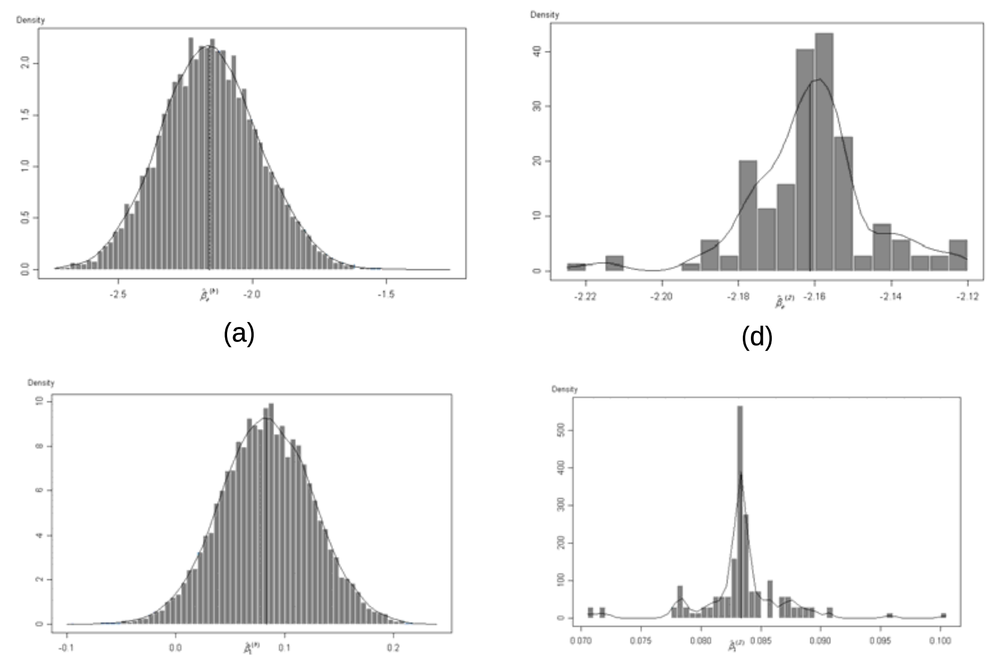
Table 4. The summary statistics of the regression coefficients for bootstrap and jackknife regression (n=100, B=10000)

	Variables	Observed	Average	S.E.	Bias	95% Confidence Interval	5%, 95% Persentile Interval
Bootstrap	Constant	-2.16133	-2.1589	0.18273	0.002457	-2.459, -1.858	-2.460, -1.850
	TL	0.08336	0.0834	0.04229	0.000069	0.0138, 0.1529	0.0137, 0.153
	OL	0.49573	0.4954	0.10250	-0.000333	0.3267 ,0.6640	0.3290, 0.663
J	Constant	-2.16133	-2.16132	0.18733	0.0007837	-2.469, -1.853	-2.19, -2.13

	TL	0.08336	0.08335	0.04326	-0.0005290	0.0122, 0.1545	0.078, 0.089
	OL	0.49573	0.49574	0.10488	0.0013688	0.3232, 0.6683	0.483, 0.506

B=10000 bootstrap samples are generated randomly to reflect the exact behavior of the bootstrap procedure and the distributions of bootstrap regression parameter estimations ($\hat{\beta}^{(b)}$) are graphed in Figure 2(a), 2(b), 2(c). The histograms of the bootstrap estimates conform quite well to the limiting normal distribution for all regression coefficients. Hence, the confidence intervals should therefore be based on that distribution, where B is sufficiently large(B=10000). And jackknife samples are generated omitting each one of the n observation sets and estimated the regression coefficients as $\hat{\beta}^{(J_i)}$. To reflect the exact behavior of the jackknife procedure and the distributions of jackknife regression parameter estimations ($\hat{\beta}^{(J_i)}$) are graphed in Figure 2(d), 2(e), 2(f). The histograms of the jackknife estimates conform quite atypical to the limiting normal distribution for all regression coefficients.

The bootstrap standard errors of the TL and OL coefficients are substantially larger than the estimated asymptotic OLS standard errors, because of the inadequacy of the bootstrap in small samples (Fox, 1997, Karlis, 2004). The confidence intervals based on the bootstrap standard errors are very similar to the percentile intervals of the TL and OL coefficients; however, the confidence intervals based on the OLS standard errors are quite different from the percentile and confidence intervals based on the bootstrap standard errors. Comparing the bootstrap coefficients averages $\overline{\hat{\beta}}_o^{(br)}$, $\overline{\hat{\beta}}_1^{(br)}$ and $\overline{\hat{\beta}}_2^{(br)}$ with the corresponding OLS estimates $\hat{\beta}_o$, $\hat{\beta}_1$ and $\hat{\beta}_2$ shows that there is a little bias in the bootstrap coefficients.



(a) (d)

(b) (e)

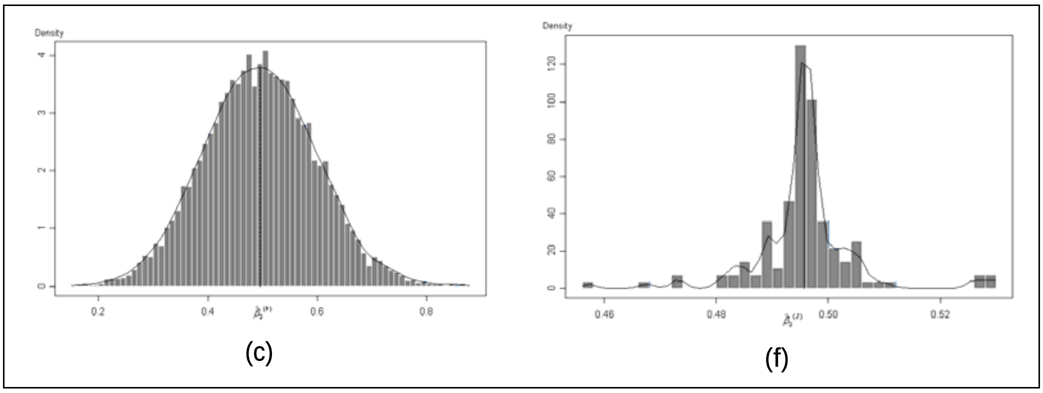


Figure 2. Histogram of bootstrap (B=10000, (a), (b), (c)) and jackknife ((d), (e), (f)) regression parameter estimates.

The shape of these graphs show that a histogram of the replicates with an overlaid smooth density estimate and the skewness of the distribution of regression parameter estimate from the bootstrap and jackknife replicate. A solid vertical live is plotted at the observed parameter value, and a dashed vertical line at the mean of the replicates

Discussion and conclusions

It is known from the statistical theory of the bootstrap that a finite total of n^n possible bootstrap samples exist. If it was computed the parameter estimates for each of these n^n samples, it would obtain the true bootstrap estimates of parameters but such extreme computation is wasteful and unnecessary (Stine, 1990). By making B large enough, it is seek to ensure that the bootstrap estimates of the regression parameter is close to the true bootstrap estimates of parameters which based on the all n^n bootstrap samples (Fox, 1997). It was suggested the bootstrap replications sufficient to be for estimating of variance $50 \leq B \leq 100$, $B \cong 1000$ for estimating of standard errors, perhaps it is not enough for confidence intervals, (Leger et al, 1992; Efron, 1990, Karlis, 2004). The number of bootstrap replications B depends on the application and size of sample and computer availability. Disadvantages of bootstrap method are; i) $\hat{F}$ (bootstrap distribution of $\hat{\beta}$) is not a good approximation of F in case of small data sets and existing of outliers in the sample, ii) so bootstrap is based on the independent assumption that it is not suggested for dependence structures like time series models, iii) bootstrap based on the error procedure assumes the fitted regression model is correct and the errors are identically distributed but is preferable to the bootstrap based on the resampling of observation, for violating the assumption for constant design matrix (Karlis, 2004). In addition, the most important advantages of the bootstrap regression method are to need smaller sample than ordinary least squares method and its practical performance is frequently much better but this is not guaranteed (Hawkins and Olive, 2002). Because of this it is a mistake to hope that bootstrap regression method always gives confident results. The confidence depends on the structure of the data and distribution function.

Fan and Wang, (1995) stated that due to the fact that sample size does impose a limit on the number of resamples, the jackknife may not be appropriate for small samples, but when the sample size is large, the bootstrap and jackknife would give similar results. Heltshe and Forrester, (1985) also reported that not only sample size but also the total number of individuals in the sample is important in improving the jackknife estimators. Hence, the jackknife bias, the standard errors and confidence intervals of the TL and OL coefficients based on the distribution $F_{(\hat{\beta}^{(J)})}$ are substantially larger than the bootstrap and estimated asymptotic OLS standard errors. The jackknife percentile intervals also are larger than to the bootstrap percentile intervals of the TL and OL coefficients.

The bootstrap and jackknife methods estimate the variation of a statistic from the variation of that statistic between sub samples, rather than from parametric assumptions and may yield similar results in many situations. In addition, they provide a way of decreasing bias and obtaining standard errors in situations where the standard methods might be expected to be inappropriate. But when bootstrap is used to estimate the standard error of a statistic, it gives very little different results when repeated on the same data, whereas the jackknife gives exactly the same result each time. The bootstrap is a more general technique and preferred to the jackknife. However the jackknife is easier to apply to complex sampling schemes than the bootstrap. Application of both techniques depends on development of computer technologies and would also more frequently use if statistical computer packages featured these analyses.

As a conclusion, bootstrap method is preferable in linear regression because of some theoretical properties like having any distributional assumptions on the residuals and hence allows for inference even if the errors do not follow normal distribution.

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[1] (Corresponding Author)

ROMANIAN PUBLIC SERVANT PROFESSIONAL LIFE: A QUANTATIVE APPROACH

Miruna MAZURENCU MARINESCU[1]

Ph.D., M.B.A., University Reader, Statistics and Econometrics Department

University of Economics, Bucharest, Romania

Main published books: Valuation and Feasibility, Oscar Print, Bucharest, 2004

Strategies for Maximizing the Value of the Companies, Oscar Print, Bucharest, 2004

E-mail: miruna@ase.ro

Ileana Gabriela NICULESCU-ARON

PhD, University, Senior Lecturer, Statistics and Econometrics Department

University of Economics, Bucharest, Romania

Main published books: Sampling techniques in work force analysis, Editura ASE, Bucharest, 2005

Statistical-economic analysis and valuation methods of the companies, (co-author), Editura Didactica si Pedagogica, Bucharest, 2002

E-mail: gabriela_aron@yahoo.com

Constanta MIHAESCU

PhD, University Professor, Statistics and Econometrics Department

University of Economics, Bucharest, Romania

Main published books: Population and employment. Past , present and future, 2001

Quantitative methods in demography and social statistics, 2005

E-mail: cmihaesc@yahoo.com

Abstract: *The perception of the Romanian Public Service Officer came out rather negative according to the most recent public opinion barometers. Our paper sets out to investigate using –inquiry technique- how it feels to be on the other side of the counter, and to what extent the multifaceted professional life of the Romanian Public Service Officer is influenced by various determinants like: work related conditions, education and training, citizens behaviour when facing administration staff, general social environment, specific laws and regulations, politics, aspects of personal life etc. in order to better satisfy their ultimate goal, namely to best serve the citizens.*

Key words: *public servant; professional life; quantitative approach; Romania*

Romania, as a EU member state, has an obligation to fulfil, in due time, the stipulations committed to, in this regard, through the Adhesion Treaty. One of the top priority fields of the post-adhesion strategy is the continuance of public administration reforms. Its fundamental objective is to consolidate the administrative capacity through the implementation of the “acquis communautaire”. The “administrative values” of the European Space, namely: transparency, predictability, responsibility, adaptability, accountability and efficiency; have to be implemented and integrated into all the administrative institutions and activities at all levels, and their application should be verified by an independent control system.

The foreign and local investors' decisions greatly depend on the quality, efficiency and credibility of the public sector, hence the need for its continuous adaptation to the rapid changes of the economy and society.

Even in the international arena, the public administration became an essential factor that determines the nation's competitive advantage.

Starting with September 2001, the complex process of public administration reforms has recorded important progress, not only at the institutional level, but also legal wise[2]. In a nutshell, the efficiency improvement efforts have to adhere to the design of a new framework for public administration, the supply of high quality public services, the modernization of institutional structures and the increase in transparency of their transactions, the development of a citizen oriented administration and the convergence towards the EU standards for the public service quality.[3].

Unfortunately, the implementation process, especially the practical application of the proposed reforms has not recorded sufficient advance. The mere setting up of a legal framework does not automatically imply solutions for the identified faults. Public administration with its own mechanisms and regulations often intervene between the promulgation of a law and its expected impact; and this might either enhance or block the beneficial effects of a specific law or governmental policy.

The reform of the administrative system does not necessarily mean the elaboration and the improvement of a legal framework, nor the mere design of an appropriate institutional framework, not even the design of recruitment, career management, standards and control mechanism of the civil servants performance programs. One important ingredient has to be particularly addressed: a new perspective on civil servants' perception, a new approach to assess their activity through perceived efficiency and effectiveness. This need is easily understandable as the human resource of administration is the backbone of the reformed framework, its warrant and the portent of profound changes.

Practically, the reform's pulse and its progress should be sought at the desk counter, as interface between the administration and the citizens, between the civil servant and the beneficiary of his or her effort. That is why the reform's success will be felt only when the citizens' needs are fulfilled through the efficient performance of public administration, as well as through the provision of better information to be made available to the public at large.

Consequently, a modern administration requires high professional standard that manifests itself through the quality of work, the obtained outcomes and the results produced, through a positive administrative culture, the stability and the political neutrality of civil servants. The creation of a professional public function, and the provision of continuous training for all employees of the public administration are essential requirements for such professional up-grading process.

We consider that the whole activity of the human resource management has to focus on the promotion and the maintenance of an organizational culture able to enforce values like: respect for citizens, teamwork collaboration, initiative encouragement, professionalism and assured certainty.

In order to achieve a continuous improvement of individuals' performance, the human resources management has to employ the following leverages:

- Employees motivation;
- Weaknesses identification;
- Well-structured work teams;
- Continuous staff training

The majority of the studies carried out up to date, indicate that more than half of the urban population of Romania perceive as weak, or very weak, their relationship and interface with authorities and public institutions that offer services to the public whether directly or indirectly. The majority of the citizens are not satisfied with the general way of communication system, the organization system of the public relations and the imperfections of the juridical system. A recent poll regarding the perception of the citizens of public function has been carried out in 2005, financed by Phare funds[4]. The conclusions confirm the same, unfavourable view of the public of their public service and its systems and processes, as well as its operatives.

The consultation of the staff of public administration on a series of particularities of the public function, took place for the first time in Romania[5] in 2004. Since then, the Public Function Barometer *(BFP)-2004,* has been drafted.

Since the BFP – 2004, more than two years have passed. In the meantime, on political arena many changes have occurred and the public administration reform has continued, mainly due to the country commitments to the process of EU adhesion. Even if only for these two reasons, we have qualified it as appropriate to carry out an inquiry aimed at capturing the most recent perceptions, attitudes and values regarding the different components of the public administration activity.

Even though the comparability criteria with the BFP 2004 are not met, and despite the fact that we shall refrain from making the results available to the country's population, and, despite the methodological drawbacks and limitations that have just been highlighted, we consider that this inquiry will prove to be a valuable tool for various decision making level for both local and central public administration. Through the newly collated results, it offers an updated image of the issues regarding the public administration staff, as they are perceived at their workplace, and their inner thoughts regarding the modernisation process of the public administration system.

The stated objectives of the inquiry were:

- To characterize the human relationships, the working environment and the promotion policy of the public administration;
- To asses the contentment level of the civil servant with respect to the type of activity he/she undertakes and the obtained income;
- To analyse the employees attitude towards the periodical evaluation and the life long learning concept;

The inquiry sample comprised 1939 civil servants from city halls, financial administrations, police stations and prefectures, all located in Bucharest and surrounding counties, as well as from the central administration (various ministries, Parliament, etc.). Due to the large size of the sample we can safely consider that the conclusions would be deemed relevant for the whole country. The method for gathering information was the direct interview and the period was between 1st and 15th March 2007. The interview operators were the students from the Faculty of Management, Public Administration Section from the Bucharest Academy of Economic Studies.

The Analysis and Interpretation of the Inquiry's Results

1. The characterization of interpersonal relationships, of the working environment and of the promotion policies of public administration

The way in which public civil servants appreciate interpersonal relationships inside public administration institutions is presented in the chart no. 1.

As it can be gleaned from the chart, the majority of respondents, reckon the relationship with the boss and with their colleagues to be good or very good (86,6%), as well as with their direct boss (80,6%).

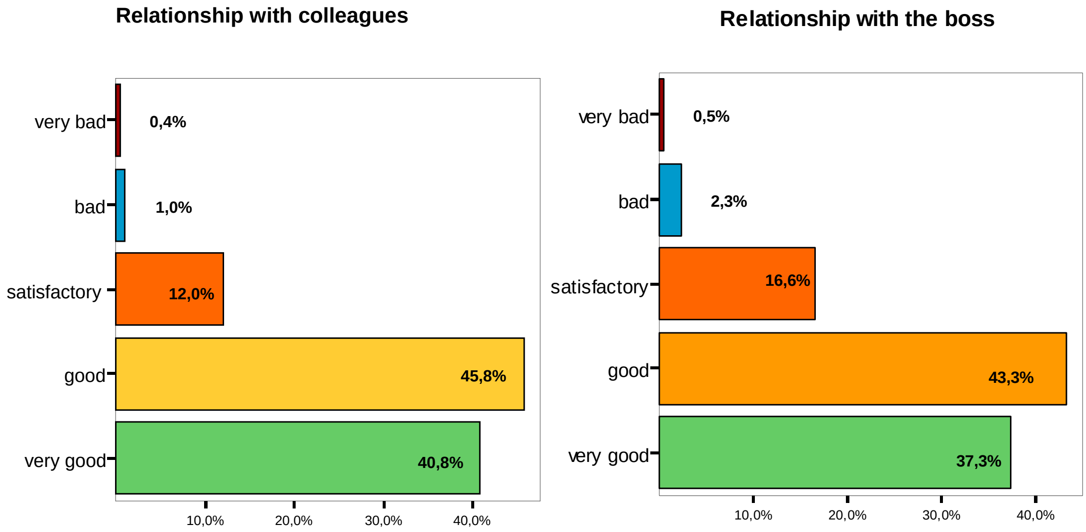

Chart no. 1. Appreciating interpersonal relationships

In order to get a better picture of the relationship with the direct boss, concerning whether they perceive that direct boss to be corrupt or not, especially in view of the fact that the Corruption Perception Index for Romania is a rather negative one at 3.1 out of 10.00. That was considered to be a delicate and somewhat sensitive question and was thus placed in the middle of the questionnaire. It was feared that soliciting a direct answer to such a tough question may have induced evasive answers.

Out of six positive attributes: (friendly, hard working, methodical, perseverant, helpful, honest), and six negative ones: (aggressive, corrupt, conservative, contemptuous, negligent, selfish), respondents were asked to mark only the ones that he/she reckons it closely associates with their boss's conduct.

In summary; by adding one point for each positive attribute and subtracting one point for each negative attribute; a final score was calculated.

The repartition of civil servants according to the score given to the direct boss is shown in the chart no. 2. The mean value of the score is 2,344, which suggest an inclination of the balance towards the appreciation of positive attributes.

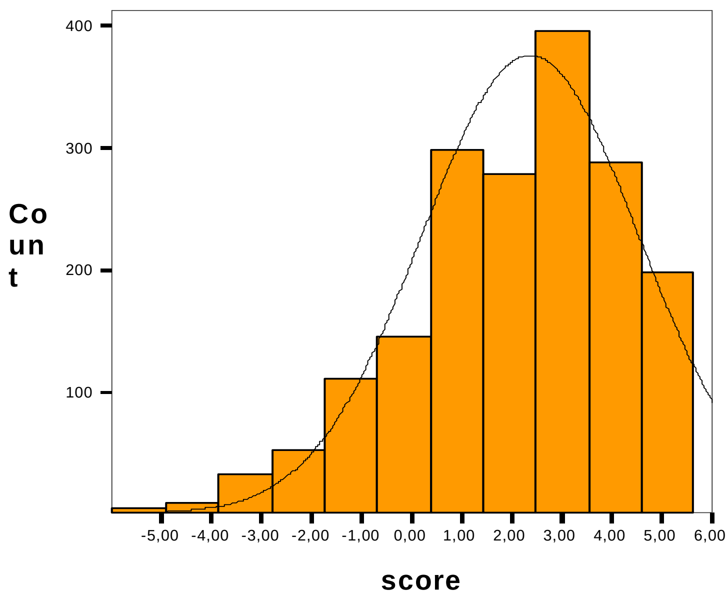

Chart no. 2. Repartition of respondents after the score obtained

Further, the study undertook the analysis of the way in which civil servants perceive the importance given to the promotion policy based on the following criteria: the level of qualification; performance, time served experience at post, personal relationships and attention given to the boss.

The respondents were asked to evaluate, according to their own opinion, on a scale from 1 to 10, each of the above-mentioned criteria. Beginning with the grade given, a mean score was calculated for each characteristic. In order to establish the importance of the advantages, the statistical significance of the difference between the mean scores was tested. The Student test for pair observations was used for dependent[6] samples. Based upon the results obtained using SPSS test, the hierarchy of advantages was established, as shown in the chart no. 3.

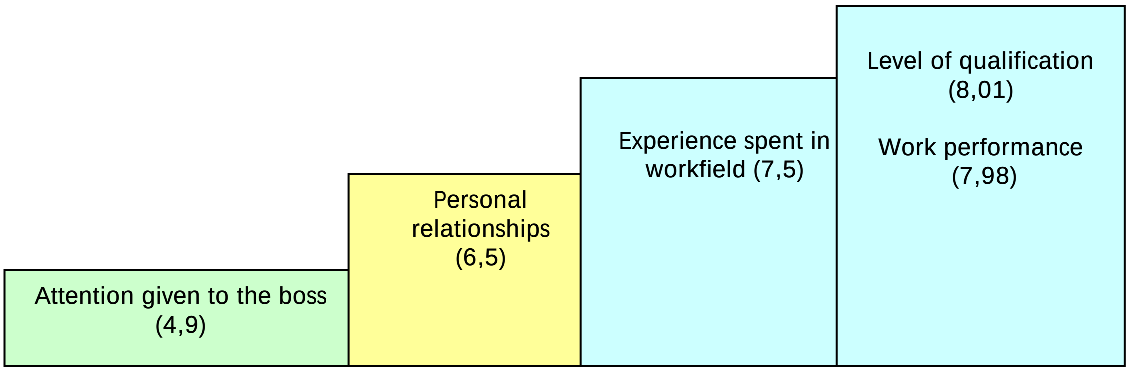

Chart no. 3. The Hierarchy of criteria in the promotion policy

After applying the Student test for the characteristics „Level of qualification" and „Work performances", the calculated value was t=1,016 (α=0,310), therefore the hypothesis of score equality was accepted. The values obtained after applying Student test for the other characteristics point to the existence of significant differences between them, corresponding to a level of significance of at least 0,01.

2. Assessing civil servants' contentment level regarding both activity and income

The civil servants' contentment level was measured considering the following aspects:

1. Work schedule
2. Work conditions
3. Incomes
4. The job
5. Horizontal communication existent between different departments or services
6. Vertical communication (tasks on a superior hierarchic line)

The respondents presented a level of contentment generally good or very good about the majority of the aspects analyzed, with the notable exception of the income factor, for which 48% of respondents stated that they are not contented or very discontented. The average wage recorded at the end of the year 2006 in public administration was about 510 RON, (150 euros a month), considerably lower than in other sectors and in comparison with the wages of European civil servants. In this context, the relative low wages would constitute the main reason that would determine them to search for a better-paid place to work. (47,8%).

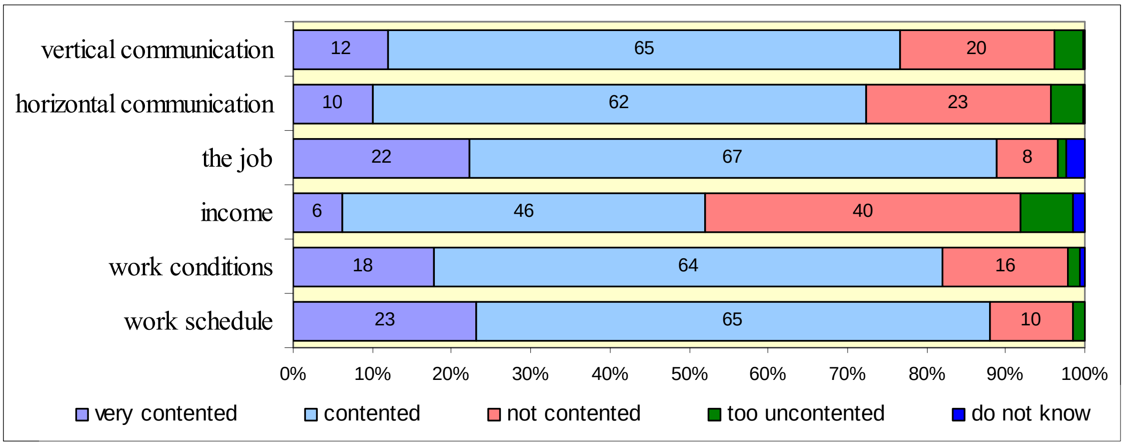

Chart no. 4. The level of contentment about different aspects related to the place of work

The evaluation of job satisfaction contentment level about is indicates an average score on a scale from 0 to 10. Data analysis based on the contingency tables highlight that the age influences the general level of contentment regarding the job (the calculated value of χ^2 test - 114,2 is statistically significant for a confidence level of 97,3%.

50% of young civil servants aged under 30 years old count themselves as very contented about their job, whilst 53% of those aged 30-49 years old are very contented, rising to 65.3%, of those aged above 50 years old.. The larger extent of contentment of older aged civil servants could be explained through the fact that either they come from the administrative apparatus of older regime, or they come from the disposed laborers following restructuring of the industrial colossus.

There are categories of persons still active who value the preservation of a certain social status and assurance of a stabile living, safe from the professional challenges specific to a the private sector of a market economy.

As for the influence of auxiliary variables „gender” and „studies” on the level of contentment, it can be stated that it is not statistically significant.

In order to complete the picture the respondents have concerning their place of work, they were asked to specify which from the following statements are true:

1. It is a stressful job;
2. It is a safe job;
3. It is generally respected by people;
4. I have a convenient working schedule;
5. My initiative is being encouraged;
6. I can take a leave off in no matter what period of the year (flexible holiday);
7. I feel it is useful for people;
8. It is a job with several responsibilities;
9. What I do is interesting;

The majority of the respondents (90,9%) believe that the activity they develop involves responsibility, and. 79,2% say that what they do is interesting, while 75,5% reckon that the job is appropriate for them, in accordance with their own capabilities.
Unfortunately, only 51,8% say that their initiative is being encouraged, which does not match the objectives of the new strategic management concepts of human resource development in public administration.

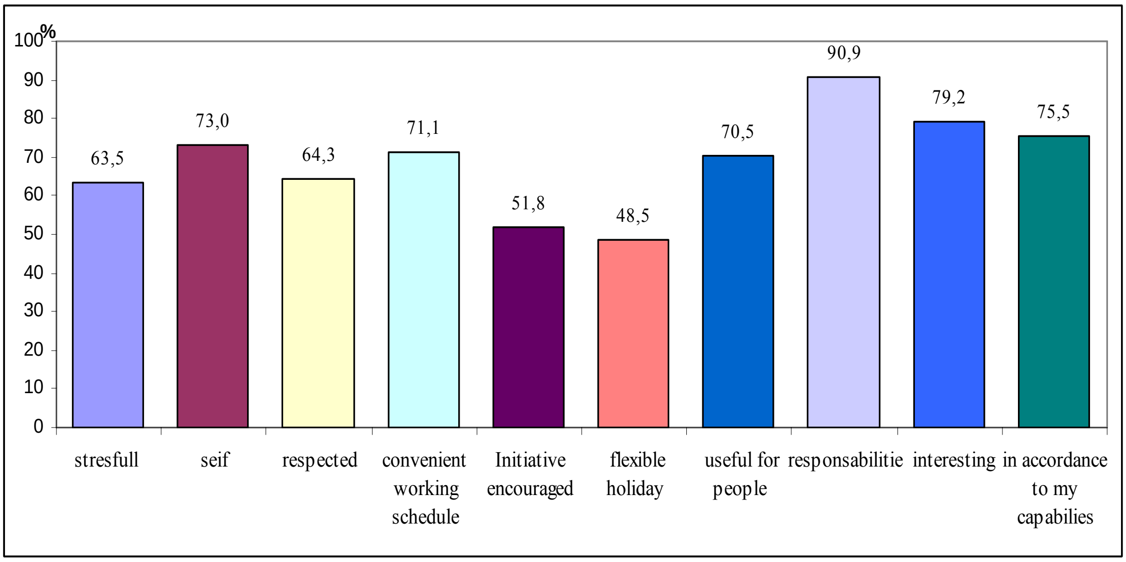

Chart no. 5. Characteristics of the work place

Although the majority of the respondents rate their wages as relatively low, their perception regarding the level of living assured by the income obtained, place them in a privileged situation compared to the general situation at the country level. For comparisons, information was taken from the database of the Public Opinion Barometer of October 2006.

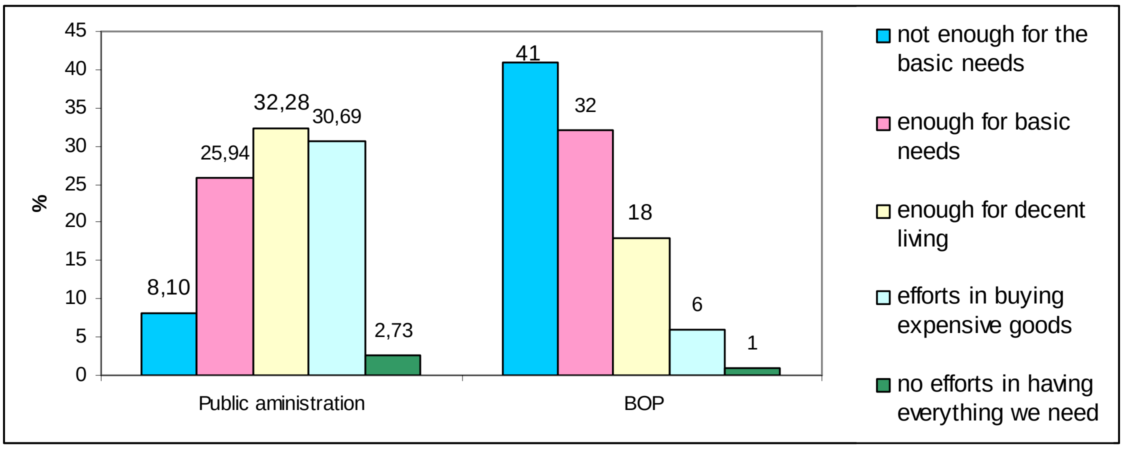

Chart no. 6. Appreciating the level of living assured by the income obtained

3. The analysis of employees attitude towards the periodical evaluation and continuous professional training

The evaluation of employees' performance is the „fundamental activity of the human resources management developed in order to determine to what extent the employees of an organization efficiently accomplish the tasks and responsibilities they have".[7]

Even though it is essential, the periodical evaluation represents for the employees „a potential threat, being also an activity difficult enough and, sometimes, controversial or even detested, and more than often, the preoccupations in this field are sources of discontent, because they are associated with the sacking of personnel".[8]

The agreement with periodical evaluation can be observed in chart no. 7. The percentage of the ones that agreed is 74% of respondents.

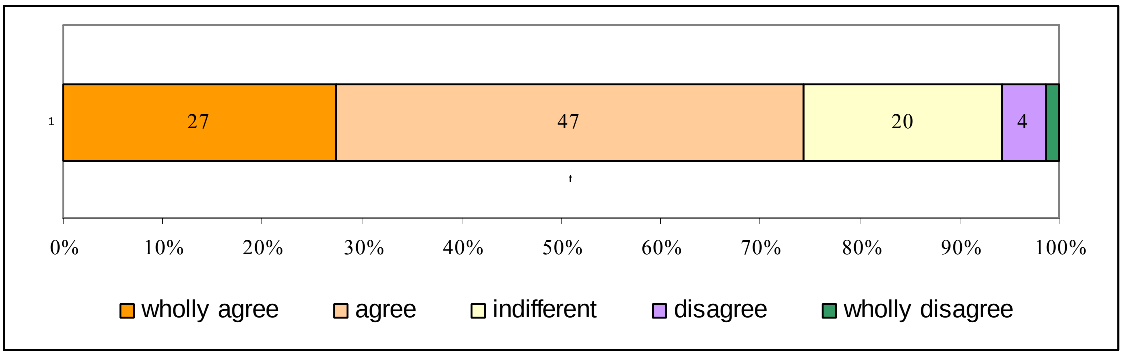

Chart no. 7. The agreement with periodical evaluation

The greater part of the civil servants interviewed think that the evaluation would have to be made by their bosses or managers and in a smaller proportion by internal evaluation commissions (chart no. 8).

This option could be explained by and through the relationship with direct boss, appreciated positively by most part of the wage earners. It is possible that this appreciation to be the proof of some practices of traditional administration which functions on a *clientele* based system. The calculated value of test χ^2 is very high - 249,8, which points out to the existence of a connection between these variables. The value of Cramer's coefficient V[9] is 0,59, suggesting that this correlation is strong.

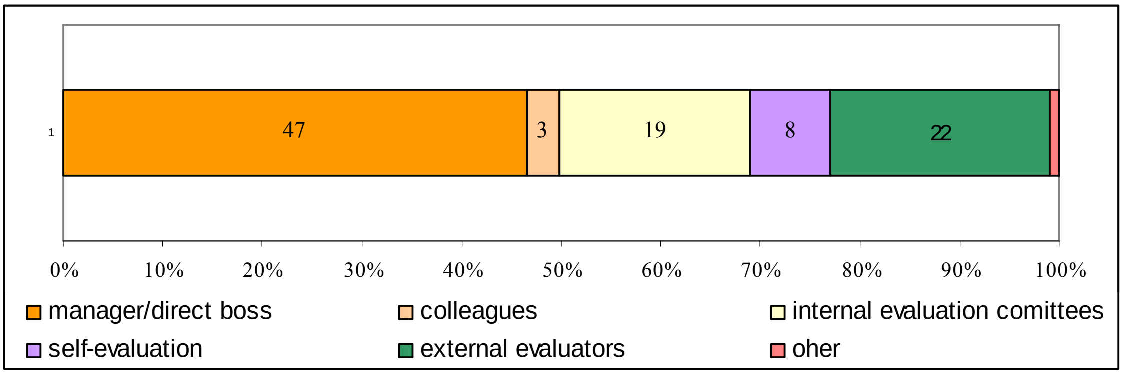

Chart no. 8. The option for evaluator

The reform in the public administration sector aims at improving the efficiency of their activity by assuring a high level of the personnel instruction, e.g., by organizing training. The wish to follow such trainings is high (see chart no. 8). 66% of the respondents consider that foreign languages courses as useful. The courses for achieving/improving the knowledge in the informatics domain are considered important, while. 63% of the respondents consider it a necessity to participate in at least one course.

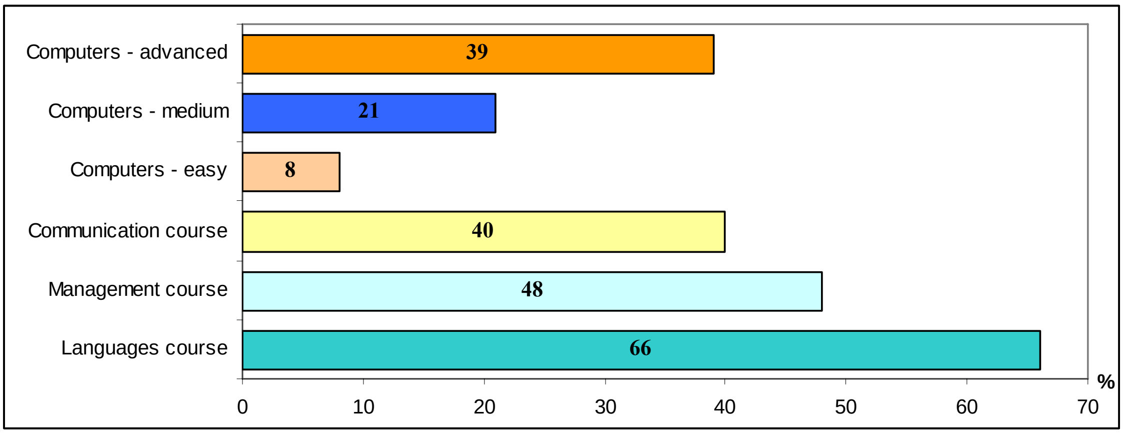

Chart no. 9. The options for training courses

Conclusions

The results of the inquiry pointed out a series of specific aspects of public administration's activity.

Positive aspects:

- ***The working environment*** is quoted as being a favorable one, the majority of respondents considering it as being good or very good, both relationships with colleagues as well as with their direct boss;
- The most important ***promotion criteria*** are perceived to be the level of qualification and attained performance;
- ***The level of contentment*** is in general good or very good towards the majority of specific aspects of the undertaken activity;

- The manifested interest of the respondents towards raising their ***level*** of ***instruction***, through participating at different courses of professional improvement.

Negative aspects:

- Although they do not occupy top places in the hierarchy, the relatively high scores obtained by the criteria – „personal relationships" and „attentions given to the boss" prove the ***maintenance of clientele promotion based practices***;
- ***The perception of the existence of corruption*** at the level of direct boss – only 55,4% of the respondents perceive the direct boss as being correct;
- ***The discontentment of the obtained income*** (48% of the respondents declare that they are not contented or very not contented), assuring for approximately 67% of the respondents the minimum requirements;
- ***Discouragement of personal initiative***, only 51,8% of the respondents considering that this is promoted at the management level of organization they work for;
- ***The instability of management of public function*** under the influence of the modifications of the ***political sphere*** (48% of the respondents indicated the change of the leading team after the change of the ruling political force).

"On the other side of the barricade" the most disturbing aspects are the ones related to the citizen's behavior (not respecting the behavior and the cleanliness rules of the public space; physical and verbal violence) and less the ones regarding their lack of knowledge (the lack of knowledge on the problem issues, the low level of general instruction; the lack of civic culture).

After two years of accelerated reforms in public administration, the perceptions and values of the civil servants reflect the positive effects of the modernization process, as measured against some practices and values related to a traditional administration still lingering on.

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[1] Miruna Mazurencu Marinescu
Reader, Ph.D., M.B.A., Statistics and Economic Forecast Department
Academy of Economic Studies of Bucharest
Local Phare and F.A.O. expert
Main published books:
- Strategies for maximizing the value of the companies, Oscar Print, Bucharest, 2004
- Valuation and feasibility, Oscar Print, Bucharest, 2004
- Statistical-economic analysis and valuation methods of the companies, Gh. Vasilescu, M. Hurduzeu, F. Wagner, I. G. Niculescu-Aron. O. Zaharia, Editura Didactica si Pedagogica, Bucuresti 2002

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[6] Test Statistics is: $t = \frac{\bar{d}}{S_d / \sqrt{n}}$ having: $S_d = \sqrt{\sum_{i=1}^{n} \left(d_i - \bar{d}\right)^2 / n-1}$ where: $\bar{d}$ = means of differences $d_i = x_{1i} - x_{2i}$; n = the number of pair observations; S_d = standard deviation of d_i variable.

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[8] Idem, p. 386

[9] $V = \sqrt{\chi^2 / n(c-1)}$ where n is the sample size and c the smallest value from the number of lines and columns.

ANALYZING THE STUDENTS' ACADEMIC INTEGRITY USING QUANTITATIVE METHODS[1]

Daniel TEODORESCU
PhD, Emory University, Atlanta, USA

E-mail: dteodor@emory.edu

Tudorel ANDREI
PhD, University Professor, Statistics and Econometrics Department
University of Economics, Bucharest, Romania
(Co)Author of books: Reforma administratiei publice in Romania in perspectiva integrarii Romaniei in Uniunea Europeana (2006), Statistica si Econometrie (2003), STATISTICA- teorie si aplicatii (2003)
E-mail: andreitudorel@yahoo.com

Erika TUSA
PhD, University Reader, Statistics and Econometrics Department
University of Economics, Bucharest, Romania
(Co)Author of the books: Statistics for Economists (2005), Aplicatii statistice (2002)
E-mail: erika_tusa@yahoo.com

Claudiu HERTELIU
PhD, University Lecturer, Statistics and Econometrics Department
University of Economics, Bucharest, Romania
Co-author of the books: Sistemul national de indicatori pentru educatie (2005), Finantarea invatamantului preuniversitar de stat (2000)
E-mail: claudiu.herteliu@mec.edu.ro, **Web page:** http://www.hertz.ase.ro

Stelian STANCU
PhD, University Professor, Department of Economic Cybernetics
University of Economics, Bucharest, Romania
Co-author of the books: Teoria jocurilor pentru economiesti (2005), STATISTICA- teorie si aplicatii (2003)
E-mail: stelian_stancu@yahoo.com

Abstract: *The transition period in Romania has generated a series of important changes, including the reforming of the Romanian tertiary education. This process has been accelerated after the signing of the Bologna treaty. Important changes were recorded in many of the quantitative aspects (such as number of student enrolled, pupil-student ration etc) as well qualitative aspects.*

The article aims to identify and analyze the main aspects related to the academic fraud in tertiary education, within Bucharest University Center, by using a statistic-survey-based assessment performed in November 2005. The research components rely on the students' and professors' academic behavior analysis, in close accordance with education performance factors.

Key words: *academic fraud, logistic model, tertiary education*

1. Introduction

The transition has generated a series of important changes at tertiary education level also. The reform process of the Romanian tertiary education has been accelerated after the signing of the Bologna treaty. After 1989 the number of students increased greatly, by almost four times. However, during this period, the pupil – student number ratio decreased significantly.

Along the aspects related to the changes in the number of enrolled students, aspects like ethics in the academic system is one of the most important as well as discussed issues

The ethics in the Romanian universities was subject to previous studies – such as *"Sociological research – ethics in universities"* coordinated by Ana Bulai or *Barometer of students' opinion"* carried out by Team Work in The University of Bucharest in 2005. This research is addressing the whole tertiary education and is underlying the favoritism as the most significant ethical problem in the academic environment. According to the study, favoritism is signaled by 36% of the interviewed professors, 29% of the students, 24% of the auxiliary staff and 33% of the PhD students.

The paper is aiming to continue and deepen the analysis of several aspects related to academic fraud in the Romanian Universities, using statistic-survey-based assessment performed in November 2005.

2. Sample characteristics

The sample comprised 1025 students and has a 2% error tolerance. For generating the study, the following sampling variables have been used: study curricula, year of study and students' age.

The main characteristics of the three sampling variables are as follows:

1. **Study curricula**. Regarding this criteria, students have been grouped in the following categories: university (with a share of 32,3% in the total number of students), tehnical studies (27,6%), economical studies (22,2%), medicine (6,7%), law (5,5%), agriculture studies (4,0%), art, sports (1,8%). Students' allocation within the sample was proportionally with the total number of students attending day-study for each curricula. The study includes all tertiary education institutions in Bucharest, except of "Politehnica" University Bucharest, where the questionnaires where banned.
2. ***The year of study*** The sample didn't include the first year students. The distribution of the students was as following: 38.6% from the second year, 33.0% from the third year, and the difference of 28,4% from the forth and fifth year (the fifth year students are from the university of medicine.
3. ***The age of students.*** The sample's distribution of the students according to their age is presented in the following figure. The average age is 23.5 years and 69.8% are at most 25 years old. The smallest weight is corresponding to the students with the age between 30-34 years.

The analysis of the sample provided information that allowed to characterize the distribution of the students according to the following criteria: the year of high school graduation, the region of graduation (Bucharest, Moldavia, Transilvania, Dobrogea, Oltenia, Muntenia or abroad), the average mark from the previous university year, the gender distribution, the percentage of contribution from self-earned incomes to the university taxes and expenses, the type of dwelling/accommodation (living with the parents, in a university residence, in a rented or own place).

3. Defining the variables of the logistic model

This study is defining *the student's non-academic behaviour as a fraudulent intent or the actual fraud in a written examination or the copying of projects achieved throughout the academic year in an attempt to pass the exam or get a higher grade in the exam.*

The following forms of academic behaviour breach were identified as regards the attempt of exam fraud: copying from a colleague or various prohibited sources during an exam (a), direct intervention or by intermediaries with the training professor to pass an exam or get a higher grade (b), copying of the projects developed throughout a semester from various books, scientific works (c) or directly from colleagues (d) and taking of private paid training classes with the titular professor (e). For every exam fraud procedure the frequency among faculty colleagues was registered. The distribution of the answers to this question is shown in the table hereunder:

Table 1. The distribution of the main methods of academic fraud (%)

	A	b	c	d	e
Nobody	3,3	16,3	22,8	5,9	11,0
Small proportion – under 10%	24,1	25,9	12,3	20,4	24,4
Significant proportion – 11-50%	30,1	7,7	3,3	19,2	19,8
Most of the colleagues – 51 -90%	23,1	2,7	1,9	17,2	11,3
Almost everybody – over 90%	7,2	0,4	0,7	8,9	3,2
I don't know	10,8	45,2	57,5	26,9	29,0
No answer	1,3	1,9	1,6	1,5	1,3
Total	100,0	100,0	100,0	100,0	100,0

The most frequent exam fraud method is the copying of paragraphs from books, articles, Internet or from projects submitted by students over the academic year (37.9 % of the students on an average prefer their projects to be worked-out that way – the std. deviation is of 33.12). The exam copying represents the method most frequently used by students in their attempt to pass an exam or get a higher grade (37.9 % of the students on an average resorted to such a method to pass an exam or get a higher grade – the std. deviation is of 30.21). The exam fraud by intervening with the professor or taking private paid training classes are methods less-frequently used by the students. Thus, on an average 11.1 % of the students answered that they intervened with the professor – std. deviation of 18.4, while the mean of the students taking private paid training classes to prepare for an exam is of 8.7 % - std. deviation of 19.44.

The independent variables of the model

When defining the logistic model attention must be paid to the major factors determining infringements of the academic integrity standards such as: educational process quality at each university level, the student's critical attitude towards the infringements of the academic standards by colleagues and professors, the academic integrity level of the professors and their position in relation to the exam fraud by students, time devoted by students to individual learning, as well as to other non-professional activities, etc. To an equal extent, several attributive characteristics influencing upon the student's behaviour like the academic year, the student's gender, the allowance received for paying his/her tuition, etc.

A. The student's attitude towards the exam fraud by students is measured by means of two variables:

- ***To what extent a student encourages the exam fraud by colleagues***. The questionnaire included questions meant to measure to what extent a student allowed to be copied by a colleague during an exam (a) or his projects be copied along an academic year (b). Three answering variants were defined for each variable: 0 – never, 1 – sometimes, 2 – often enough. The results are presented I the following table:

Table 2. Frequency of academic fraud (%)

	Exam copying	Extra-classroom projects
Never	12.3	39.1
Sometimes	58.2	39.9
Often enough	26.8	15.3
Non-response	2.7	5.6
Total	100.0	100.0

These results allow the following conclusions to be drawn: i) as a rule the students allow the exam copying by colleagues; ii) the students are much more favourable to the copying of a written examination than of the projects developed along an academic year.

Moreover, the colleagues who did not allow copying during a written examination have a very bad image among the peers. A significant proportion of students (75.3 %) had a negative projection about their colleagues who did not allow exam copying: 15.7 % think that those not allowing exam copying were selfish while the opinion of 3.15 % was that "*they were no true students*".

- ***To what extent the students report to the faculty's leadership the non-academic attitude noticed in colleagues or the teaching staff at courses and seminars***. Three behavioural cases not complying with the academic standards were identified for the students: the student offers money/gifts to a professor to pass the exam or get a higher grade (a); the student copies during the exam from a colleague or from other unallowed sources (b); a colleague pays for the service of graduation diploma or project drawing up during the year (c). As far as the professors are concerned the following three situations not complying with the academic standards were identified: the professor asks for or receives money from students (d); the professor plagiarized the course he/she delivers or his/her published works (e); the professor asks his/her students to buy his/her published works (f).

Table 3. Students' behavior on reporting academic fraud (%)

	Reporting of the colleagues' non-academic attitude			Reporting of the professors' non-academic attitude		
	a	***b***	***C***	***D***	***e***	***f***
Yes	9.4	3.9	5.8	23.4	18.9	12.8
No	49.5	72.3	64.8	37.6	37.2	54.1
Do not know	38.8	20.5	26.1	36.8	40.5	30.1
Non-response	2.3	3.2	3.2	2.2	3.4	3.0

By using the six above original variables one can define the following variables derived for measuring the students position in relation to the corruption cases noticed among colleagues or professors in connection with matters associated to passing an exam or getting a higher grade:

- ***the students intent to report to the faculty's leadership a non-academic behaviour among the colleagues in order to pass an exam and work out graduation papers or projects***. In this respect, we must calculate the arithmetic mean of the three original variables used for measuring the students' intent to report the non-academic behaviour of the colleagues;
- ***the students intent to report to the faculty's leadership a non-academic behaviour among the professors***. The new variable is defined by computing the arithmetic mean of the three original variables measuring a student's intent to report to the faculty's leadership a non-academic behaviour of a professor in giving grades in an exam, in plagiarizing delivered courses and published works;
- ***the students intent to report to the faculty's leadership a non-academic behaviour among the colleagues or the professors***. This variable is a mean of the six original variables.

B. Professors benefiting from illicit gains offered by students

Three cases of illicit gains got by a professor from students by taking advantage of his/her position in the candidate/student assessment process within the higher education system. Thus three questions were added to the questionnaire in order to establish to what extent the professor asked for and accepted money from students to pass an exam or get higher grades in the entrance examination or in a customary faculty exam or specifically required his/her students to buy a manual that he/she published.The answers to the questions are summarised in Table 4.

Table 4. Perception of illicit gains of professors (%)

	A professor requested/accepted money in an exam	A professor requested/accepted money for an entrance examination	A professor specifically required the purchasing of one of his/her works
No case	37.4	45.3	18.1
1-3 cases	20.6	13.5	35.6
4-6 cases	6.4	3.3	13.0
7-10 cases	2.4	1.8	4.8
More than 10 cases	6,5	2.6	15.5
Do not know	23.3	29.9	9.7
Non-response	3.3	3.6	3.3

The following conclusions may be drawn: i) the exam fraud perception or the exam fraud suspicion is insignificant among the students; ii) the most frequent case of professors using their authority for personal purposes is obliging students to buy their works in order to get prepared for an examination. Thus, 68.9% of the students pointed to at least one such case; iii) more than 35% of the interviewed students knew of at least one case of intervention with a professor to pass an exam or get a higher grade in an exam.

The values of the three above variables that are not used to define the variables within the regression model are the following: 0 – for no case or for a "I do not know"answer, 2 – for 1-3 cases noticed, 5 – for 4-6 cases, 8.5 – for 7-10 cases, 13 – more than 10 cases. An optimistic situation was imagined namely when the individuals having answered "I do not know" are inclined to believe that there is no major corruption at academic level for the elements considered.

The variable measuring the academic professors inclination of benefiting from their position within the system is defined based on the arithmetic mean of the following two variables: *"A professor requested or accepted money/gifts in return for a successful examination"* and *"A professor specifically required his students to buy a book or a manual he/she published"*.

C. Frequency of students attendance at courses and seminars and number of hours devoted to individual learning

The attendance at courses and seminars is measured by a variable defined as an arithmetic mean of the following two questions: *"Throughout the academic year 2004-05, how often did you attend the courses within your faculty?"*(a) and *"Throughout the academic year 2004-05, how often did you attend the seminars/laboratories within your faculty?"*(b). Thus the variable *FSCS* is determined. After processing the answers to the two questions the results in the table hereunder were obtained:

Table 5. Attendance of clases by students (%)

	I only came for the exams	I attended less than half of the courses	I attended more than half of the courses	I attended all courses	Non-response
How often did you attend the courses	0.7	9.6	54.2	35.4	0.1
How often did you attend the seminars	-	4.4	39.4	55.9	0.3

The results in the table above show a good attendance by students at the courses and seminars.

The questionnaire also included a question meant to measure the number of hours devoted on the average by every student to individual learning. The following conclusions were drawn: i) 46.3 % of the students do not devote to individual learning more than 5 hours per week while the share of those devoting more than 16 hours is of only 18.0 %; ii) the average time devoted by a student to individual learning per week is of only 8.9 hours; iii) almost 5% of the students get prepared for their examinations only during the examining session and they actually devote less one hour to individual learning over one week.

D. Educational system quality

Within this study the educational system quality is measured according to its purpose at student level. Thus three aspects were retained: i) educational system contribution to the development of the student's personality; ii) extent to which it contributes to the development of the student's integrity; iii) usefulness of the studies perceived by the students. In this respect one has to define the variables contributing to the definition of the linear regression model used for the analysis of the students non-academic behaviour:

i) ***The extent to which the studies already followed at the faculty contributed to the development of the student's personality.*** To define this variable the extent to which the faculty educational system was taken into account as regards its contribution to the acquisition of a general culture (a), the development of specialised knowledge in the field (b), the development of clear and efficient communication skills in writing (c), the verbal communication skills (d), the development of a critical and analytical thinking (e), the use of computer and information technology (f), the solving of complex practical issues (g) and the capability of working efficiently in a team (h).

The correlation matrix of the variables defined based on the questions written down in the questionnaire looks like a positive and significant linear correlation among those variables. All the matrix values differ significantly from zero at a significance threshold of 1%.

Table 6. The correlation matrix

	A	B	c	D	e	f	g	h
a	1	0.511**	0.467**	0.480**	0.357**	0.240**	0.391**	0.373**
b		1	0.475**	0.445**	0.470**	0.361**	0.398**	0.311**
c			1	0.769**	0.497**	0.277**	0.438**	0.382**
d				1	0.538**	0.325**	0.492**	0.425**
e					1	0.449**	0.500**	0.389**
f						1	0.521**	0.538**
g							1	0.587**
h								1

** Correlation is significant at the 0.01 level (2-tailed)

ii) ***The extent to which the university studies contributed to the development of integrity*** can be measured based on three elements: extent to which the assessment and grading methods for each course are obvious (a), objectivity of the assessment and grading system (b) and extent to which the university studies contribute to the development of students integrity (c). Five numerical values were defined for the three variables, as follows: 1-leaves to be desired, 2-mediocre, 3-acceptable, 4-good, 5-excellent.

iii) ***Usefulness of the studies finished in a faculty***. In this study the usefulness of the faculty studies perceived by the students is defined in relation to his/her option of choosing to follow again the same faculty if such a possibility existed (a) and of continuing studying for a master's or doctor's degree at the same institution after graduating (b).

The questionnaire also included a question meant to measure a graduate student's intent of continuing studying for a master's or doctor's degree at another faculty (c). The answers to the three questions are shown in the table below:

Table 7. Graduate student's intent of continuing studying

	a	b	c
Undoubtedly YES	32.2	36.4	18.4
Probably YES	46.7	49.7	40.1
Probably NO	14.2	10.4	29.1
Undoubtedly NO	6.9	3.5	12.4
Total	100.0	100.0	100.0

When interpreting the data in the table above we should also consider the students chances of getting a job in their field of study. Thus, 31.7% of the interviewees feel that their chances are high (more than 60%), 40.6% that they are moderate (between 40-60), while 26.1% are pessimistic about finding a job after graduation (less than 40%).

E. Extra-professional activities

Based on the questions provided in the questionnaire three variables are derived to characterise the time devoted to extra-professional activities:

i) ***Time devoted to work outside the campus***. This variable was chosen for several reasons: the number of hours devoted on an average per student to activities outside the campus is relatively high. This represents 6.75 hours/student over a week; the linear correlation between this variable and the grades got in an exam is a negative one and differs significantly from zero at a significance threshold of 0.01 (Pearson coefficient of -0.1); more than 50% of the students carry out an off-campus activity on a regular basis over the week;

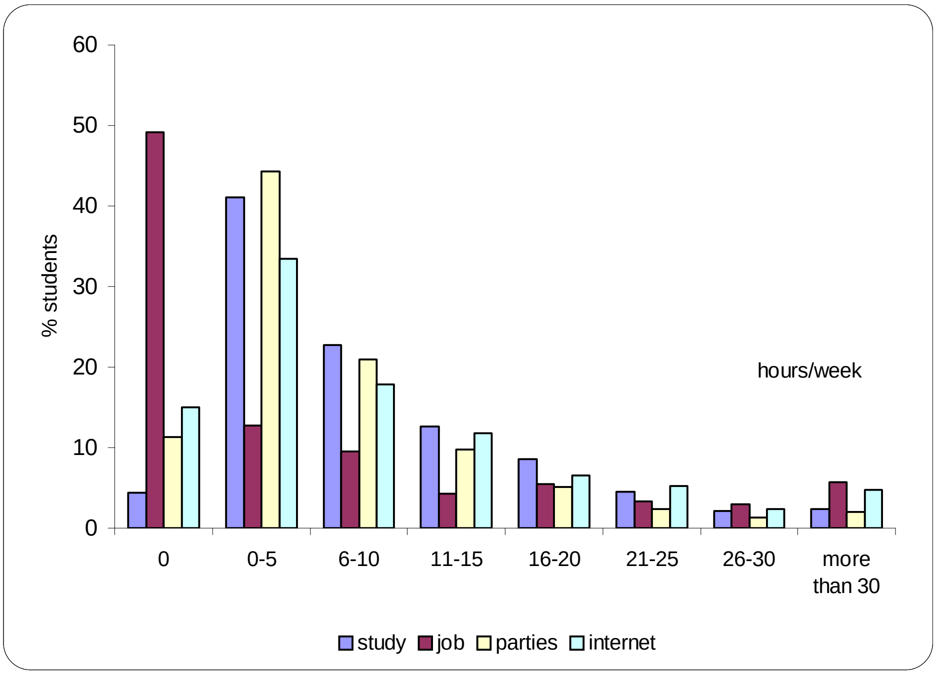

Figure 1. Breakdown of students by time dedicated to certain activities

ii) ***Time devoted to partying with friends and video or Internet games*** computed as the sum of three original variables resulting from the three questions in the questionnaire. The linear correlations between the examination results and the three variables are partially negative and significant at a significance threshold of 0.01. Thus the Pearson coefficients have values of -0.123 for parties with friends or colleagues, -0.076 for internet surfing and -0.130 for video or computer games. The correlation between the newly defined variable and the grades got in the examinations is of -0.135 being a significant value at a significance threshold of 0.01;

iii) ***Time devoted to other extra-professional activities*** computed as the sum of the time given to TV relaxation, fun reading and sports or physical training.

Table 8. Time allocation per activity hours/week

Number of hours devoted per week	Mean	Std. Deviation
Individual learning	8.85	7.84
Off-campus work	6.75	10.41
Student activities outside the courses	1.77	4.69
TV relaxation	6.23	7.04
Fun reading	6.17	6.18
Sports or physical training	5.15	6.33
Partying with friends or colleagues	7.01	7.19
Internet surfing	8.93	9.22
Video or computer games	3.81	6.90

F. General characteristics including the individual's gender (*SEX*), data relative to the payment of the university tuition fees (*PT*) and the average of the last academic year student's grades (*MED*).

4. Using the logistic model for analyzing academic fraud

The model is defined starting from the following assumption: the fraud of an exam, as dependant variable (y) is a function of the following independent variables: i) gender (x_1); ii) the level of corruption in the university induced by the behavior of the professors (x_2); iii) the performance level of the student, defined by the weekly time spent, in average for study (x_3) and the students' appreciation of their colleagues performance level (x_4), iv) the quality of the academic activity in the university, measured by the course relevance (x_5) and the course attendance (x_6); v) the free time spent outside the campus (x_7) and in extraprofessional activities (x_8); vi) the predilection to cheat an exam given similar practices during the high school (x_9).

The estimations for the logit model and its characteristics are presented in the next table:

Table 9. Characteristics logistic model

Variable	B	S.E.	Wald	Sig.	Exp (B)
1. Gender (x_1)	-0,0261	0,191	1,867	0,172	0,770
2. level of corruption in the university (x_2)	0,203	0,089	5,237	0,022	1,225
3. Level of students' academic performance					
31. Individual study (x_3)	-0,238	0,067	12,630	0,000	0,788
32. Assessment of colleagues' academic performance (x_4)	0,183	0,110	2,759	0,097	1,201
4. Quality of teaching activity					
41. Relevance of courses (x_5)	-0,224	0,087	6,605	0,010	0,799
42. Attendance at classes (x_6)	-0,543	0,164	10,934	0,001	0,581
5. Free time					

51. Extra-campus work (x_7)	-0,133	0,044	9,067	0,003	0,876
52. Extraprofessional activities (x_8)	0,268	0,070	14,631	0,000	1,307
6. Predilection to cheat in high school (x_9)	0,268	0,094	3,434	0,064	1,190
Regularity	0,488	0,749	0,424	0,515	1,628

The logistic model will be defined as follows:

P(exam fraud by cheating)=

$$=\frac{\exp(0,488\text{-}0,0261x_1+0,203x_2\text{-}0,238x_3+0,183x_4\text{-}0,224x_5\text{-}0,543x_6\text{-}0,133x_7+0,268x_8+0,268x_9)}{1+\exp(0,488\text{-}0,0261x_1+0,203x_2\text{-}0,238x_3+0,183x_4\text{-}0,224x_5\text{-}0,543x_6\text{-}0,133x_7+0,268x_8+0,268x_9)}$$

5. Conclusion

The characteristics that quantify the number of hours allocated to individual study, during a week, the relevance (perceived importance) of the course and the class attendance generate a reduction in the probability of cheating at an exam.

The students that have to work outside campus on a regular basis are less tempted to fraud an exam. On the other hand, the extraprofessional activities, such as parties and gathering with friends, internet surfing, video games tend to increase the probability of cheating at an exam. More time a student allocates to these activities, more likely to fraud the exam.

The gender has low relevance with respect to the probability to fraud an exam. Nevertheless, the female students are more inclined to cheat at an exam compared to their male colleagues.

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AN APPLICATION OF THE FRANK-WOLFE ALGORITHM AT MAXIMUM LIKELIHOOD ESTIMATION PROBLEMS

Ciprian Costin POPESCU
PhD Candidate, University Assistant
Department of Mathematics, University of Economics, Bucharest, Romania
Calea Dorobantilor 15-17 Street, Sector 1, Bucharest, Romania

E-mail: cippx@yahoo.com

Lia POPESCU

Department of Mathematics, Economic College
Rosiori de Vede, Romania

Abstract: *This paper tackles the problem of maximum likelihood estimation [2] under various types of constraints (equalities and inequalities restrictions) on parameters. The initial model, which is in fact a maximization problem (here are a few methods available in literature for estimating the parameters: ERM (expectation-restricted-maximization) algorithms, GP (gradient projection) algorithms and so on) is change into a new problem, a minimization problem. This second form is suited to a variant of Frank-Wolfe method for solving linearly restricted nonlinear programming problems [5]. In this way, some difficulties from the previous approaches are removed.*

Key words: *Constrained maximum likelihood; Nonlinear programming; Frank-Wolfe algorithm*

Developments and algorithms[1]

There are many situations in statistical computation which implies maximum likelihood estimation. The aim of this work is to generalize a model developed by Jamshidian [2] (by introducing a supplementary inequality constraint at the left) and to solve them using a regularization of FW-algorithm [5]. Thus, consider the optimization problem:

$$\max l(\boldsymbol{\theta})$$

$$\text{subject to } \begin{cases} \mathbf{a}_i^T\boldsymbol{\theta} = b_i, i \in I_1 \\ b_i^- \le \mathbf{a}_i^T\boldsymbol{\theta} \le b_i^+, i \in I_2 \end{cases} \qquad (1)$$

where $\boldsymbol{\theta} = (\theta_1,...,\theta_p) \in R^p, \boldsymbol{\theta} \ge \mathbf{0}$, $\mathbf{a}_i = (a_{i_1},...,a_{i_p}) \in R^p$, $I_1 = \{i_1,...,i_m\}$ and $I_2 = \{i_{m+1},...,i_{m+n}\}$, $\text{card}I_1 = m$, $\text{card}I_2 = n$.

From (1) we get:

$$\max l(\boldsymbol{\theta})$$

subject to $\begin{cases} \mathbf{a}_i^T\boldsymbol{\theta} = b_i, i \in I_1 \\ \mathbf{a}_i^T\boldsymbol{\theta} \le b_i^+, i \in I_2 \\ -\mathbf{a}_i^T\boldsymbol{\theta} \le -b_i^-, i \in I_2 \end{cases}$ (2)

Now we denote

$$\begin{cases} \mathbf{a}_{i_1} = \mathbf{u}_{i_1} \\ \vdots \\ \mathbf{a}_{i_m} = \mathbf{u}_{i_m} \end{cases}, \begin{cases} b_{i_1} = v_{i_1} \\ \vdots \\ b_{i_m} = v_{i_m} \end{cases}, \begin{cases} \mathbf{a}_{i_{m+1}} = \mathbf{u}_{i_{m+1}} \\ \vdots \\ \mathbf{a}_{i_{m+n}} = \mathbf{u}_{i_{m+n}} \end{cases}, \begin{cases} b^+_{i_{m+1}} = v_{i_{m+1}} \\ \vdots \\ b^+_{i_{m+n}} = v_{i_{m+n}} \end{cases} \quad (3)$$

and

$$\begin{cases} -\mathbf{a}_{i_{m+1}} = \mathbf{u}_{i_{m+n+1}} \\ \vdots \\ -\mathbf{a}_{i_{m+n}} = \mathbf{u}_{i_{m+2n}} \end{cases}, \begin{cases} b^-_{i_{m+1}} = v_{i_{m+n+1}} \\ \vdots \\ b^-_{i_{m+n}} = v_{i_{m+2n}} \end{cases} \quad (4)$$

Let $I_3 = \{i_{m+1},...,i_{m+n},...,i_{m+2n}\}$.
The problem (2) is equivalent with:

$$\max l(\boldsymbol{\theta})$$

s.t. $\begin{cases} \mathbf{u}_i^T\boldsymbol{\theta} = v_i, i \in I_1 \\ \mathbf{u}_i^T\boldsymbol{\theta} \le v_i, i \in I_3 \end{cases}$ (5)

or

$$\max l(\boldsymbol{\theta})$$

s.t. $\begin{cases} \mathbf{u}_{i_k}^T\boldsymbol{\theta} = v_{i_k}, k = \overline{1,m} \\ \mathbf{u}_{i_k}^T\boldsymbol{\theta} \le v_{i_k}, k = \overline{m+1, m+2n} \end{cases}$ (6)

Under the assumptions that $\theta_1 = \mu_1,...,\theta_p = \mu_p$ and $f : R^{p+2n} \to R, f(\boldsymbol{\mu}) = (l(\boldsymbol{\theta}) + 0\cdot\mu_{p+1} + ... + 0\cdot\mu_{p+2n})$ where $\boldsymbol{\mu} \in R^{p+2n}, \mu = (\boldsymbol{\theta}, \mu_{p+1},...,\mu_{p+2n}) = (\theta_1,...,\theta_p,\mu_{p+1},...,\mu_{p+2n}) =$
$= (\mu_1,...,\mu_p,\mu_{p+1},...,\mu_{p+2n})$
we may formulate (6) as:

$$\min f(\boldsymbol{\mu})$$

$$\text{s.t.} \begin{cases} \mathbf{u}_{i_k}^T \boldsymbol{\theta} = v_{i_k}, k = \overline{1,m} \\ \mathbf{u}_{i_k}^T \boldsymbol{\theta} + \mu_{p+k-m} = v_{i_k}, k = \overline{m+1, m+2n} \end{cases} \tag{7}$$

In the matricial form we have:

$$\min f(\boldsymbol{\mu})$$
$$\text{s.t.} \begin{pmatrix} \mathbf{u}_{i_1}^T & 0 & \dots & & 0 \\ \vdots & \vdots & & & \vdots \\ \mathbf{u}_{i_m}^T & 0 & 0 & \dots & 0 \\ \mathbf{u}_{i_{m+1}}^T & 1 & 0 & \dots & 0 \\ \mathbf{u}_{i_{m+2}}^T & 0 & 1 & \dots & 0 \\ \vdots & \vdots & & & \vdots \\ \mathbf{u}_{m+2n-1}^T & 0 & \dots & 1 & 0 \\ \mathbf{u}_{i_{m+2n}}^T & 0 & \dots & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \mu_1 \\ \\ \\ \vdots \\ \vdots \\ \\ \\ \mu_{p+2n} \end{pmatrix} = \begin{pmatrix} v_{i_1} \\ \\ \\ \vdots \\ \vdots \\ \\ \\ v_{i_{m+2n}} \end{pmatrix} \tag{8}$$

or

$$\min f(\boldsymbol{\mu})$$
$$\begin{pmatrix} \mathbf{U}_1 & \mathbf{0} \\ \mathbf{U}_2 & \mathbf{I}_{2n} \end{pmatrix} \boldsymbol{\mu} = \mathbf{v} \tag{9}$$

where

$$\mathbf{U}_1 = \begin{pmatrix} \mathbf{u}_{i_1}^T \\ \vdots \\ \mathbf{u}_{i_m}^T \end{pmatrix}, \ \mathbf{U}_2 = \begin{pmatrix} \mathbf{u}_{i_{m+1}}^T \\ \vdots \\ \mathbf{u}_{i_{m+2n}}^T \end{pmatrix}.$$

If

$$\mathbf{A} = \begin{pmatrix} \mathbf{U}_1 & \mathbf{0} \\ \mathbf{U}_2 & \mathbf{I}_{2n} \end{pmatrix}$$

then (9) is equivalent with

$$\min f(\boldsymbol{\mu})$$
$$\text{s.t.} \tag{10}$$
$$\mathbf{A}\boldsymbol{\mu} = \mathbf{v}$$

where

$\boldsymbol{\mu} \in R^{p+2n}, \mathbf{A} \in M_{m+2n,p+2n}(R), \mathbf{v} \in R^{m+2n}$

or

$$\min f(\boldsymbol{\mu})$$
$$\text{s.t.} \qquad (11)$$
$$\boldsymbol{\mu} \in X$$

where

$$X = \{\mathbf{x} \in R^{p+2n} / \mathbf{Ax} = \mathbf{v}, \mathbf{x} \geq \mathbf{0}\}$$

Discussion and conclusion

The problem in the form (11) is suitable for applying a variant of Frank-Wolfe method (the regularized algorithm-RFW) (see Migdalas [5]):

For $\boldsymbol{\mu}^k \in X$, the objective function f is approximated by $\nabla f(\boldsymbol{\mu}^k)^T \boldsymbol{\mu}$ and (11) becomes:

$$\min \nabla f(\boldsymbol{\mu}^k)^T \boldsymbol{\mu}$$
$$\text{s.t.} \qquad (12)$$
$$\boldsymbol{\mu} \in X$$

The regularization of the problem means that an additional term appears in the objective function such that the distance between the iteration point $\boldsymbol{\mu}^k$ and the solution $\tilde{\boldsymbol{\mu}}^k$ is restricted. It is proved [2] that the point $\boldsymbol{\mu}^k$ is a solution for (11) if and only if it verifies the regularized subproblem:

$$\min \nabla f(\boldsymbol{\mu}^k)^T \boldsymbol{\mu} + t_k \phi(\boldsymbol{\mu}, \boldsymbol{\mu}^k)$$
$$\text{s.t.} \qquad (13)$$
$$\boldsymbol{\mu} \in X$$

Moreover, the regularized Frank-Wolfe algorithm, given below, is convergent [4,5].

-Step 1: consider $\boldsymbol{\mu}^0 \in X, t_0 = t > 0, k = 0$.

-Step 2: consider $\boldsymbol{\mu}^k$ the solution for (11) and let $\mathbf{d}^k = \tilde{\boldsymbol{\mu}}^k - \boldsymbol{\mu}^k$. If $\mathbf{d}^k = \mathbf{0}$, stop.

-Step 3: for $\tilde{\alpha}^k = \max\{\alpha / \boldsymbol{\mu}^k + \alpha^k \mathbf{d}^k\} \in X$ seek after $\alpha^k \in \operatorname{argmin}\{f(\boldsymbol{\mu}^k + \alpha^k \mathbf{d}^k), \alpha \in [0, \tilde{\alpha}^k]\}$. Let $\boldsymbol{\mu}^{k+1} = \boldsymbol{\mu}^k + \alpha^k \mathbf{d}^k, t_{k+1} = t_k, k = k+1$. Go to step 2.

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STRUCTURAL MODELING AND ANALYSIS OF INTELLIGENT MOBILE LEARNING ENVIRONMENT: A GRAPH THEORETIC SYSTEM APPROACH

Nitin UPADHYAY[1]

Computer Science & Information Group, BITS-PILANI, Goa Campus, India

E-mail: upadhyay.nitin@gmail.com, nitinu@bits-goa.ac.in

Vishnu Prakash AGARWAL[2]

PhD, Mechanical Engineering Group, BITS-PILANI, Goa Campus, India

Abstract: *This paper presents a new methodology using graph theory and matrix algebra to analyze software architecture based on systems engineering approach. It proposes a set of analytical tool to capture the notion of structural model as the basis to analyze characteristics of software architecture. In the present work, architecture (structure) modeling and analysis of intelligent mobile learning environment (iMLE) are presented that describe characteristics of performance, quality and reliability.*

Key words: *intelligent mobile learning environment; mobile agent; m-learning; agent; intelligent tutoring system; system structure; graph theory; matrix approach; variable permanent function (VPF)*

1. Introduction

Much work has been done during the last two decades in modeling and analyzing software architecture for various characteristics such as- performance, quality, reliability etc. Systems engineering has evolved as a novel approach to model software architectures. It is proposed that the structural/system modeling technique **[Saradhi M., 1992]** acts as a framework through which components, attributes, inter-relationship, and inter-dependencies within and across the system are expressed. It has been shown by researchers that the overall performance of a system depends upon the interaction/interdependence of its systems and subsystems **[Maes et al., 1998; Gray, 1997; Papaionnou and Edwards, 1998; Nick et al., 2000; Maes and Guttman, 1998]**.

The work reported here addresses the fundamental issue of how to analyze software architecture based on systems engineering approach. Our contribution is to propose a mathematical model, which analyzes without loss of generality all the flow, information, control, semantics, static and dynamic behaviors of systems and sub-systems using graph theory, matrix algebra, and permanent function. The present work, deals with the modeling and analysis of intelligent mobile learning environment. The methodology proposed here can work for all other software (system) architecture as well. The methodology is so strong that it can analyze all aspects of ***iMLE*** architecture without losing any information and optimize the characteristics associated with it.

2. Literature survey & related work

The process of learning has undergone revolutionary changes. The system of education has now crossed its geographical and time limit only because of the availability of high bandwidth infrastructure (such as 3G, GPRS and UMTS networks), advances in wireless technologies **[Chen and Nahrstedt, 2000; Chiang, et al., 1998; Johnson and Maltz, 1996; Chen and Lai, 2000; Lin and Liu, 1999]** and acceptance of handheld devices [**Microsoft, 2001**]. Now, e-learning system is moving from first generation to second generation. The integration of artificial intelligence and e-learning is identified as second generation learning or ***ITS*** **[Upadhyay, 2006]**. Integrating mobile computing with e-learning has given rise to new promising field known as mobile learning (m-learning) **[Upadhyay, 2006; Lehner and Nösekabel, 2002]**. In order to improve efficiency and performance of education systems various architectures have been developed and deployed.

In most of the analysis of intelligent m-learning education systems the researchers mainly consider the optimization of the characteristics of education systems from the aspects of autonomous behavior, quality and security. This may be a time bound solution. But in the long term, the performance of other sub-systems will affect the performance of the ***iMLE*** as a whole and hence whatever has been optimized may not be good. Therefore, an appropriate systems approach is best for identifying a permanent solution over the expected life cycle of the ***iMLE.***

The authors are not aware of any study that integrates all the subsystems and system of ***iMLE***. Researchers have identified that the performance of any system is a function of its basic architecture (i.e. layout and design). The understanding of systems architecture and its connectivity and interactions between different systems and down to component level is useful for estimating the contribution of different attributes of the performance of the system. The performance of complete ***iMLE*** (e.g. intelligence, adaptability, quality, availability, reliability) depends upon the performance of its macro level systems and interconnections in an integrated manner. Currently no effective mathematical model is present for studying these aspects in relation with each other or independently.

An attempt is made in this paper to represent the architecture of intelligent mobile learning environment mathematically and a methodology to model complete structure of ***iMLE*** consisting of its macro systems. This is achieved with the help of graph theory, matrix-algebra, and permanent function. This tool has so far been used by various authors to study a sub system for a particular attribute of the performance of a system in thermal power plant **[Mohan et al., 2003]**, nuclear plant **[Sacks et al., 1983],** selection of rolling elements of bearings **[Seghal et al., 2000],** maintainability index **[Gandhi et al., 1991],** but so far it has not been used to model and analyze software architectures with special emphasis to an intelligent mobile learning environment.

For rapid software development, software designers are encouraged to integrate commercial-off-the-shelf (COTS) components in their software systems. Component-based software engineering, in particular, **[Cai et al., 2000; Kozaczynski W., and Booch G., 1998]** has drawn tremendous attention in developing cost-effective and reliable applications to meet short time-to-market requirements. Performance, reliability, quality and other characteristics of software architectures are mostly analyzed and measured only at the time of implementing artifacts. It has been identified by industry and academia that investing in architecture design in the early phase of lifecycle is of paramount importance to a project's success **[Bosch J., 2000; Clements et, al., 2002; Kruchten P., 1995; Shaw M., and Garlan D., 1996]**. The basic structure of the system (software architecture) contributes approximately 30% value to various attributes associated to it. For instance, in order to obtain the value of performance of software architecture following formula can be evaluated:

Performance = (structure, $\boldsymbol{a}_i$)

Where $\boldsymbol{a}_i$ (i = 1,…,n; Load balancing, Priority, Assignment, Scheduling etc.,) are the attributes other than the basic structure of the software architecture which can affect it.

Most of the research is done in optimizing these attributes **[Goel A.L., and Okumoto K., 1979; Jelinski, Z. and Moranda, P. B., 2001; Littlewood, B.A., and Vernall, J.L., 1973; Musa J.D., and Okumoto K., 1984;]**. The optimization of characteristics such as reliability and testing is based on software development and testing rather than on complete software structure is addressed in **[Lyu et. al., 2002]**. Some research point out that software reliability and performance cannot be assessed at the architectural level **[Medvidovic N., and Taylor R., 1998]**. In structural statistical software testing (SSST) model reliability issues are evaluated by considering components independently **[Lyu M. R., 1996; May J. H. R., and Lunn A. D., 1995; May J. H. R., and Lunn A. D., 1995]**. Issues such as reliability, safety, security and availability comprise software dependability **[Littlewood B., and Strigini L., 2000; Randell B., 1995]**. However, there is no standard representation for dependability in model driven architecture thus lacking in complete optimizing of software architecture characteristics. The reliability estimation is also proposed using modular approach **[Woit D., 1997]**. It supposed that software system can be divided into independent components and each component has associated reliability as provided by the vendor. The overall system reliability can be calculated using well know Markov analysis techniques in software system. However the approach does not take into account the reliability of interactions between pair of components.

Our mathematical model preserves all inter-relationships, inter-dependencies, interactions within and across the systems and sub-systems in a single multinomial function. This model also permits us to evaluate various characteristics such as-performance, quality, reliability etc., associated with software architectures. In the previous works researchers were mainly concern about the evaluation of systems/sub-systems (components) and interactions independently. The limitation to these approaches results in not fully optimizing the overall system characteristics as the approaches do not analyze or evaluate all information (components and interactions) together. Our contribution is the major break through in optimizing the overall system characteristics by giving special emphasis to evaluate and analyze structural modeling aspects of software architectures based on system engineering approach. No study deals with the aspect of modeling and analyzing characteristics of software architectures concurrently but our mathematical model does it efficiently.

3. Identification of system

A top-level system is viewed as a combination of various systems and subsystems. The structure of ***iMLE*** is dependent on the elements contained in the boundary and their interconnections. In order to perform complete designing and analysis of ***iMLE***, we also have to consider contributing factors other than the main physical sub-systems and their interconnections. A subsystem is a system in itself.

To define an intelligent mobile learning environment engineering process, an outline of the necessary tools and procedure to support it is required. Initially, system requirement is identified which is broken down for further analysis, generating its own set of requirements. The whole process is repeated containing more detailed view of the system and sub-systems, until the component level is reached. The prime objective of system approach is to facilitate through evaluation and proper accommodation of new concepts and technology in ***iMLE*** design. On the basis of critical review **[Fabiano et al., 2003; Capuano et al., 2000 ; Oana et al**., **2005; Pesty and Webber, 2004; Tang and Wu, 2000]**, different sub-systems are identified which are further combined to produce five generic sub-systems as shown in Figure 1.

1. Intelligent Tutoring System (***ITS***).
2. Multiagent Intelligent System (***MIS***).
3. Mobile Dimension System (***MDS***).
4. Environment and Human Aspect System (***EHAS***).
5. Mobile Agent System (***MoAS***).

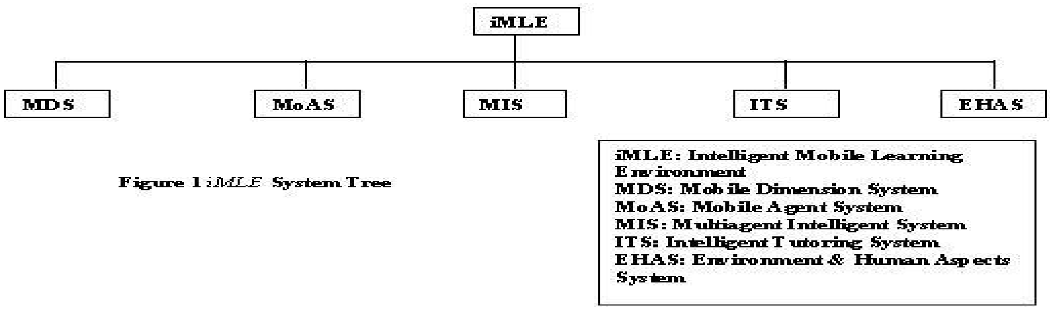

Figure 1 *iMLE* **System Tree**

Mobile learning application industry is free to identify a different set of subsystems as per its requirements, aims and objectives. Interaction and interdependency of various subsystems from the point of view of business, researches, maintenance etc. is the basis to understand the function and performance of ***iMLE***.

Figure 1 does not show interactions between sub-systems. In real application interactions are present among these sub-systems. An attempt is made to identify different types of interactions/interdependencies or information flow between these sub-systems under different situations. For better understanding the system tree diagram, Figure 1 is modified to include all the interactions and is shown as block diagram, Figure 2.

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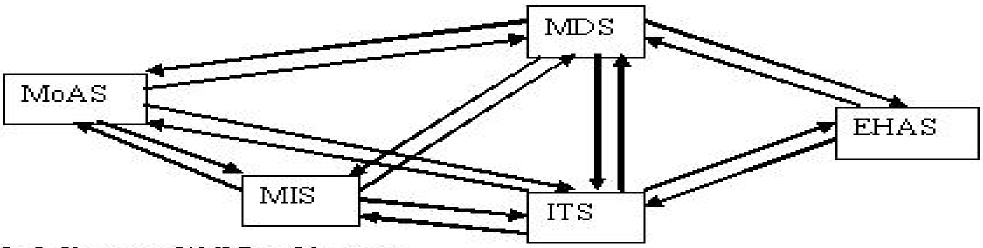


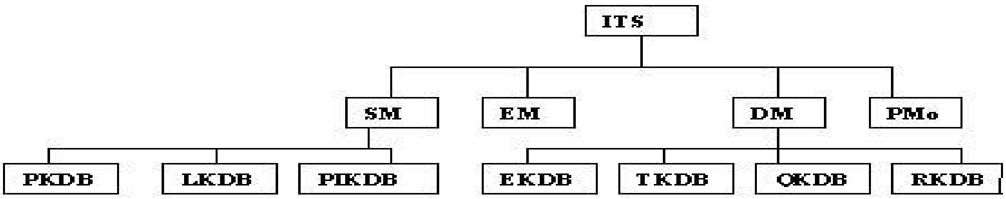
Figure 2 Block diagram of *iMLE* architecture.

3.1. Composition of subsystems and sub-subsystems of imle

The following sub-sub systems of ***iMLE*** sub-systems are proposed:

3.1.1. iMLE Subsystem - Intelligent Tutoring System (*ITS*):

Three major subsystems characterize the ***ITS*** **[Upadhyay, 2006],** Figure 3 - the **Student Model**, the **Domain Model** and the **Pedagogical Module**. A new subsystem **Education Model** adds functionality for the teacher.



iMLE subsystem - ITS
SM: Student Model
DM: Domain Model
PMo: Pedagogical Module
EM: Education Model

PKDB: Profiles Knowledge Database
LKDB: Learning Knowledge Database
PIKDB: Personal Information Knowledge Database
RKDB: Reinforcement Knowledge Database
QT KDB: Questions test Knowledge Database
EKDB: Exercises Knowledge Database
TKDB: Theory Knowledge Database

Figure 3 *subsystems* of Intelligent Tutoring System

Student Model

In Student Model, knowledge about the students is maintained, which is obtained by means of their profile and interaction with the system. It consists of three subsystems - knowledge databases (KDBs):

Personal Information KDB: It maintains personal identification, which allows access control to learning perspective through system.

Profile KDB: It manages student level (beginner, intermediate and advanced) and presentation styles (font style, color, size, background etc.)

Learning KDB: It maintains information about students learning history such as page visited, scrolls performed, number of hits/clicks, exercise and tests attempted so far etc.

Domain Model

In Domain Model, knowledge about the contents to be taught is stored. It consists of four subsystems i.e. knowledge databases:

Content KDB: It manages content pages to be used for teaching purpose.

Test KDB: It maintains test questionnaire on the specific contents for different levels.

Exercise KDB: It maintains exercises on the specific contents for different levels.

Reinforcement KDB: It maintains information to be shown to students for better learning. This is done by analyzing information from pedagogical module.

Pedagogical Module

In Pedagogical module, critical analysis is done for effectively presenting the subject matter to the student. It performs three main tasks:

- It provides learning guidelines.
- It updates domain model statistics.
- It keeps record of reinforcement information in learning KDB.

Education Model

In Education Model, functions necessary for teaching are managed. Using this model teacher can change the contents of the subject matter on the basis of information obtained from the Student Model and Domain Model. For effective teaching, teacher can change preferences (presentation styles, color, background etc.), give reinforcement to students, obtain statistics and consult the subject matter.

3.1.2. iMLE Subsystem – Multiagent Intelligent System (MIS)

The MIS comprises four subsystems, Figure 4, as follows:

Exercise agent: The exercise agent looks after the exercise that a student has to deal with depending upon student level of understanding and the content that student has covered. The exercise agent by its own means (pro-active) also provides link to the subject content pages relevant to the proposed exercises.

Preference agent: The preference agent is responsible for maintaining the student (user) choice state of interaction as compatible with MUI.

Account agent: The accounting agent perceives the interaction between user and the MUI when the student accesses content page. This agent keeps track of the scroll and time spent on each page of content. When the student shifts to some other content then account agent stores all parameters in learning KDB.

Test Agent: The test agent is responsible for proposing test as per student level. The test agent by its own means (pro-active) works for the designing of test for the particular topic/content. The test is shown to the student in the form of questionnaires. The test and exercise agent both work synchronously.

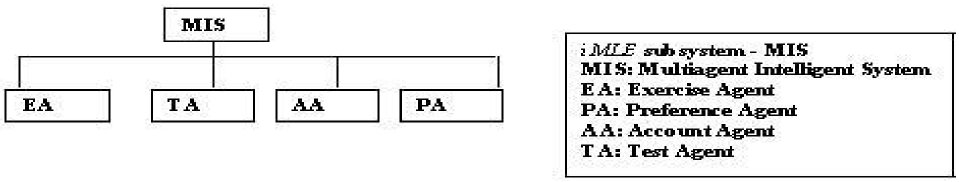


Figure 4 subsystems of Multiagent Intelligent System

3.1.3. iMLE Subsystem – Mobile Dimension System (MDS)

The critical subsystems of MDS, Figure 5, are:

Multimodal User Interface (MUI)

For desktop/PC applications, use of keyboard, mouse and monitor have been widely accepted. But mobile application needs additional mode of interaction such as voice user interfaces, smaller displays, stylus and other pointing devices, touch screen displays, and miniature keyboards.

Platform (PF)

The scalability issue in mobile devices leads to manufacturing of small size mobile devices. Theses devices are composed of less hardware in comparison to PC/Desktops. It is advisable to write program/application for different compatible platforms only if not needed for specific one for some performance reasons.

Device Capability (DC)

The physical size limitation imposes boundaries on volatile storage, non-volatile storage, and CPU on mobile devices. Storage and processing issues are largely addressed by the various operating systems and platforms on the mobile devices. Limited power supply results by putting constraints on limited size and usage on batteries instead of AC power supply.

Active Behavior (AB)

The two main subsystems of active behavior are:

Synchronous system: These behaviors are time dependent transactions. Here transaction is used in data storage and other systems to indicate boundaries for roll-back and committing of a series of actions that must be executed successfully,` in some predefined manner, for the completion of transactions.

Asynchronous system: These behaviors are time independent transactions.

Wireless Environment (WE)

Whether wired or wireless connectivity is used, mobility means loss of reliability in network connectivity. In the case of wireless network connectivity, physical conditions can significantly affect the quality of service (QoS). For example bad weather, solar flares, and a variety of other climate-related conditions can negate QoS.

Context Awareness (CA)

Context awareness consists of various subsystems as follows:

Location awareness: It deals with the sensing of desired location service in mobile applications.

Environment awareness: Collects information related to environmental conditions and variations such as humidity scale etc.

Situation awareness: It is responsible for specific situation such as light condition, sound and orientation of display.

User recognition awareness: It manages the automatic identification of user login.

Personalization awareness: It perceives the personal information of the user for example font style and color, theme, background color and avatar.

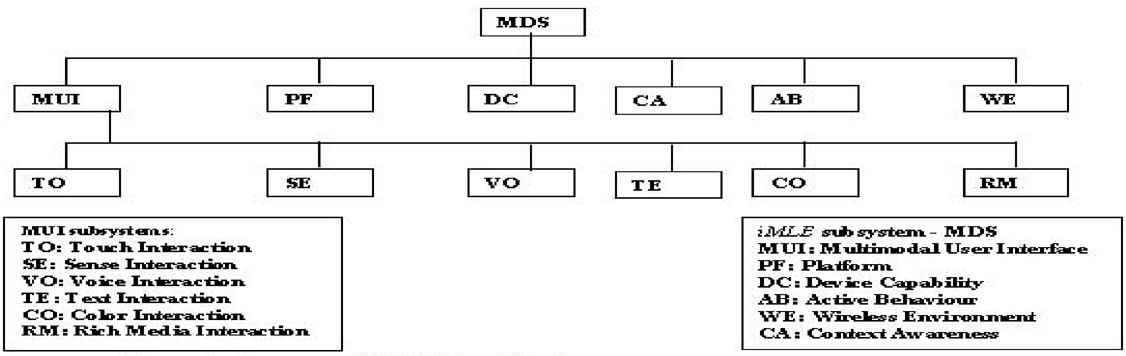


Figure 5 *subsystems* **of Mobile Dimension System**

3.1.4. iMLE Subsystem - Environment & Human Aspects System (EHAS)

Two main subsystems have been identified, Figure 6, as **EC**: Environment Condition and **HH**: Human Handling systems. Both theses subsystems affect the functionality of ***iMLE*** architecture. To ensure quality, performance and reliability these two subsystems have to be perfectly sound and fixed from all internal and external disturbances.

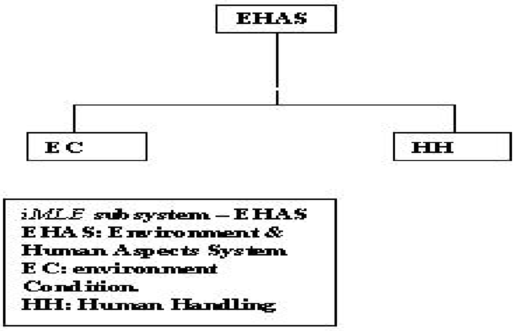


Figure 6 *sub systems* **of Environment & Human Aspects System**

3.1.5. iMLE Subsystem - Mobile Agent System (MoAS)

Two types of agents as shown in Figure 7: **agent wrappers** and **mediation agents.** They are the subsystems identified **[Bee-Gent and Plangent, 2003]** for **MoAS.** The functionality of both is as follows:

Agent Wrappers: They are used to incorporate agents on existing application. Wrapper agents manage the states of applications. They are also responsible for invoking the application as required.

Mediation Agents: They are responsible for all sorts of inter-application communication among applications. They migrate from an application site to another where they interact with remote agent wrappers.

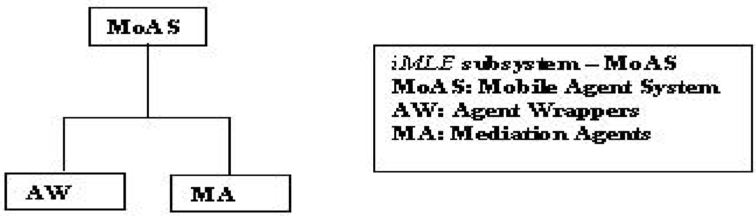


Figure 7 *sub systems* **of Mobile Agent System**

4. Hierarchical tree structure of iMLE

To compute overall designing and analysis of ***iMLE*** system, a "top-down" approach is used. In this, systems, sub-systems, sub-sub-systems etc are identified up to the component level. This tree structure allows all the parts to be designed from components level to the system level in the hierarchical order by using "bottom-up" approach. This helps to ensure design and geometric compatibility in the system. In general, the hierarchical tree structure may have (n+1) levels as given below:

Level – 0: Complete ***iMLE*** (system)
Level – 1: Sub systems (s-systems)
Level – 2: Sub sub systems (ss-systems)
Level – 3: Sub sub sub systems (sss-systems)
|
|
Level – n: Component level (Component)
A four level tree structure of typical ***iMLE*** system and ITS sub system is proposed in Figure 3 as:
Level – 0: Complete ***iMLE*** (system)
Level – 1: ***ITS, MoAS, MIS*** etc., sub systems (s-system)
As an example for ***ITS*** subsystem (Figure 3)
Level – 2: ***Student Model, Domain Model*** etc., sub sub systems (ss-system)
Level – 3: ***Profile KDB, Learning KDB*** etc., is components' level (Components)
Similarly level 2 and 3 are developed for remaining four subsystems as shown in Figure 4-7.
The hierarchical trees of ***iMLE*** structure may differ depending upon the choices of the distinct systems up to the component level. Identification of tree structure helps in full understanding of ***IMLE*** system engineering process and acts as an asset in improving efficiency, quality, maintainability and reliability of the system.

5. Graph theoretic modelling of system architecture

A system graph $\mathbf{G_s}$ = [**S, E**] is used to model system architecture by applying graph theory using linear graph. Let each of the five systems of ***iMLE*** be represented by $\mathbf{S_i}$ (i=1,…,5) and interconnections between them ($\mathbf{S_i}$, $\mathbf{S_j}$) as edge set **E** by edges $\mathbf{e_{ij}}$ (i,j = 1,…,5) connecting the two vertices $\mathbf{S_i}$ and $\mathbf{S_j}$. The graph theoretic representation [**S, E**] of vertex and edge sets of the five-system of ***iMLE*** is called the ***iMLE*** system structure graph. Various types of edges and weights can differentiate the type of connections and interconnections. The undirected edges show the connectivity between (sub) systems or components and the directed edges represent the flow of information or interaction.

The system structure graph (**SSG**) of ***iMLE*** is shown in Figure 8. The five nodes represent respective systems of ***iMLE*** and edges corresponding to the connections/interactions between the subsystems. Connectivity, interdependence and interactions between systems are shown by undirected, directed and dashed edges is shown. If the two systems are interdependent on each other then the relation is shown by opposite arrow edge. If one system is influencing the other then directed edge characterizes this influence. Physical connectivity is represented simply by undirected edge. Dashed edge represents weak or indirect connection.

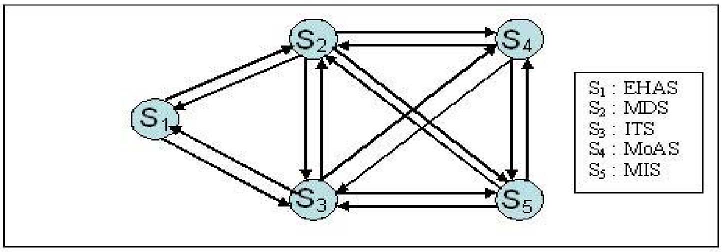


Fig. 8(a). Directed Graph

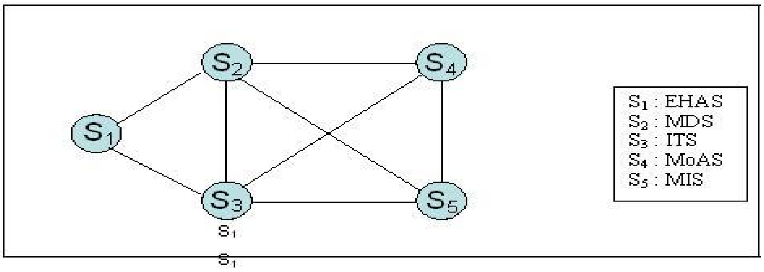


Fig. 8(b). Undirected Graph

Figure 8. System Structure Graph of ***iMLE***

These systems of the ***iMLE*** are also connected physically or indirectly at the level of their sub-systems. Graph theoretic architecture model is used to represent direct, undirected or hybrid interactions between subsystems. A real life ***iMLE*** is represented graphically by directed graph Figure 8a and undirected graph Figure 8b. The connectivity may be directed or undirected depending upon the structural, functional or performance considerations. The ***iMLE SSG*** is capable of updating, modifying and deleting of systems or sub-systems based on different design aspects as per real life situation. The proposed ***SSG*** representation is suitable for understanding and visual analysis, but not appropriate for computer processing. If the number of systems is more, then the overall system becomes more complex for understanding and visual analysis. Moreover, changing of labels of vertices/systems results into new ***SSG***. In view of this, we present computer efficient representation. Many matrix representations are available in the literature **[Deo, 2004; Upadhyay, 2004],** for example, adjacency and incidence matrices. The adjacency matrix is a square matrix and used for this purpose. Using this ***iMLE*** is represented in matrix form.

6. Matrix models

The adjacency matrices of the ***SSG*** are defined to find out which matrix is more suitable to represent ***iMLE***. The matrix should be flexible enough to incorporate the structural information of subsystems and interconnections between them.

6.1. System structure matrix [adjacency matrix] (VAM- iMLE) of iMLE

An incidence matrix can be used to understand the number of connections and how these connect the sub systems. As the resultant matrix is non-square matrix, its further use for system analysis or its derivatives is not very useful. An alternative to incidence matrix, adjacency matrix representation is used to show the connectivity and graph representation. The adjacency matrix **[Deo, 2004; Jurkat and Ryser, 1996]** of a graph **G** = [**V, E**] with '**n**' nodes is an '**n**' order symmetric binary (**0, 1**) square matrix, and e_{ij} representing the connectivity between systems i and j such that:

$\mathbf{e_{ij}}$ = 1, if the sub system '**i**' is connected/interacted to the sub system '**j**' and
= 0, otherwise.

However, $\mathbf{e_{ii}}$ = 0, as subsystem is not connected to itself. In a case where it is connected to itself $\mathbf{e_{ii}}$ = 1. This implies a self-loop at node '**i**' in the graph.

In the (0, 1) adjacency matrix each row and column of the system structure matrix corresponds to a subsystem. The off-diagonal elements $\mathbf{e_{ij}}$ in the matrix represent connection between systems i and j. In this matrix, $\mathbf{e_{ij} = e_{ji} = 1}$ as only connections between systems are considered. The adjacency matrix for a graph as shown in Figure 8 (b) is given below as:

1 2 3 4 5 Subsystems

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \qquad (1)$$

The (**0, 1**) adjacency matrix does not contain properties/attributes characterizing different interactions/ connections between different subsystems. It only represents the system connectivity. As the matrix is a square matrix its evaluation is possible. In order to get information about the structural characteristics of the system, we associate variable with the elements of adjacency matrix.

In order to show connectivity/interconnection/interdependence between different systems '**i**' and '**j**' of the ***iMLE***, let off-diagonal elements be represented by a symbol $\mathbf{e_{ij}}$ whose function will depend upon type of connection/interconnection. Adjacency matrix **A** = [$\mathbf{a_{ij}}$] will be (**0,** $\mathbf{e_{ij}}$) instead of (**0, 1**) matrix. The $\mathbf{e_{ij}}$ also provides information about the flow from one subsystem to the other. Variable adjacency matrix (VAM- ***iMLE***) of the system shown in Fifure 8 is proposed below assuming $\mathbf{e_{ij} = e_{ji}}$ as:

1 2 3 4 5 Subsystems

$$V_A = \begin{bmatrix} 0 & e_{12} & e_{13} & 0 & 0 \\ e_{12} & 0 & e_{23} & e_{24} & e_{25} \\ e_{13} & e_{23} & 0 & e_{34} & e_{35} \\ 0 & e_{24} & e_{34} & 0 & e_{45} \\ 0 & e_{25} & e_{35} & e_{45} & 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \qquad (2)$$

The $\mathbf{e_{ij}}$, of (VAM- ***iMLE***) apart from representing connectivity also represents influence of structural performance characteristics of $\mathbf{i^{th}}$ subsystem on $\mathbf{j^{th}}$ sub system, change of $\mathbf{i^{th}}$ subsystem affecting the structural performance of $\mathbf{j^{th}}$ sub system etc., according to the particular analysis of ***iMLE***. Hence, this is the complete representation of interconnection/interdependence of ***iMLE***. As this matrix also does not infer anything about the characteristic features of the systems a new matrix called '***characteristic system structure matrix***' is defined.

6.2. Characteristic system structure matrix (CSSM- iMLE)

By defining characteristic system structure matrix '**C**', realization of the presence of different systems (based upon system structure) can be done. The ***iMLE*** characteristic system structure matrix (CSSM- ***iMLE***) corresponding to the systems graph in Figure 8 is given below:

C = {**SI – A**}

$$\mathbf{C} = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & \text{Subsystems} \end{matrix} \\ \begin{bmatrix} S & -1 & -1 & 0 & 0 \\ -1 & S & -1 & -1 & -1 \\ -1 & -1 & S & -1 & -1 \\ 0 & -1 & -1 & S & -1 \\ 0 & -1 & -1 & -1 & S \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \end{array} \quad (3)$$

Where ***I*** is the identity matrix and ***S*** is used as a variable to represent systems characteristic features of the basic structure. This matrix is similar to the characteristic matrix defined in graph theory **[Deo, 2004]**. The characteristic of a system can be reliability, security, availability etc. It can be inferred from the matrix that, it is capable of representing the presence of systems and interconnection between them. It does not include information about the attributes of the connections among subsystems. The determinant of CSSM- ***iMLE*** is called characteristic system structure polynomial (**CP- s**). The **CP-s** of the matrix is shown below:

Det (C) = $S^5 - 8S^3 - 10S^2 - S + 2$

The **CP-s** of the matrix is invariant of the system **[Deo, 2004]** as it does not change by modifying labeling of systems (vertices) and is the characteristic of the systems structure. It can be inferred that CSSM- ***iMLE*** is not an invariant of system, as new matrix can be obtained by changing labels of systems. Also, diagonal elements show that identical systems are present in the basic structure. This is one of the reasons that make the **CP-s** of CSSM- ***iMLE*** non-unique and incomplete representation of any real system. It has been identified in literature that many graphs belong to the same family known as co-spectral graphs on the basis of having same **CP-s**. To present distinct information of different systems and interconnections between them, a matrix called a ***variable characteristic system structure matrix*** (VCSSM- ***iMLE***) is proposed.

6.3. Variable characteristic system structure matrix (VCSSM- iMLE)

A variable characteristic system structure matrix $\mathbf{V_c}$ is defined by taking into consideration distinct characteristics of subsystems and their interconnections defined by ***SSG***. Let the off-diagonal elements matrix **F** consists of $\mathbf{e_{ij}}$ rather than **1** to represent interaction/connectivity (system '**i**' is connected to system '**j**') and also $\mathbf{e_{ij} = e_{ji}}$. Let us also define diagonal matrix **D** with its variable diagonal elements $\mathbf{S_i}$ (i = 1, 2…., 5) representing the characteristic structure features of five distinct systems.

The VCSSM- ***iMLE*** $\mathbf{V_c}$ = [**D – F**] is written as:

$$\mathbf{V_C} = \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \text{ Subsystems} \\ \left[\begin{array}{ccccc} S_1 & -e_{12} & -e_{13} & 0 & 0 \\ -e_{12} & S_2 & -e_{23} & -e_{24} & -e_{25} \\ -e_{13} & -e_{23} & S_3 & -e_{34} & -e_{35} \\ 0 & -e_{24} & -e_{34} & S_4 & -e_{45} \\ 0 & -e_{25} & -e_{35} & -e_{45} & S_5 \end{array}\right]\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \end{array} \qquad (4)$$

The determinant of this (VCSSM- ***iMLE***) is known as variable characteristic multinomial and is written as VCM- ***iMLE***, the variable characteristic multinomial of the ***iMLE***.

Det($\mathbf{V_c}$) =

$$S_1S_2S_3S_4S_5 - e_{12}^2S_3S_4S_5 - e_{13}^2S_2S_4S_5 - e_{23}^2S_1S_4S_5 - e_{24}^2S_1S_3S_5 - e_{25}^2S_1S_3S_4 - e_{34}^2S_1S_2S_5 - e_{35}^2S_1S_2S_4$$
$$-e_{45}^2S_1S_2S_3 - 2e_{12}e_{13}e_{23}S_4S_5 - 2e_{23}e_{24}e_{34}S_1S_5 - 2e_{23}e_{25}e_{35}S_1S_4 - 2e_{24}e_{25}e_{45}S_1S_3 - 2e_{34}e_{35}e_{45}S_1S_2$$
$$-2e_{23}e_{34}e_{25}e_{45}S_1 - 2e_{23}e_{35}e_{24}e_{45}S_1 - 2e_{24}e_{35}e_{25}e_{34}S_1 - 2e_{12}e_{24}e_{13}e_{34}S_5 - 2e_{12}e_{25}e_{13}e_{35}S_4 + e_{13}^2e_{45}^2S_2$$
$$+e_{24}^2e_{35}^2S_1 + e_{25}^2e_{13}^2S_4 + e_{24}^2e_{13}^2S_5 + e_{23}^2e_{45}^2S_1 + e_{25}^2e_{34}^2S_1 + e_{12}^2e_{45}^2S_3 + e_{12}^2e_{34}^2S_5 + e_{12}^2e_{35}^2S_4 + 2e_{12}^2e_{34}e_{35}e_{45}$$
$$+2e_{45}^2e_{12}e_{23}e_{13} + 2e_{13}^2e_{24}e_{25}e_{45} - 2e_{12}e_{24}e_{13}e_{35}e_{45} - 2e_{12}e_{25}e_{13}e_{34}e_{45} \qquad (5)$$

The VCM- ***iMLE*** multinomial contains terms both of positive and negative signs. It is the comprehensive tool for analysis in symbolic form. While calculating VCM- ***iMLE*** value for ***iMLE*** analysis, some information about system, sub-systems, components and their connectivity is lost. This is due to the cancellation of some terms and subtraction operation in the process of computing VCM- ***iMLE***. In order to avoid loss of information during structural analysis and structural performance evaluation in critical cases, we propose a new matrix function, which will retain all the multinomial terms with no subtraction operation and hence preserve information about the system, sub systems, components and their interconnectivities, i.e. permanent/permanent function of matrix **[Mohan et al., 2003; Luo and Huang, 2005]**.

6.4. Variable permanent system structure matrix (VPSSM- *iMLE*)

In order to describe proper characterization of ***iMLE*** systems as derived from combinatorial considerations, a permanent matrix **P**, is proposed. The matrix function/permanent **Per(P)** of **VPSSM- *iMLE*** is capable of describing whole ***iMLE*** system i.e. system graph in a single multinomial equation **[Jurkat and Ryser, 1996]**. Let the complete permanent matrix of five-subsystem ***iMLE*** system with all possible interactions present be defined as

$$\mathbf{P} = \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \text{ Subsystems} \\ \left[\begin{array}{ccccc} S_1 & e_{12} & e_{13} & e_{14} & e_{15} \\ e_{12} & S_2 & e_{23} & e_{24} & e_{25} \\ e_{13} & e_{23} & S_3 & e_{34} & e_{35} \\ e_{14} & e_{24} & e_{34} & S_4 & e_{45} \\ e_{15} & e_{25} & e_{35} & e_{45} & S_5 \end{array}\right]\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \end{array} \qquad (6)$$

A variable permanent system structure matrix (VPSSM- ***iMLE***) '$\mathbf{V_p}$' of **SSG** with $\mathbf{e_{ij}} = \mathbf{e_{ji}}$ in Figure 8(b) is written as:

$\mathbf{V_p} = \{\mathbf{D} + \mathbf{F}\}$

$$\begin{array}{c} \begin{array}{cccccl} 1 & 2 & 3 & 4 & 5 & \text{Subsystems} \end{array} \\ V_p = \begin{bmatrix} S_1 & e_{12} & e_{13} & 0 & 0 \\ e_{12} & S_2 & e_{23} & e_{24} & e_{25} \\ e_{13} & e_{23} & S_3 & e_{34} & e_{35} \\ 0 & e_{24} & e_{34} & S_4 & e_{45} \\ 0 & e_{25} & e_{35} & e_{45} & S_5 \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \end{array} \qquad (7)$$

It is a complete representation of ***iMLE***, as it does not contain any negative sign. This means that it preserves all the structural information about dyads, loops of systems, or system attributes such as reliability, availability etc even in numerical form. The only difference between VCSSM- ***iMLE*** and VPSSM- ***iMLE*** is in the signs of off-diagonal elements. The VPF- ***iMLE*** for matrix is written as:

Per($\mathbf{V_p}$) =

$$\begin{aligned} & S_1S_2S_3S_4S_5 + [e_{12}^2S_3S_4S_5 + e_{13}^2S_2S_4S_5 + e_{23}^2S_1S_4S_5 + e_{24}^2S_1S_3S_5 + e_{25}^2S_1S_3S_4 + e_{34}^2S_1S_2S_5 + e_{35}^2S_1S_2S_4 + e_{45}^2S_1S_2S_3] \\ & + [2e_{12}e_{23}e_{31}S_4S_5 + 2e_{23}e_{34}e_{42}S_1S_5 + 2e_{23}e_{35}e_{52}S_1S_4 + 2e_{24}e_{45}e_{52}S_1S_3 + 2e_{34}e_{45}e_{53}S_1S_2] \\ & + \{[2e_{23}e_{34}e_{45}e_{52}S_1 + 2e_{23}e_{35}e_{54}e_{42}S_1 + 2e_{24}e_{43}e_{35}e_{52}S_1 + 2e_{12}e_{24}e_{43}e_{31}S_5 + 2e_{12}e_{25}e_{53}e_{31}S_4] + [e_{13}^2e_{45}^2S_2 + e_{24}^2e_{35}^2S_1 \\ & + e_{25}^2e_{13}^2S_4 + e_{24}^2e_{13}^2S_5 + e_{23}^2e_{45}^2S_1 + e_{25}^2e_{34}^2S_1 + e_{12}^2e_{45}^2S_3 + e_{12}^2e_{34}^2S_5 + e_{12}^2e_{35}^2S_4]\} \\ & + \{[2e_{12}^2e_{34}e_{45}e_{53} + 2e_{45}^2e_{12}e_{23}e_{31} + 2e_{13}^2e_{24}e_{45}e_{52}] + [2e_{12}e_{24}e_{45}e_{53}e_{31} + 2e_{12}e_{25}e_{54}e_{43}e_{31}]\} \end{aligned} \qquad (8)$$

It can be inferred that the terms present in VCM- ***iMLE*** and VPF- ***iMLE*** are the same but they differ in the signs. In VCM- ***iMLE*** terms consist of both positive and negative sign. But VPF- ***iMLE*** only contains terms of positive sign.

The above equation (multinomial) uniquely represents the ***iMLE*** of Figure 2 irrespective of labeling of subsystems. Every term of these equations represents a subset of the ***iMLE*** system. It is possible to write these equations simply by visual inspection of the ***iMLE*** system of Figure 8 as every term corresponds to a physical subsystem of thee complete system. To achieve this objective, the permanent function of Equation (8) is written in a standard form as (N + 1) groups. All these distinct combinations of subsystems and interactions of the macro system are shown graphically in Figure 9. The multinomial, i.e., the permanent function when written in (N + 1) groups, presents an exhaustive way of analysis of ***iMLE*** at different levels. It helps in identifying different critical components and links to improve reliability, fault tolerance, performance, quality, security, autonomy and availability of system.

On critical analysis of permanent function (8) it is inferred that this multinomial contains only distinct subsystems – $\mathbf{S_i}$, dyads - e_{ij}^2 and loops – $\mathbf{e_{ij}}$ $\mathbf{e_{jk}}$ …… $\mathbf{e_{ni}}$. A complete permanent function has been written in a systematic manner for unambiguous and unique interpretation. In short it can be represented as:

Per ($\mathbf{V_p}$) = g ($\mathbf{S_i}$, e_{ij}^2, $\mathbf{e_{ij}}$ $\mathbf{e_{jk}}$ $\mathbf{e_{ki}}$ etc) { if $\mathbf{e_{ij}} = \mathbf{e_{ji}}$}

= g (Vertices, dyads, loops)
= g (structural components)

Per (V_P) = g' (S_i, , $e_{ij}e_{ji}$, e_{ij} e_{jk} e_{kl} e_{li} , e_{ij} e_{jk} e_{kl} e_{lm} e_{mi}) { if $e_{ij} \neq e_{ji}$}
= g' (Vertices, 2-vertex loops, loops)
= g' (structural components)

The terms of the permanent function **Per (V_P)** are arranged in (n+ 1) groups in the decreasing order of number of vertices/sub-systems S_i present in each term. The first group contains terms with (n – 1) **S_i's.** Second group will contain terms with (n – 2) **S_i's** and remaining as dyad e_{ij}^2 or $e_{ij}e_{ji}$ and so on. The last group does not contain any S_i in its terms. It contains only terms such as e_{ij}^2, e_{ij} e_{jk} e_{ki} , etc.

Group 1: The first term (grouping) represents a set of **N** unconnected ***iMLE*** subsystems, i.e., S_1, S_2, …, S_n.

Group 2: Group is absent as a particular subsystem has no interaction with itself (absence of self-loops) i.e. any of the subsystem ***MDS, MoAS, ITS, MIS*** **or** ***EHAS*** is not connecting itself.

Group 3: Each term of the third grouping represents a set of two-element ***iMLE*** system loops (i.e.,S_{ij} S_{ji})and is the resultant ***iMLE*** system dependence of characteristics **i** and **j** and the ***iMLE*** system measure of the remaining (**N-2**) unconnected elements/subsystems. Group has eight terms, each term is a set of one dyad, e_{ij}^2 or a two-subsystem loop i.e. $e_{ij}e_{ji}$ and three independent subsystems (dyad is a system of two subsystems **i** and **j** , considered as one entity).

Group 4: Each term of the fourth grouping represents a set of three-element ***iMLE*** subsystem interaction loops (e_{ij} e_{jk} e_{ki} or its pair e_{kj} e_{ji}) and the composite system measure of the remaining (**N-3**) unconnected elements. Group has (2*5) 10 terms in all. Each term has a set of one 3-subsystem loop (e_{ij} e_{jk} e_{ki}) and independent subsystems. The three-subsystem loop is a system, to be considered as one entity.

Group 5: The fifth grouping contains two subgroups. The terms of the first subgrouping consist of two-element ***iMLE*** subsystem interaction loops (i.e., e_{ij} e_{ji} and e_{kl} e_{lk}) and ***iMLE*** constituent e_m .The terms in the second grouping are a product of four-element ***iMLE*** subsystem interaction loops (i.e., e_{ij} e_{jk} e_{kl} e_{li}) or its pair (i.e., e_{il} e_{lk} e_{kj} e_{ji})and ***iMLE*** constituent S_m . Group has two subgroups: Group 5(i) has ten terms; each term is a subset of two independent dyads (e_{ij}^2, e_{kl}^2) or two-subsystem loops and one independent subsystem. Group 5(ii) has nine terms; each term is a set of 4-subsystem loop (e_{ij} e_{jk} e_{kl} e_{li}) and one independent subsystem.

Group 6: The terms of the sixth grouping are also arranged in two sub-groupings. The terms of the first sub-grouping are a product of a two-element ***iMLE*** subsystem interaction loop (i.e., e_{ij} e_{ji}) and a three-element ***iMLE*** subsystem interaction loop (i.e., e_{kl} e_{lm} e_{mk})or its pair (i.e., e_{km} e_{ml} e_{lk}).The second sub-grouping consists of a five-component ***iMLE*** subsystem interaction loop (i.e., e_{ij} e_{jk} e_{kl} e_{lm} e_{mi})or its pair (e_{im} e_{ml} e_{lk} e_{kj} e_{ji}). Group has again two subgroups: Group 6(i) has one 3-subsystem loop and a dyad or two-subsystem loop while Group 6(ii) has three 5-subsystem loops.

By providing/associating proper physical meaning to the VPF-***iMLE*** structural components, appropriate interpretation is obtained:

- e_{ij}^2 is interpreted as a two-system structural dyad, for example, e_{35}^2 represents the dyad of interaction between **ITS** and **MIS** systems.
- $\mathbf{e_{ij}\ e_{jk}\ e_{ki}}$ is a three system structural loop, for example, $\mathbf{e_{12}\ e_{23}\ e_{31}}$ represents the three system structural loop between ***EHAS, MDS*** and ***ITS*** systems.
- $\mathbf{e_{ij}\ e_{jk}\ e_{kl}\ e_{li}}$ is a four system structural loop, for example, $\mathbf{e_{23}\ e_{34}\ e_{45}\ e_{52}}$ represents four system structural loop, between ***MDS, ITS, MoAS*** and ***MIS*** systems.

In all, a general 5-subsystem permanent function will have **5!** i.e., 120 terms (subsets) arranged in (N + 1) groups. Figure 9 gives graphical/physical interpretation of terms of different groups for visual understanding, analysis, and improvement of a ***iMLE*** system architecture. It is therefore possible for the system analyst and designer to carry out **SWOT** (strength-weakness-opportunities-threats) analysis of their complete ***iMLE*** system and take strategic decisions to their advantage as per policy.

7. Modular design and analysis of iMLE system

Different terms of permanent function, equation (8) of ***iMLE*** system Figure 8 represent different subsets Figure 9 of the system. As these terms consist of structural terms $\mathbf{S_i}$, e_{ij}^2 **/**$\mathbf{e_{ij}\ e_{ji}}$, $\mathbf{e_{ij}\ e_{jk}\ e_{kl}\ e_{li}}$**/**$\mathbf{e_{il}\ e_{lk}\ e_{kj}\ e_{ji}}$ **etc.,** global ***iMLE*** system solution providers offer different alternative solutions for each of these structural modules/subsystems. If there are **n** distinct terms in the permanent function, there are **n** ways of designing and analyzing the ***iMLE*** system. If the system is already in place, the **SWOT** analysis can help in improving the existing system. If the designer is using the proposed methodology to develop optimum design solution at conceptual stage, it can be done in the presence of available standard solutions of modules e.g. subsystems ($\mathbf{S_i}$), dyads (e_{ij}^2) and loops ($\mathbf{e_{ij}\ e_{jk}\ e_{kl}\ e_{li}}$ **)** etc. Thus physical representation of Figure 9 helps in analyzing and designing ***iMLE*** system comprehensively using the structural modules. This permanent function and its interpretations become basis of modular analysis and design of ***iMLE*** system.

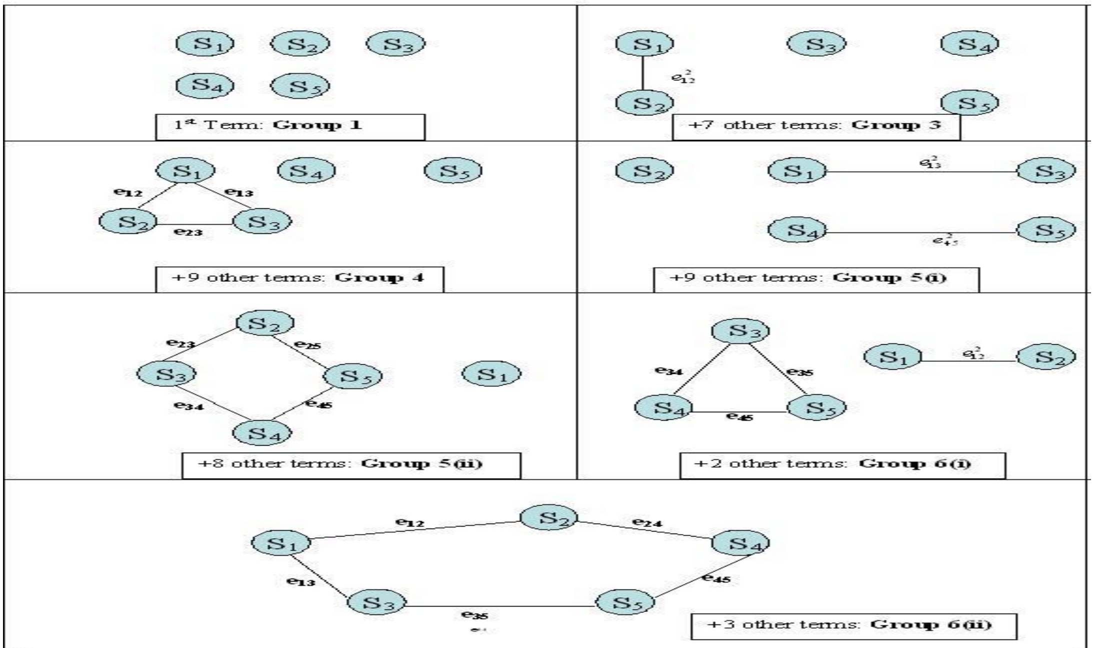


Figure 9 Graphical/Physical representation of Permanent Function Expression for *iMLE*

8. Evaluation of V_P

The diagonal elements of the matrix in equation (**7**) correspond to the five subsystems that constitute a ***iMLE*** system. The values of these diagonal elements S_1, S_2... S_5 are calculated as:

S_1= Per(V_PS_1) S_2= Per(V_PS_2) S_3= Per(V_PS_3)
S_4= Per(V_PS_4)S_5= Per(V_PS_5) (8a)

Where **V_P**S_1, **V_P**S_2, **V_P**S_3, **V_P**S_4, **V_P**S_5 are the variable permanent matrices for five subsystems of the ***iMLE*** system. The procedure for calculating **S_1**, **S_2**... **S_5** is the same as for calculating **Per**(**V_p**)of equation (**8**).For this purpose, the subsystems of ***iMLE*** system are considered, and the procedure given below is followed:

1. The schematics of these subsystems are drawn separately by considering their various sub-sub-systems.
2. Identify the degree of interactions, interconnections, dependencies, connectivity, etc. between different subsubsytems.

Digraph representations (like Figure 7) of five subsystems are drawn first separately to obtain their matrix equations (like Equation (**8**)) i.e. **V_pS_i** and then their permanent functions **Per(V_PS_i)**, **$S_{i,}$ i** = 1,...,5. The off-diagonal terms **e_{ij}** (**i ,j** = 1,2,...,5) of matrix equation (**7**) gives the connections between the systems **S_i** and **S_j** . Depending upon the type of structural analysis, **S_{ij}** can be represented as multinomial, graph, and matrix or by some analytical model. To get the exact degree of interactions, interconnections, dependencies, connectivity, etc. between subsystems or subsubsystems we may have to consider the views of technical team experts. A team of experts selected from system analyst, design, software engineering, computer science, information systems etc. to consider all the issues involved from the point of view of engineering, science, technology, and business strategy. The final decision on the values of **S_i** and **S_{ij}** may be taken on the recommendations of the team. Thus, following the top-down approach and the step-by-step procedure given below will give the complete structural analysis of the ***iMLE*** system.

9. Compact representation of permanent function

The variable permanent function (**VPSSM- *iMLE***) being the characteristic of ***iMLE*** system of any industrial product is a powerful tool for its evaluation and analysis. The **VPSSM- *iMLE*** system expression, which corresponds to the five-factor digraph and matrix, equation (**6**), is written in a compact sigma ($\sum$) form.

VPSSM- *iMLE* = Per(**V_P**)

$$\prod_{1}^{5} S_i + \sum_i\sum_j\sum_k\sum_l\sum_m \left(e_{ij}e_{ji}\right)S_kS_lS_m + \sum_i\sum_j\sum_k\sum_l\sum_m \left(e_{ij}e_{jk}e_{ki} + e_{ik}e_{kj}e_{ji}\right)S_lS_m$$
$$+\left(\sum_i\sum_j\sum_k\sum_l\sum_m \left(e_{ij}e_{jk}\right)\left(e_{kl}e_{lk}\right)S_m + \sum_i\sum_j\sum_k\sum_l\sum_m \left(\left(e_{ij}e_{jk}e_{kl}e_{li}\right)+\left(e_{il}e_{lk}e_{kj}e_{ji}\right)\right)S_m\right)$$
$$+\sum_i\sum_j\sum_k\sum_l\sum_m \left(e_{ij}e_{ji}\left(e_{kl}e_{lm}e_{mk} + e_{km}e_{ml}e_{lk}\right) + \sum_i\sum_j\sum_k\sum_l\sum_m \left(e_{ij}e_{jk}e_{kl}e_{lm}e_{mi} + e_{im}e_{ml}e_{lk}e_{kj}e_{ji}\right)\right) \quad (9)$$

The above equation is a generalized mathematical expression in symbolic form corresponding to five-factor digraph representation. It ensures an estimate of the ***iMLE*** system of any industrially integrated product. The above equation contains **5!**terms. Each term is useful for a ***iMLE*** designer as each term serves as a test for the effectiveness of the relevant group in **Per**($\mathbf{V_P}$).

10. Generalization of methodology

Suppose a system consists of **N** subsystems in place of proposed five subsystems and is represented as a digraph, then the most general way of matrix representation is shown below. This matrix is also known as the variable permanent matrix (**VPSSM- *iMLE***) corresponding to the **N** subsystems.

$$
\begin{array}{cccccccc|c}
1 & 2 & 3 & . & . & . & . & N & \text{Subsystems} \\
S_1 & e_{12} & e_{13} & . & . & . & . & e_{1N} & 1 \\
e_{21} & S_2 & e_{23} & . & . & . & . & e_{2N} & 2 \\
e_{31} & e_{32} & S_3 & . & . & . & . & e_{3N} & 3 \\
. & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . \\
e_{N1} & e_{N2} & e_{N3} & . & . & . & . & S_N & N
\end{array}
\qquad (10)
$$

Permanent for the above matrix, i.e., **Per**($\mathbf{V_P}$)is called variable permanent function (**VPSSM- *iMLE***).The **VPSSM- *iMLE*** for the above matrix is written in sigma form as

$$
\begin{aligned}
\text{Per}(V_P) = {} & \prod_{x=1}^{N} S_x + \sum_i \sum_j \sum_k \sum_l \cdots \sum_N \left(e_{ij}e_{ji}\right) S_k S_l \ldots S_N + \sum_i \sum_j \sum_k \sum_l \cdots \sum_N \left(e_{ij}e_{jk}e_{ki} + e_{ik}e_{kj}e_{ji}\right) S_l S_m \ldots S_N \\
& + \left(\sum_i \sum_j \sum_k \sum_l \cdots \sum_N \left(e_{ij}e_{jk}\right)\left(e_{kl}e_{lk}\right) S_m \ldots S_N + \sum_i \sum_j \sum_k \sum_l \cdots \sum_m \left(\left(e_{ij}e_{jk}e_{kl}e_{li}\right) + \left(e_{il}e_{lk}e_{kj}e_{ji}\right)\right) S_m \ldots S_N \right) \\
& + \sum_i \sum_j \sum_k \sum_l \cdots \sum_N \left(e_{ij}e_{ji}\left(e_{kl}e_{lm}e_{mk} + e_{km}e_{ml}e_{lk}\right) S_n S_o \ldots S_N + \sum_i \sum_j \sum_k \sum_l \cdots \sum_m \left(e_{ij}e_{jk}e_{kl}e_{lm}e_{mi} + e_{im}e_{ml}e_{lk}e_{kj}e_{ji}\right) S_n S_o \ldots S_N \right) \\
& + \ldots
\end{aligned}
\qquad (11)
$$

The number and composition of groups and subgroups will be the same as discussed earlier. So it is possible to write the permanent function of any ***iMLE*** system in (**N + 1**) groups. It may be noted that a permanent function will contain **N!** terms only, provided e_{ij} are not 0. In certain cases, designers and/or developers team may decide that some of e_{ij} are 0 because of insignificant influence of one subsystem over the other subsystem. Substitutions of corresponding $\mathbf{e_{ij}}$ equal to 0 in general permanent function (equation (**9**)) or in general **VPM** (equation (**8**)) gives the exact number of terms with modified permanent function.

11. Step-by-step procedure

The step-by-step methodology is proposed which can permit industry, university and organizations to modify, extend, and improve quality of their ***iMLE*** products. Various marketing and strategic decisions can also be taken as per the competitiveness of ***iMLE*** products in global market. It will also give an insight to researchers, designers and developers to identify, select and create critical systems integration process. A generalized procedure for the complete design and analysis of ***iMLE*** system architecture is summarized below:

Step 1: Consider the desired *iMLE* product. Study the complete *iMLE* system and its subsystems, and also their interactions.

Step 2: Develop a block diagram of the ***iMLE*** system Figure 2, considering its sub-systems and interactions along with assumptions, if any.

Step 3: Develop a systems graph of the ***iMLE*** system Figure 8 with sub-systems as nodes and edges for interconnection between the nodes.

Step 4: Develop the matrix equation (10) and multinomial representations equation (11) of ***iMLE*** system.

Step 5: Evaluate functions/values of diagonal elements from the permanent functions of distinct sub-systems equation (8a) of the composite and repeat Steps 2 – 4 for each sub-system.

Step 6: Identify the functions/values of off-diagonal elements/interconnections at different levels of hierarchy of the ***iMLE*** amongst systems, sub-systems, sub-sub-systems, etc.

Step 7: Carry out modular design and analysis of ***iMLE*** products while purchasing off the shelf from the global market.

The visualization of the step-by-step procedure for the complete design and analysis of ***iMLE*** system is shown in Figure 10.

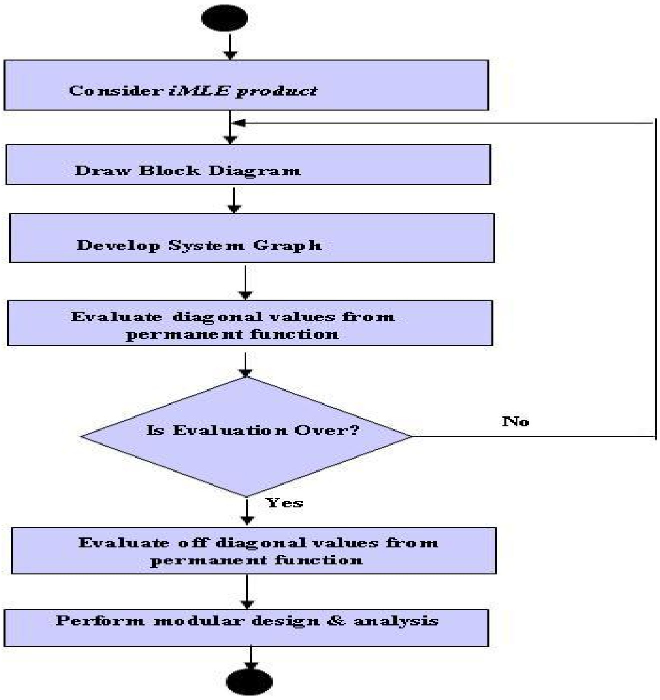


Figure 10. Visualization Model

The values (or functions) of interactions $\mathbf{e}_{ij}$ (i ,j = 1,2,...,N) between different subsystems $S_1,S_2,...,S_N$ can be written as a multinomial or a matrix, depending upon the type of interaction/reaction between the two subsystems. The sub-subsystems can again be treated as systems, as every sub-subsystem is a system in itself. Following the above procedure, these subsystems can be broken down into sub-subsystems and different graphs, matrices, and permanent representations can be obtained. Depending upon the depth of analysis required, the process could be taken to the constituent level and further ahead. In certain cases, it may be possible to evaluate $\mathbf{e}_{ij}$ **'s** experimentally or using available mathematical models. With the help of this data, complete multinomial for the ***iMLE*** system can be evaluated. Using/available standard modules of ***iMLE*** architectural sub-systems (e.g. dyads and loops of different subsystems) in global market, designers can develop alternative designs of ***iMLE*** products and carry out analysis and improvement of existing ***iMLE*** products. Work is in progress to carry out performance analysis of any ***iMLE*** system architecture from different perspectives using the structural model presented in this article.

12. Conclusions

The following concluding remark highlights the contributions of the present study.

1. The proposed ***iMLE*** system architecture is developed using system methodology and graph theoretic model. They represent its structural information, including its systems, their subsystems and their interconnections.
2. The systems methodology consists of the ***iMLE*** system digraph, the ***iMLE*** system matrix, and the ***iMLE*** system permanent function. These permit us to derive and exploit a number of results, which are useful to analysts, designers and developers of the system for quality products.
3. The ***iMLE*** digraph is the mathematical representation of the structural characteristics and their interdependence, useful for visual modeling and analysis. The ***iMLE*** system matrix converts digraph into another mathematical form. This matrix representation is a powerful tool for storage and retrieval of subsystems in computer database and also for computer processing. The ***iMLE*** system permanent function is a mathematical model characterizing the structure of the ***iMLE*** product and also helps one to determine the ***iMLE*** system index.
4. The permanent function of the ***iMLE*** system architecture at a particular level of hierarchy represents all possible combination of its subsystems. The terms of permanent function not only represent different subsets of ***iMLE*** system architecture but also guide the analysts, designer, developer, manager, decision maker and purchaser to generate large number of alternative design solution before selecting an optimum system.
5. The present work emphasizes the numerical methodology of ***iMLE*** that can also optimize the design and the development parameters.
6. The proposed systems model is a very a powerful tool from the commercial point of view in this highly competitive world. As the industry, university and organization has complete knowledge of every sub-system and all process parameters and their interactions through this systems model, they have a number of choices to shape its designing and developing strategy based on market dynamics.

7. As it is an integrated systems approach, all the subsystems up to the component level are modeled and evaluated to be used as inputs for diagonal elements at next higher level and so on. It can be inferred that to get the structural performance level (i.e. permanent index) of the overall system, the structural performance level of each subsystem at the lower level need to be calculated and substituted as diagonal elements of the variable permanent adjacency matrix at higher level.
8. The proposed structural methodology is comprehensive enough to deal with different structural and performance issues of **iMLE** system architecture at different levels of its life cycle.
9. A generalized methodology is also proposed to model a system consisting of **N** sub-systems and their interactions.
10. Current undergoing research deals with correlation of structural models with the desired performance parameters and associated characteristics. The outcome will be reported in future publications.

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[1] Nitin Upadhyay is an acknowledged teacher and prolific writer. He is currently working as Faculty in the Department of Computer Science and Information Systems, BITS, PILANI-GOA Campus. He has created a definite niche for himself in the field of Computer Science by contributing eight books. His research and creative zeal has enabled him to contribute research papers for journals and conferences. His major research areas are mobile learning and mode of education, mobile computing, system engineering, object oriented system, graph theory, software architecture and software engineering.

[2] Dr. V. P. Agrawal is working as a visiting professor of Mechanical Engineering at the Birla Institute of Technology and Science, Pilani, Goa campus, Goa, India after his retirement from IIT Delhi, India. He had been at IIT Delhi for the last 28 years and scaled from lecturer to professor in the Mechanical Engineering Department. He completed his graduation in 1966, post graduation in 1969 and PhD in 1983 from Jiwaji University Gwalior, University of Roorkee, and IIT Delhi respectively. He has published around 120 papers in International Journals and Conferences. He has guided successfully a large number of BTech, MTech projects and supervised a number of PhD theses. He has worked extensively in the areas of mechanisms, systems approach, and machine design.

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DECISIONAL MODELS AND ASPECTS FOR OPTIMAL MANAGE OF SOME PRODUCTION PROCESSES

Anatol GODONOAGA
Academy of Economic Studies, Kishinev, Moldova

E-mail: anatgo@ase.md

Anatolie BARACTARI
Academy of Economic Studies, Kishinev, Moldova

E-mail: anatolie_baractari@yahoo.com

Abstract: *Decisions about supplied goods depend of used technologies, of possibilities to acquire necessary factors, of quality of products, of demand level etc. Usually, firms cannot get a 100% level of qualitative goods, dividing them in some groups or categories. We research three generalizations of the classical model, where expected profit depends of quantity of good of first quality, of prices for each category, of demand level. We propose to solve described models using method of projection of generalized gradient. There are present some experimental results for different demand behavior.*

Key words: models; decisions; generalized gradient; stochastic

There is a follow linear model that expresses hypothetic value of maximal revenue of an industrial enterprise:

$$V(y) = \sum_{j=1}^{n} v_j y_j \rightarrow \max_y \qquad (1)$$

subject to:

$$\sum_{j=1}^{n} a_{ij} y_j \leq b_i, i = \overline{1,m} \qquad (2)$$

$$y_j \geq 0, j = \overline{1,n} \qquad (3)$$

Significance of used notations:

v_j - price per unit of good *j*, $j = \overline{1,n}$;

y_j - quantity of good *j* – level of that will be determinate

$V(y)$ - total revenue, that firm will have obtain if will sell all amounts of produces $y_1, y_2, ..., y_n$;

b_i - available quantity of resource *i*, $i = \overline{1,m}$;

a_{ij} - technological coefficient that represents necessity of resource *i* to create a unit of product *j*;

Nearly all linear models inclusive model (1)-(3) can be solved using an universal method SIMPLEX. The model (1)-(3) represents adequate that situation when all quantity of production is sold. Therefore is reasonable to develop the shown model, which will describe different situations from an activity of production firm. Next, is present three generalizations of described model [1].

Generalization 1

Decision about quantity of each category of goods that follow to produce depends certainly of demand level on market for respective product. In such situations model (1)-(3) can take under consideration the demand for each good or service. Let us now admit that demand is represented by vector $Y = (Y_1, Y_2, ..., Y_n)$. While firm makes a quantity of goods greater than demand then, it accounts additional expenses that are determinated by so-called "phenomena of overproduction". These expenses will be included in the next model. The vector $p = (p_1, p_2, ..., p_n)$ define the losses per unit that system of production accounts for each sort of good of overproduce ($y > Y$).

Taking into consideration mentioned conditions, the total net revenue is represented as follows:

$$V(y;Y) = \sum_{j=1}^{n} \left[v_j \cdot \min\{y_j; Y_j\} - p_j \cdot \max\{0; y_j - Y_j\} \right] \quad (4)$$

Defining $\varphi_j(y_j;Y_j) = v_j \cdot \min\{y_j;Y_j\} - p_j \cdot \max\{0; y_j - Y_j\}$ that represents net revenue obtained from realization of good *j* and is a nondifferentiable function in relation with deciding factor y_j, then, evident:

$$\varphi_j(y_j;Y_j) = \begin{cases} v_j \cdot y_j, & for \quad y_j \le Y_j \\ (v_j - p_j)Y_j - p_j y_j, & for \quad y_j > Y_j \end{cases}$$

Thus, taking in consideration new data, result the next:

$$V(y;Y) = \sum_{j=1}^{n} \varphi_j(y_j;Y_j) \to \max_y$$

subject to: $\sum_{j=1}^{n} a_{ij} y_j \le b_i, i = \overline{1,m} \qquad y_j \ge 0, j = \overline{1,n}$

Generalization 2

Every manager takes such decisions that will guarantee the lowest level of rejects (defective articles/goods) in total amount of manufactured goods. However, in spite of this, is unrealizable to get the 100% of qualitative goods and this situation can be analyzed just as a theoretical one in the almost cases. From this reason are justifiable the following improvement of the model obtained at the first generalization.

Further, it is assumed that $k_j \cdot 100\%$ $(0 < k_j \leq 1)$ from produced good y_j represents competitive production, the rest, $\alpha_j y_j$ $(\alpha_j = 1 - k_j)$ represents defective goods and cannot be sale. In this case, net revenue is represented as follows:

$$V(y;Y) = \sum_{j=1}^{n}\left[v_j \cdot \min\{k_j y_j; Y_j\} - p_j \cdot \max\{y_j - Y_j; \alpha_j y_j\}\right]$$

and function respective $\varphi_j(y_j;Y_j) = \begin{cases} v_j \cdot Y_j - p_j(y_j - Y_j) & if \quad Y_j \leq k_j y_j \\ v_j k_j y_j - p_j \alpha_j y_j & if \quad Y_j > k_j y_j \end{cases}$

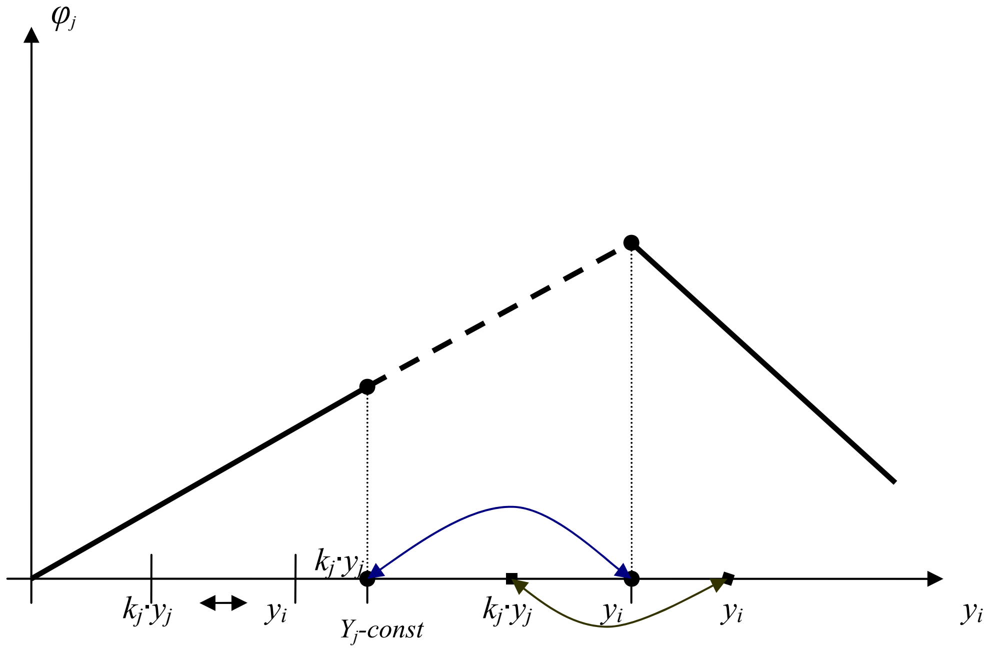

Figure 1. Dependence of net revenue in relation with product *j*, for fixed demand Y_j and coefficient of competitiveness $0<k_j<1$

Generalization 3

There is a similar situation as in previous stage, but where $k_j y_j$ represents production of "superior" quality (A), with price per unit $v_j = v_j^A$, and $\alpha_j y_j$ - production of "lower" (B) price per unit equal with v_j^B and $0 \leq v_j^B < v_j^A$. Concurrently we admit that good *j* of the "second" quality can be sold only if were sold all quantity of product of quality A. Supplement at revenue will be:

a) 0, if $Y_j \leq k_j y_j$

b) $v_j^B(Y_j - k_j y_j)$, if $k_j y_j \leq Y_j \leq y_j$

c) $v_j^B(y_j - k_j y_j)$, if $Y_j \geq y_j$

Reunify these three cases we obtain value of addition in form:

$$\min\left\{ \max\{0; v_j^B(Y_j - k_j y_j)\}; v_j^B \alpha_j y_j \right\}$$

In addition, function $\varphi_j(y_j;Y_j)$ obtains the following aspect:

$$\varphi_j(y_j;Y_j) = v_j^A \cdot \min\{k_j y_j;Y_j\} + v_j^B \cdot \min\left\{ \max\{0;Y_j - k_j y_j\};\alpha_j y_j \right\} - p_j \cdot \max\{y_j - Y_j;0\}$$

Besides modification of objective function will be changed constraints of non-negativity of variables, that more adequately reflect behavior of an economic system:

$$0 \le y_j^{\min} \le y_j \le y_j^{\max} \quad j = \overline{1,n}$$

Necessity of such conditions appears in situations when firm cannot (constraints of different nature) exceed volumes of some kinds of goods.

Finally, we obtained nest model [2]:

$$V(y;Y) = \sum_{j=1}^{n} \varphi_j(y_j;Y_j) \to \max_y \qquad (5)$$

where

$$\varphi_j(y_j;Y_j) = v_j^A \cdot \min\{k_j y_j;Y_j\} + v_j^B \cdot \min\left\{ \max\{0;Y_j - k_j y_j\};\alpha_j y_j \right\} - p_j \cdot \max\{y_j - Y_j;0\} \qquad (6)$$

Subject to:

$$\sum_{j=1}^{n} a_{ij} y_j \le b_i, i = \overline{1,m} \qquad (7)$$

$$0 \le y_j^{\min} \le y_j \le y_j^{\max} \quad j = \overline{1,n} \qquad (8)$$

Evident, respective model is non-linear (objective function is non-linear) and cannot be solved using traditional methods. In this situation we can use the method of generalized gradient [4].

Analyzing relations between demand vector *(Y)*, total produced quantity *(y)* and manufactured amount of quality A ($k \cdot y$) we obtain 3 situations that determine 3 correspondence forms of function $\varphi_j(y_j, Y_j)$:

a) $Y_j \le k_j y_j$ - demand do not exceed volume of good *j* of quality A (competitive) in this case function φ_j has form: $\varphi_j(y_j,Y_j) = v_j^A \cdot Y_j - p_j \cdot (y_j - Y_j)$;

b) $k_j y_j \le Y_j \le y_j$ - quantity of unsatisfied demand, evident is equal with $Y_j - k_j y_j$, in this situation φ_j is represented as:

$$\varphi_j(y_j, Y_j) = v_j^A k_j y_j + v_j^B \cdot (Y_j - k_j y_j) - p_j \cdot (y_j - Y_j);$$

c) $y_j \le Y_j$ then $\varphi_j(y_j, Y_j) = v_j^A \cdot k_j \cdot y_j + v_j^B \cdot \alpha_j \cdot y_j$.

Fixing demand quantity Y_i (figure 1) in a point and varying y_i in certain limits, we obtain graphical image of function $\varphi_j(y_j, Y_j)$ in dependence of output y_i (for 3 situations as described above).

Researching price of good of quantity *B* (v_j^B) and its contribution in value of net income we observe that in situation ***a***) this contribution represents *0* monetary units, because sold amount of quality *B* the same is zero. In situation ***c***) all quantity of good is sold and total income increases due to products of quality *B* with $v_j^B \cdot \alpha_j \cdot y_j$ monetary units. The most interesting is situation ***b***) here we can see three moments related to contribution of goods of quality B in total income and these three cases depends of price v_j^B :

1. $v_j^B > 0$ but inessential, on graphic this situation corresponding to ***case I*** and optimal volume of supply is $y_j = Y_j / k_j$;
2. $v_j^B > 0$ and of such nature that $v_j^A \cdot Y_j - p_j \cdot (y_j - Y_j) = v_j^A k_j y_j + v_j^B \cdot (Y_j - k_j y_j) - p_j \cdot (y_j - Y_j)$ that is accordingly maximal income and is equal for every y_j that satisfies condition $Y_j \leq y_j \leq \frac{Y_j}{k_j}$. In figure 1 respective situation corresponds to ***case II*** ;
3. $v_j^B > 0$ for values of v_j^B when contribution of quality B is ***considerable***, then y_j optimal supply will be equal with demand Y_j and respective maximal income $\varphi_j^{\max}$ is $v_j^A k_j Y_j + v_j^B \cdot (Y_j - k_j Y_j)$, ***case III*** from figure 2.

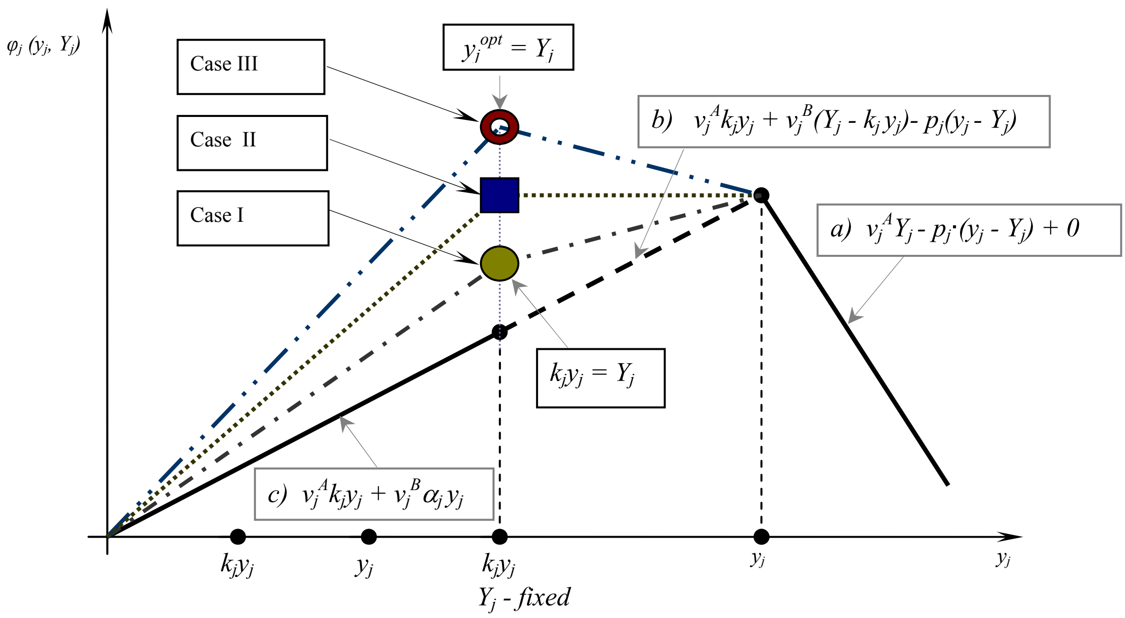

Figure 2. Graphical illustration of net income in dependence of relations between demand, supply and values of v_j^A și v_j^B, p_j, k_j

Further, will make an evaluation of price v_j^B , in function of values that were presented in those 3 cases from situation ***b***). Reasoning from equality $v_j^A \cdot Y_j - p_j \cdot (y_j - Y_j) = v_j^A k_j y_j + v_j^B \cdot (Y_j - k_j y_j) - p_j \cdot (y_j - Y_j)$ or from that angular coefficient of function $\varphi_j(y_j, Y_j) = v_j^A k_j y_j + v_j^B \cdot (Y_j - k_j y_j) - p_j \cdot (y_j - Y_j)$ must be zero while function is constant, obtain following:

$$v_j^A k_j - v_j^B k_j - p_j = 0 \Leftrightarrow v_j^B k_j = v_j^A k_j - p_j \Leftrightarrow v_j^B = \frac{v_j^A k_j - p_j}{k_j} = v_j^A - \frac{p_j}{k_j}$$

Generalizing those related above we get following conclusions:

1. if $0 \le v_j^B < v_j^A - \frac{p_j}{k_j}$ then y_j optimal is $y_j^{opt} = \frac{Y_j}{k_j}$

2. while $v_j^B = v_j^A - \frac{p_j}{k_j}$ then y_j optimal is $Y_j \le y_j^{opt} \le \frac{Y_j}{k_j}$

3. if $v_j^B > v_j^A - \frac{p_j}{k_j}$ then y_j optimal is $y_j^{opt} = Y_j$

In figure 3 y_i is fixed and Y_i- variable, being represented those situations when for every level of demand obtained income is equal or lower than zero, and the same case when income is greater than 0. These situations depends by sign between relations $p_i / (p_i + v_i^A)$ and k_i . In figure 3 is represented behavior of function $\varphi_j(y_j, Y_j)$ in dependence of relations between y_i and Y_i .

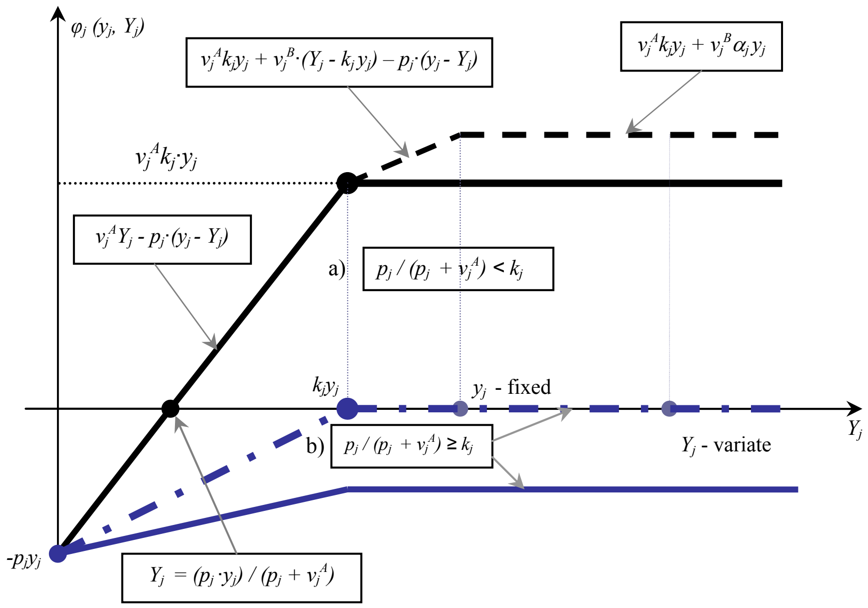

Figure 3. Dependence φ_i (y_i, Y_i) of demand Y_i (y_i – is fixed, y_i >

If $Y_j = k_j y_j$ then we obtain the following form of objective function:

$$\varphi_j = v_j^A k_j y_j - p_j(y_j - Y_j) = v_j^A k_j y_j - p_j y_j + p_j k_j y_j = y_j(v_j^A k_j + p_j(k-1)) = y_j \cdot (v_j^A k_j - p_j \alpha_j)$$

that represents maximal income from selling of good *j* of quality A, when supply of this good is y_j units.

$$v_j^A k_j - p_j \alpha_j = 0 \Leftrightarrow v_j^A k_j = p_j(1-k_j) \Leftrightarrow v_j^A k_j - p_j k_j = p_j \Leftrightarrow k_j = \frac{p_j}{v_j^A + p_j}$$

There are two cases, if:

a) $\frac{p_j}{v_j^A + p_j} < k_j$ then $\max_{Y_j} \varphi_j = (v_j^A k_j - p_j \alpha_j) \cdot y_j > 0$ while $Y_j \ge k_j y_j$;

b) $\frac{p_j}{v_j^A + p_j} \geq k_j$ then $\max_{Y_j} \varphi_j \leq 0$ so, is not reasonable supply product *j*.

Further, will describe a method of solve of obtained model at the third generalization. Method of projection of generalized gradient is based on concept of subgradient and offers an approximate solution (with admissible error).

Let us admit, first of all that demand Y has a deterministic nature, Y=const.

Evident, relations (8) determine a set of vectors *y,* that belongs to a N-dimensional parallelepiped and will be noted with ***D***. So, constraints (8) can be represented as $y \in D$.

To solve the model (5) - (8) using method of projection of generalized gradient, will introduce a function, that replace constraints (7):

$$\Phi(y) = \max\{\overline{\Phi}_1(y), \overline{\Phi}_2(y), ..., \overline{\Phi}_i(y), ...\overline{\Phi}_m(y)\} \leq 0 \qquad (9)$$

and $\overline{\Phi}_j(y) = \sum_{j=1}^{n} a_{ij} y_j - b_j \leq 0 \quad i = \overline{1,m}$

Relation (9) is true if and only if relation (7) is true. Function $\Phi(y)$ determines maximal value of deviation of restriction. If this value is positive, result that at least a constraint is not satisfied and function will indicate maximal deviation.

Idea of above mentioned method consist in following. There is generated a set of points $y^0, y^1, ..., y^k, y^{k+1}...$, initial point y^0 it is given, and is chosen by user (decision maker) in dependence of problem particularities. Having approximation y^k, a new (next) approximation y^{k+1} is determined as:

$$y^{k+1} = \Pi_D(\overline{y}^k) \qquad (10)$$

that is, we do operation of project of vector $\overline{y}$ on set ***D***, that is compute by relation:

$$\overline{y}^k = y^k + h_k \cdot \eta^k,$$

where η^k - represents direction of motion and h_k - step length. In order to converge to solution, series $h_0, h_1, ..., h_k, ...$ must satisfy following constraints:

$$h_k > 0, h_k \to 0, \sum_{k=0}^{\infty} h_k = 0 \qquad (11)$$

For this method, series h_k is calculated by formula $h_k = \frac{H}{k+1}$, $H > 0$ and will satisfy restrictions (11).

Vector of motion direction η^k is constructed by formula:

$$\eta^k = \begin{cases} subgrad(V(y^k, Y)), & \Phi(y^k) \leq 0 \\ -subgrad(\Phi(y^k)), & \Phi(y^k) > 0 \end{cases} \qquad (12)$$

In situation when vector $\eta^k = (\eta_1, \eta_2, ..., \eta_j, ..., \eta_n)$ is calculated by relation $\eta^k = subgrad(V(y^k, Y))$, then its elements can be compute by formula $\eta_j = \eta_j^1 + \eta_j^2 + \eta_j^3$, where:

$$\eta_j^1 = \begin{cases} v_j^A \cdot k_j & if \quad k_j y_j^k < Y_j \\ 0 & if \quad k_j y_j^k \geq Y_j \end{cases}$$

$$\eta_j^2 = \begin{cases} 0 & if \quad Y_j \leq k_j y_j^k \\ -v_j^B k_j & if \quad k_j y_j^k < Y_j \leq y_j^k \\ v_j^B \alpha_j & if \quad y_j^k < Y_j \end{cases}$$

$$\eta_j^3 = \begin{cases} -p_j & if \quad y_j^k - Y_j \geq 0 \\ 0 & if \quad y_j^k - Y_j < 0 \end{cases}$$

Elements of vector determinate by $subgrad(\Phi(y^k))$ represents coefficients a_{ij}, where $j = \overline{1,n}$, and *i* depends of number of constraint that maximize function $\Phi(y)$.

New obtained approximation by formula (5), we get through projection of vector $\overline{y}$ on set ***D.*** This operation of projection takes place as follows:

$$y_j^{k+1} = \begin{cases} y_j^{\min}, if \quad \overline{y}_J^K < y_j^{\min} \\ y_j^{\max}, if \quad \overline{y}_J^K > y_j^{\max} \\ \overline{y}_J^K, \quad contrarily \end{cases} \qquad j = \overline{1,n}$$

Process of construction of series continues while *k* is lower than a fixed number by user, just this moment guarantees a finite number of steps. By this value depends number of iterations and it must be significant ($>10^4$).

Of course, there is no guarantee that solution (approximation) obtained at the last iteration is the best; a better one can be at previous or next iteration but differs unessential.

To use "method of projection of generalized gradient" besides of information from model (number of constraints – *m*, number of variables – *n*, vectors v^A, v^B, Y, *k*, *p*, y^{max}, y^{min}, *b*, matrix *A*) we must get following parameters – number of iteration, interval of showing of solutions and the most important initial vector y^0.

Further, will succinctly describe aspects of some numerical methods [3] based the same on notion of subgradient, to solve decisional models like (1)-(3) for situations when factor of demand Y is stochastic or uncertain. In case of stochastic model [5] will consider objective function:

$$R_{stochastic}(y) = E_Y\left[V(y,Y)\right] \qquad (13)$$

Objective function of model where demand is uncertain will be defined in Wald aspect (minmax):

$$R_{uncertain}(y) = \min\left[V(y,Y)\right] \qquad (14)$$

In both situations as directions of movement are used random vectors that depend of simulations of demand.

In case of stochastic model vector of direction of movement on *k* iteration:

$$\eta^k = \begin{cases} subgrad(V(y^k, Y^k)), & if \quad \Phi(y^k) \leq 0 \\ -subgrad(\Phi(y^k)), & if \quad \Phi(y^k) > 0 \end{cases}$$

Here Y^k represents an independent observation (a new simulation) of aleatoric factor Y in correspondence with given low of demand distribution.
In situation when Y is uncertain, set of possible states of Y is defined as a probabilistic measure P(dY), and element Y^k at iteration k is defined as follow:

$$Y^k = \begin{cases} Y^{k-1}, \; if \;\; V(y^k, \, Y^{k-1}) \geq V(y^k, \tilde{Y}^k) \\ \tilde{Y}^k, \; for \;\; V(y^k, \, Y^{k-1}) < V(y^k, \tilde{Y}^k) \end{cases}$$

Set $\tilde{Y}^0, \tilde{Y}^1, ..., \tilde{Y}^k, ...,$ represent independent observation of factor of demand *Y* in correspondence with given low of demand distribution *P(dY)*.

Some experimental results (for the first generalization)

There are considered the follow problem. A firm can product two kind of goods. Technological matrix $A = \begin{bmatrix} 0.1 & 0.2 \\ 0.5 & 0.3 \end{bmatrix}$; vector of available resources $b = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$; vector of revenue per unit of product (v_1, v_2) = (0.5; 1.5); the cost per unit of product (p_1, p_2) = (0.25; 0.8); The minimal quantity of supply y_{min}=(0;0), and the maximal y_{max}=(20;20).

Firstly, simplex method can not be adapted for such model, indifferent of decisional situations (certain, risk or uncertain). Secondly, the model is not linear and non-differentiable, this situation been caused by relation between possibilities of output of product system and behavior of demand.

Remark. The analyzed model can be reduced at classical linear model (for certain, risk or uncertain) if quantities of demand for all sorts of goods, excel significant the capacities of system.

Further, will be analyzed the iterative behavior of supply and corresponding revenue in dependence of nature of demand, that can be deterministic, risky or uncertainly.

Deterministic case (demand is known).

Let there be given the demand Y=(3; 7) that belong to admissible domain. We considered two situations:

1. Point of start y^0= (0; 0)

Numerical results are presented in follow table:

Table 1. Numerical data for deterministic case

k	y_1	y_2	F_i	V_{max}
0	0	0	0	0
5	0	12.3	0	6.259
10	2.58	8.08	0	10.925
15	2.68	6.45	0	11.025
100	3.01	7.02	0	11.910
200	3.00	7.01	0	11.988

At given iteration *k*, F_i represents value $\max\{\Phi(yk);0\}$, where function $\Phi(y)$ is defined in relation (9)

Evident, for such level of supply the firm can not get positive revenue because the output is zero. But simultaneously with increase of number of iterations the quantity of supply approaches to the optimal variant (obvious, equal with demand), and respective income tends to the maximal value.

2. Initial supply $y^0 = (15; 18)$

In situation when, supply essentially exceeds demand, evident, revenue can get negative value. Starting the optimization algorithm of process of production, the vectorial series of consecutive supplies tends to the optimal variant of decision, which is an element of polygon of constraints (in this case studied model is reduced at linear one). This algorithm get with such approximation the same solution obtained using the simplex method.

In these circumstances, producer has to make the maximal possible quantities of goods, been assured that produced volumes will be sold.

Stochastic case

The demand is considered an aleatory vector in such a way that the first its component is a discrete random variable (with given probabilities), and the second component represents a continuous random variable, constant distributed on given interval. Using this information, we simulate different possible values of demand that alternates with iterative searching process of optimal variant of output (considering as objective maximization of average income). Obtained results are selective shown in table 2.

Table 2. Numerical data for situation of risk

k	y_1	y_2	F_i	V_{medium}
0	0	0	0	0
5	0	12.3	0	0
10	2.31	8.08	0	8.728
15	1.44	8.67	0	8.579
100	3.08	6.84	0	8.535
200	3.09	7.77	0	8.812

The following diagram (figure 4) represents dynamic of modification of average income in relation with increase number of iterations *k*. On horizontal axis with points, are marked iterations at which are obtained "fully" inadmissible solutions (when $F_i > \varepsilon$; $\varepsilon = 10^{-2}$). At iterations with admissible decisions ($F_i \leq \varepsilon$) we see an increase tendency of average revenue.

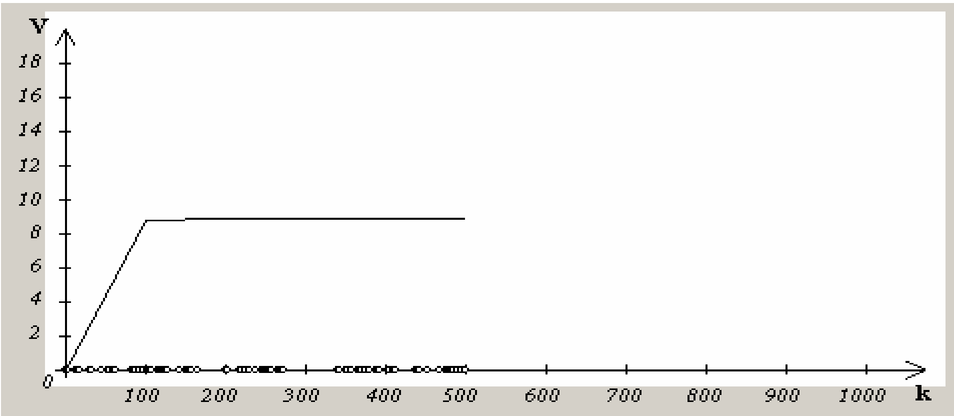

Figure 4. Graphical representation of dynamic of average income

Uncertain situation

The case is similar with cu stochastic variant (see figure 5), demand is manifested with the same possible values. The optimal decision of output level is taken in accordance with Wald criteria (maxmin).

In these circumstances corresponding algorithm is represented by following data.

Table 3. Numerical data for uncertain case

k	y_1	y_2	F_i	V_{max}
0	0	0	0	0
5	0	12.3	0	-5.194
10	1	1.96	0	2.704
15	0.38	2.99	0	2.158
100	0.001	2.06	0	2.991
200	0.003	2.11	0	2.951

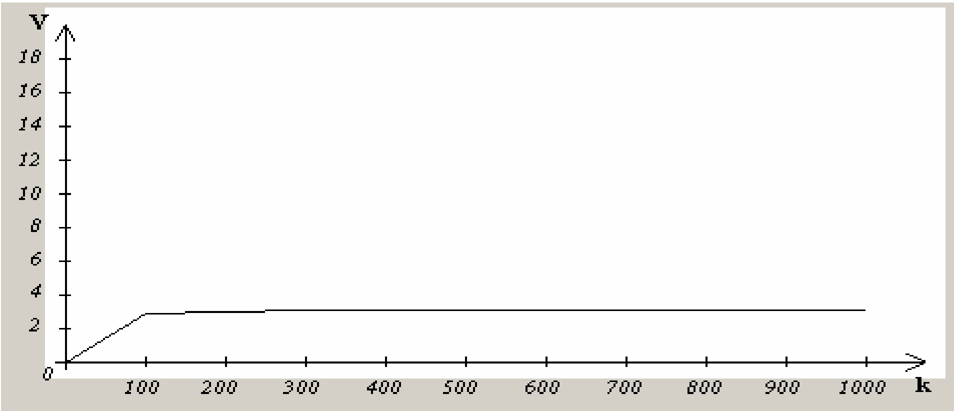

Figure 5. Graphical representation of revenue modification (Wald criteria)

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IMPROVING RESOURCE LEVELING IN AGILE SOFTWARE DEVELOPMENT PROJECTS THROUGH AGENT-BASED APPROACH

Constanta Nicoleta BODEA
PhD, University Professor, Economic Informatics Department
University of Economics, Bucharest, Romania

E-mail: bodea@ase.ro

Cristian Sebastian NICULESCU
University of Economics, Bucharest, Romania

E-mail: n.cristian@gmail.com

Abstract: *Successfully project planning, coordinating and controlling in order to deal effectively with projects sponsors, customers, unexpected risks and changing scope are difficult tasks even for the most experienced project managers. The tight deadlines, volatile requirements and emerging technologies are the main reasons for this lake of performance. This agile project environment requires an agile project manage¬ment.*
Different approaches to project planning and scheduling have been developed. The Operational Research (OR) approach provides two major planning techniques: CPM and PERT. Artificial Intelligence (AI) initially promoted the automatic planner concept. In order to plan a project, the automatic application of predefined operators is required. However, most domains are not so easily formalized in the form of predefined planning operators. The new AI approaches promote model-based planning and scheduling that are more appropriate for the agile project management.
The paper focus is on the agent-based approach to project planning and scheduling, especially in Resource Leveling issues. The authors have developed and implemented the ResourceLeveler system, an agent-based model for leveling project resources. The objective of Resource Leveler is to find a scheduling of resources similar to the optimal theoretical solution which takes into consideration all constraints stemming from the relationships between projects, activity calendars, resource calendars, resource allotment to the activities and resource availability. ResourceLeveler was developed in C# as a plug-in for Microsoft Project. Future work will focus on the development of agile software agents for resources leveling.

Key words: *gile project management; agent-based models; artificial intelligence; leveling performance; project resource leveling*

1. Introduction

Different approaches to project planning and project scheduling have been developed [2][1], [3], [4], [5]. The Operational Research (OR) approach provides two major planning techniques: CPM and PERT. Artificial Intelligence (AI) initially promoted the *automatic planner concept* [6], [7]. In order to plan a project, the automatic application of predefined operators is required. However, most domains are not easily formalized in the form of predefined planning operators. The new AI approaches promote *model-based planning and scheduling*. An important class is that of agent-based models.

An agent is an entity that can perceive its environment through sensors and act upon that environment through effectors. The goal of AI is to design the agent program: a function that implements the agent mapping percepts to actions [5]. This program runs on some sort of computing device, called agent architecture.

The Procura model was developed by S. Goldmann in cooperation with Stanford University [2]. Procura is an agent-based model which supports the planning, scheduling and execution of complex projects in an incremental and hierarchical approach. Procura uses and extends the Redux model [4].

2. Definition of the resource leveling problem. It tools used in resource leveling

Starting from a well-defined resource collection allotted to a project, one can define Resource Leveling as the planning of the project's activities in a manner that respects all constraints resulting from activity dependencies and resource availability. It also minimizes the project duration. Resource Leveling implies finding the minimal solutions for the activity plan with consideration to the above mentioned constraints. We will see that there is no standard procedure in finding an optimal solution in the case of Resource Leveling. Even the recognition of a solution similar to the optimal one is problematic when dealing with complex projects that have complicated dependencies and allotments of multiple resources for their activities.

A number of IT instruments have been developed to assist project managers. The best known tools are Microsoft Project and Primavera Project Planner. Considering the market percentage, Microsoft Project is the most popular project management software. It is useful and powerful in almost every aspect of project management. This is why we will focus on the existing solutions which can be integrated with Microsoft Project.

3. Specific requirements for the resource leveling within agile software development projects. The approach of resourceleveler

The agile approach started in 1994 with some trials of *semi-formal* agile methodologies, such as RAD, DSDM, XP, Crystal, Scrum. These methodologies are based *on agile methods* [1]. Agile methods are adaptive rather than predictive. Engineering methods tend to try to plan out a large part of the software process in great detail for a long span of time, this works well until things change. So their nature is to resist change. The agile methods, however, are waiting for change.

An solution of solving the problem of resource leveling in an agile approach is the ResourceLeveler model. The objective of the Resource Leveler plug-in is to find a resource scheduling similar to the optimal theoretical solution which takes into consideration all constraints stemming from the relationships between projects, activity calendars, resource calendars, resource allotment to the activities and resource availability and has the flexibility required by the agile environment.

ResourceLeveler is based on a multi-agent system and an auction market. During the pre-leveling stage the statistic data of the project is computed (including the analysis of the critical path). Data collected in this stage will be used during leveling to compute the priority of each task. The leveling is realized by analyzing the work periods with a certain precision (hour or day) from the beginning of the project to its end. For each of these periods the program runs a negotiation round between the agents which represent the tasks in the frame of a virtual market that simulates a resource auction.

The market has the objective of deciding the winning offers and implicitly the activities which will be planned for the specified time span. Every offer received from the agents contains the desired resources and the required quantity as well as a price which characterizes the estimate value of the resources at the moment of auction for the agent.

The agents who represent the actions decide the leveling strategy because the price generated by the offers determines the task’s importance in the present context. In order to set up a price, the agent uses a database that contains all considered elements. Some characteristics are common to all agents and represent proprieties of the project (for example the dependence graph between tasks), while other characteristics are specific to the represented activity. In the following we will present the main components of ResourceLeveler.

The Auction Market

On this market resources are exchanged. The resources are sold by the auction judge (in this case the market) and bought by agents who represent the activities of the project. In case of an over-allotment these auctions are held with a deficit of resources. In this case the winning offers are the ones which have offered the best price. These winning offers are bound to an activity which will be planned for implementation in the current day of the project execution. An important characteristic of this market is the way in which the auction is held.

The implementation of a first-price auction with sealed offers has been chosen because the goal of the bid is not to encourage a competition between the participating agents but to create a hierarchy of the theoretic values of the represented activities. An important factor was the fact that such an auction takes place rapidly because it consists of only one bidding round and no negotiations. The bidding market is responsible for the coordination of the auction with sealed offers. The bidding market plays the role of the auction judge, deciding the winning offers.

The difference between the implementation used by ResourceLeveler and the classical implementation of bidding with sealed offers is the way in which goods are sold. Classically, the goods are sold one by one, every agent wishing to participate having to make an offer for the auctioned resource. Despite this, the particularities of allotting a resource for the tasks have led to an extended version of this type of auction. All auctioned goods are presented before the bidding begins and the involved agents make a single offer for all goods the whish to obtain. In this way, one has realized a natural and efficient model for the allotment of multiple resources in the same activity. By providing a single price for all auctioned goods, the agent's offers raise further problems regarding the choice of a winner because one has to find the combination of offers that maximize the market's profit.

Fig. 1 presents the market structure used by ResourceLeveler. The main steps of the auction are:

1. The first agent reads the total of available resources.
2. The first agent generates a proposal to the market.
3. The second agent reads the total of available resources.
4. The second agent generates a proposal to the market.
5. After all offers have been received, they are ordered in according to the price offered.
6. In decreasing order of the price, the necessary resources are verified and compared to the available resources. If all resources are available, the offer is accepted and the resources consumed. The next offers will be verified according to the new resource availability. The process continues until all offers are analyzed.

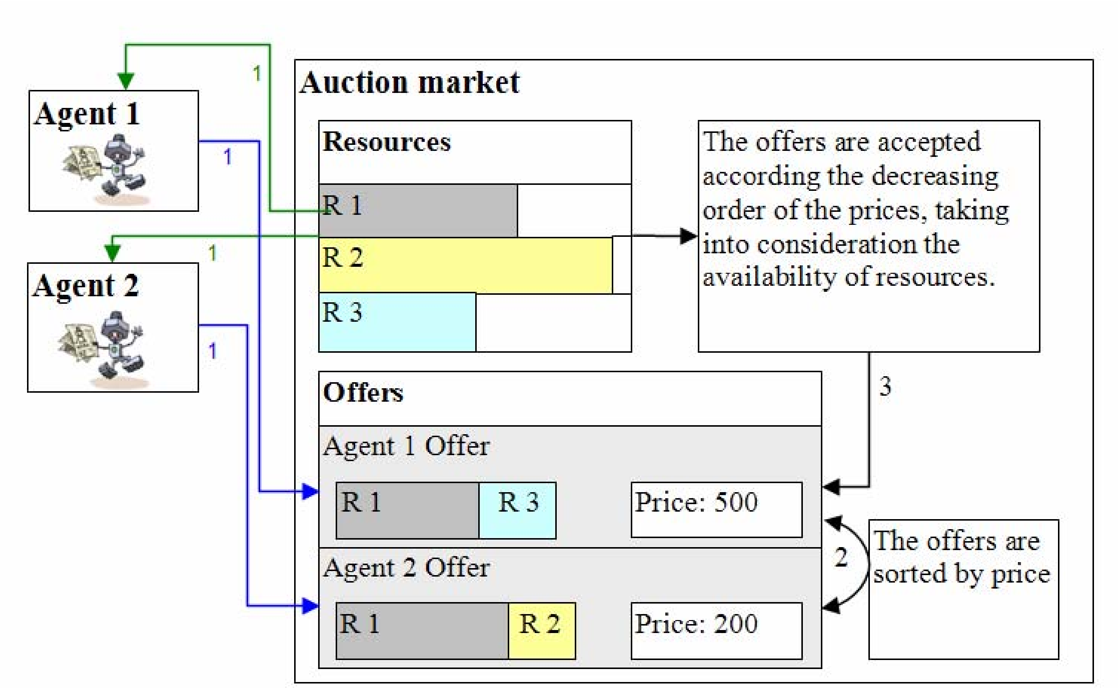

Figure 1. The structure of the auction market with sealed offers

Bidding Agents

These are the main entities of the resource leveler and have a strong impact on its behavior. By changing the types of agents used one can completely change the program's behavior. This is why it is important that these entities be carefully designed. The agents represent the component activities of the project and their interest is to gain the necessary resources for the execution of the represented activities. If an agent makes an offer and wins the resource bid, the represented activity can be executed on the same day. From case to case the starting date of the task will be modified or a new section for the planning of the activity (split) will be created.

Because a system of sealed offers is used, the bidding agents use the estimated value of the resources as price. The value estimation of the necessary resources represents the agent's logic and determines his behavior.

The described model supports any implementation of the agents and even a number of different implementations of the agents. The implemented agent's complexity varies from ordinary agents of level 0, who have no own models, to level 2 agents who model both the system as well as the other competitor agents. For the implementation ordinary level 0 agents were chosen, who found their reasoning on small heuristic algorithms analyzing the data of different tasks, the data extracted from the critical path analyze and the data referring to resource allotments. Three types of ordinary agents have been implemented, each of them being the representation of a specific resource leveling strategy:

a) Agent Based on the Duration of the Activities Following the Represented Task;
b) Agent Based on the Time Float of the Represented Activity;
c) Agent Based on the Number of Allotted Resources and on the Time Float of the Represented Activity.

4. The resourceleveler system: structure and implementation

ResourceLeveler system was developed in C#, considering the Microsoft Project plug-in support which is dedicated to the programmers using .NET technologies. The system has the following functional modules, which communicate through the interfaces (fig. 2):

- *Interface module*. This module is responsible for the insertion of the ResourceLeveler button into the Microsoft menu and the communication with the user;
- *Wrapper module*. This module extracts the project data offered by Microsoft Project;
- *Leveling module*. This is an intermediary module which adapts the negotiation algorithm based on bid to the leveling process
- *The auction market simulation module*.

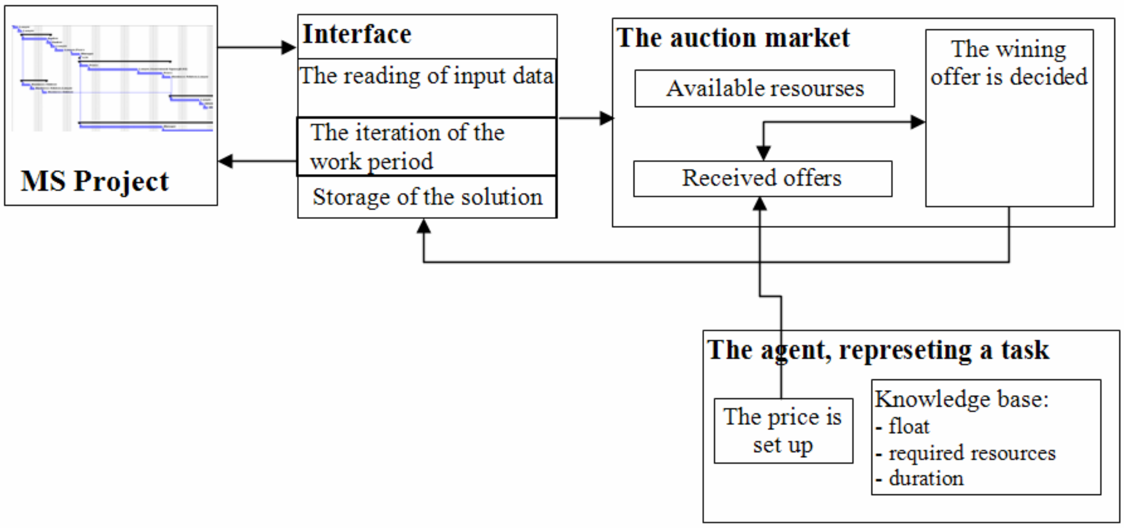

Figure 2. The structure of ResourceLeveler

5. Conclusions. Future research

From the point of view of managers, a good resource leveling tool ensures the minimal duration of a project taking the available resources into consideration. This is because a project finished early saves costs. In spite of this, managers hesitate to over-allot resources in order to speed up a project. This reasoning is based on two factors: the human factor and the financial factor. The latter takes into account the rise in costs because of the over-allotment taxes, and the human factor deals with the unwanted collateral effects of using a human resource over its normal work capacity.

In future research we intend to extend the types of agents acting on the auction market in order to increase the system's flexibility. We intend to develop agents that use an *iterative estimation* of the activities' duration and time float. We will compare current results with the ones obtained through their implementation in ResourceLeveler. Through this comparative analysis we will develop an agility indicator for software agents.

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