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THE ELASTICITY OF THE PRICE OF A STOCK AND ITS BETA

Cyriac ANTONY
MPh, Lecturer (Selection Grade) in Statistics,
Sacred Heart College Thevara, Kochi, India

E-mail: cyriacantony2003@yahoo.co.in

E.S. JEEVANAND
PhD, Reader in Statistics, Union Christian College, Aluva, Kerala, India

E-mail: radhajeevanand@sancharnet.in, radhajeevanand@yahoo.co.in

Abstract: *Systematic risk in an investment in a security is measured by the security's beta. The beta of a stock is considered as a very important parameter in asset pricing. It is used to estimate the expected return of a stock with respect to its market return. Beta is estimated by the regression method. In this paper, we consider some problems associated with the concept of beta and its estimation. We also advocate for the use of elasticity as an alternative to beta.*

Key words: *Beta; CAPM; characteristic line; price index; risk; least squares; elasticity*

1. Introduction

Estimating the expected return on investments to be made in the stock market is a challenging job before an ordinary investor. Different market models and techniques are being used for taking suitable investment decisions. The past behaviour of the price of a security and the share price index plays a very important role in security analysis. The straight line showing the relationship between the rate of return of a security and the rate of market return is known as the security's characteristic line. The slope of the characteristic line is called the security's beta. The concept of beta introduced by Markowitz(1959) is being widely used to measure the systematic risk involved in an investment. Ordinary least square (OLS) method is used by researchers and practitioners for estimating the characteristic line.

The measure beta is an inevitable tool in portfolio management. Asset pricing without beta is unimaginable. The Capital Asset Pricing Model (CAPM) is used to determine a theoretically appropriate required rate of return of an asset that is added to a portfolio. See Sharpe (1964), Linter (1965) and Mossin (1966). Beta plays a prominent role in CAPM. Therefore, the usefulness of CAPM mainly depends on the authenticity of beta.

In this paper, we explore the meaning of beta and its incapability to measure the sensitivity of return of a security to market returns. We also strongly recommend the use of the concept of elasticity of the price of a stock as an alternative to measure the sensitivity of its price corresponding to the market movements. Using the term elasticity, we modify the CAPM introduced by William Sharpe (1964), John Linter (1965) and Jan Mossin (1966).

This paper is organized as follows. Section 2 discusses the characteristic line of a stock and its limitations in measuring the relationship between the return of a security and the market return. In Section 3 we introduce the concept of the elasticity of a stock and the advantages of elasticity over beta in measuring the sensitivity of the price of a stock with respect to market movements. Section 4 is devoted to a discussion of the idea of modifying the CAPM by replacing beta by elasticity. In section 5 we give an illustration and a comparison of calculated values of beta and elasticity. Section 6 concludes the paper with the main advantages of our results.

2. Characteristic line and beta

The price 'Y' of a stock depends on a number of factors some of which are internal to the company and others external. Empirical studies show that there is a linear relation between the share price index 'X' and Y. Let (X_1, Y_1), (X_2, Y_2), … (X_n, Y_n) be 'n' observations relating to X and Y made at 'n' consecutive periods of time. If 'x' denotes the percentage rate of return of the price index and 'y' denotes that of the security, then the values of x and y are given by

$$x_i = 100\,(X_{i+1} - X_i)\,/\,X_i \quad \text{and} \quad y_i = 100\,(Y_{i+1} - Y_i)\,/\,Y_i \quad \text{for } i = 1,2,3,\ldots,n-1$$

The characteristic line representing the relationship between x and y is given in Figure(1).

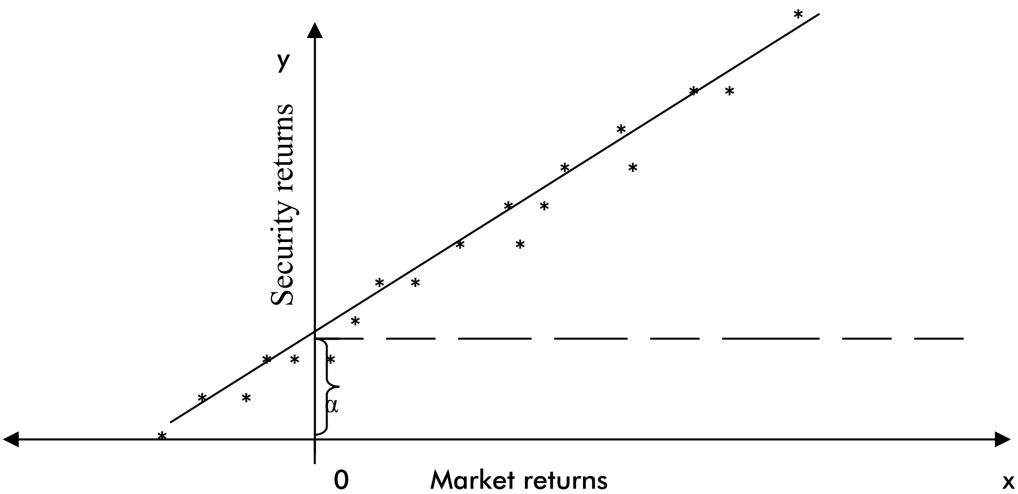

Figure 1. The characteristic line of a stock

The equation of the characteristic line can be written as

$$y = \alpha + \beta x \tag{1}$$

where α and β are constants. The slope of characteristic line β is the security's beta. The characteristic line is estimated by the least squares method. At present, beta is taken as the measure of the sensitivity of the security's price Y with respect to market changes. Beta shows how the price of a security responds to market forces. It is an indispensable tool in asset pricing.

2.1. Problems of the characteristic line and beta

As beta is the slope of a straight line, it is always a constant. Blume (1971), Hamada (1972) and Alexander & Chervani (1980) challenged the stability of beta. They argued that beta is time varying. Black (1976) linked beta to leverage which changes owing to changes in the stock price. Mandelker & Rhee (1984) related beta to decisions by the firm and thus a varying measure. The relationship between macro-economic variables and the firm's beta, as illustrated in the work of Rosenberg & Guy (1976) points to the varying character of beta. Since beta is evaluated as the covariance between the stock returns and index returns, scaled down by the variance of the index returns and the index volatility is time-varying (Bollerslev et al. 1992), beta is not constant over a period of time. Roll et al (1994) point out the inefficiency of the CAPM for estimating the expected returns using beta.

The constancy nature of beta raises doubts about the suitability of using it as a measure of the sensitivity of the security's return corresponding to market returns. This led us to think of a suitable measure that reflects instantaneous changes of the market.

Even if x and y are related by (1), beta alone cannot be used to measure the sensitivity of the price of the security. The parameter α also will play a major role unless its value is tested statistically insignificant. To measure the market sensitivity, the concept of elasticity is more useful. We discuss elasticity in the next section.

To understand and measure the response of the price of a security with respect to market changes, we need only the relationship between X and Y. Using regression method, the best linear equation can be estimated as

$$Y = a + b X \tag{2}$$

where a and b are constants.

3. Elasticity of price of a stock

The term 'elasticity' is a technical term used mainly by economists to describe the degree of responsiveness of the endogenous variable in an economic model with respect to the changes in the exogenous variable of the model. It measures the percentage change in the endogenous variable when the exogenous variable is increased or decreased by 1 %. So the concept of elasticity will be useful to measure the sensitivity of the price of a stock corresponding to market movements. If Y = f (X) is the functional relationship between X and Y, then the elasticity of Y with respect to X is given by

$$\eta = \frac{X}{Y} \cdot \frac{dY}{dX}$$

Theorem 1.

The elasticity of price of a security with respect to price index is a constant 'k' if and only if the relationship between the price Y of the security and the price index X is of the form

$$Y = C X^k \quad \text{where } C > 0,\ k > 0$$

By definition,

$$\eta = \frac{X\,C\,kX^{k-1}}{CX^{k}} = k$$

Conversely, if the elasticity is a constant, then

$$\frac{X}{Y}\cdot\frac{dY}{dX} = k$$

which implies

$$\frac{dY}{Y} = k\cdot\frac{dX}{X}$$

On integration of both sides,

$$\log Y = k \log X + \log C = \log C X^{k}$$

which implies

$$Y = C X^{k},\ C > 0, k > 0$$

Where log C is the constant of integration. If k =1, then Y = C X which represents a straight line passing through the origin. For any other value of k, the relationship is a power curve. This indicates that the sensitivity of the price of a security is a varying measure unless the relationship between the price of the security and the price index is a power curve.

If X and Y are related by (2), then

$$\eta = \frac{b\,X}{a + b\,X}$$

Note that the value of η is not a constant. It depends on the value of X. So, η varies when X varies. This means that the sensitivity of the price of a stock is not the same at all levels of the index. Further, the value of η depends on both the parameters a and b.

Case (i). $\eta = 1$.

This is the case when the price return of a stock is the same as that of the market return. This means that the price of a security increases (decreases) by 1 % when the share price index increases (decreases) by 1 %. In this case, a = 0. Since the intercept is zero, the regression line passes through the origin. See Figure 2.

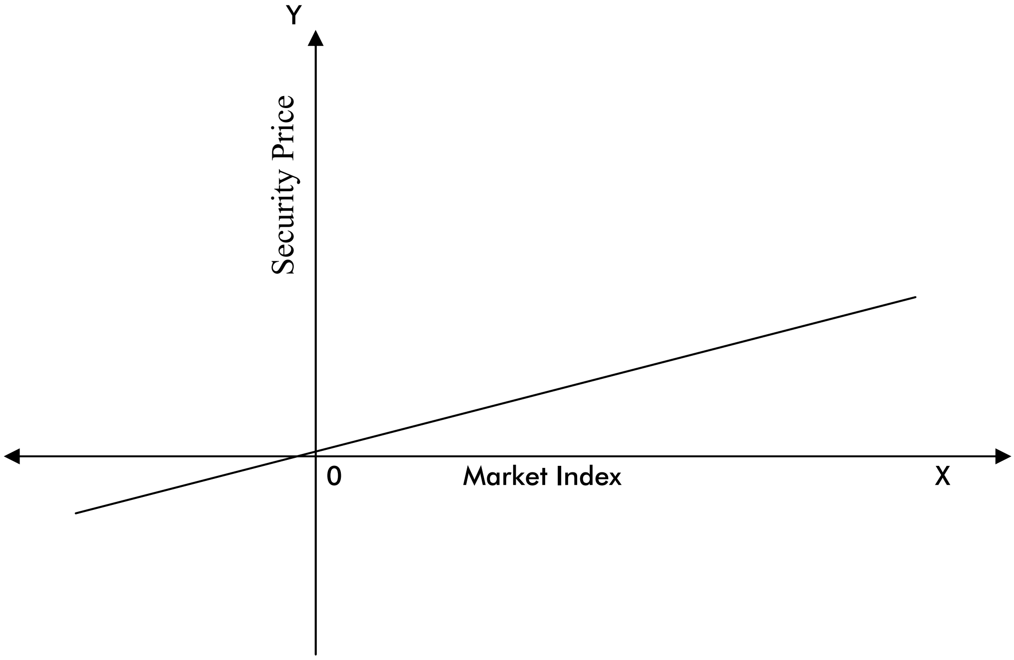

Figure 2. The regression of price of a stock on market index when $\eta = 1$.

It is very interesting to see that the elasticity remains unity for any regression line passing through the origin irrespective of the slope of the line.

Case (2). $\eta > 1$.

This is the case when the price return of a stock is more than proportional to market return. Since the price of a stock and the market index are generally positively correlated, the slope of the regression line 'b' is positive. Therefore, $\eta > 1$ only if 'a' is negative. See Figure 3.

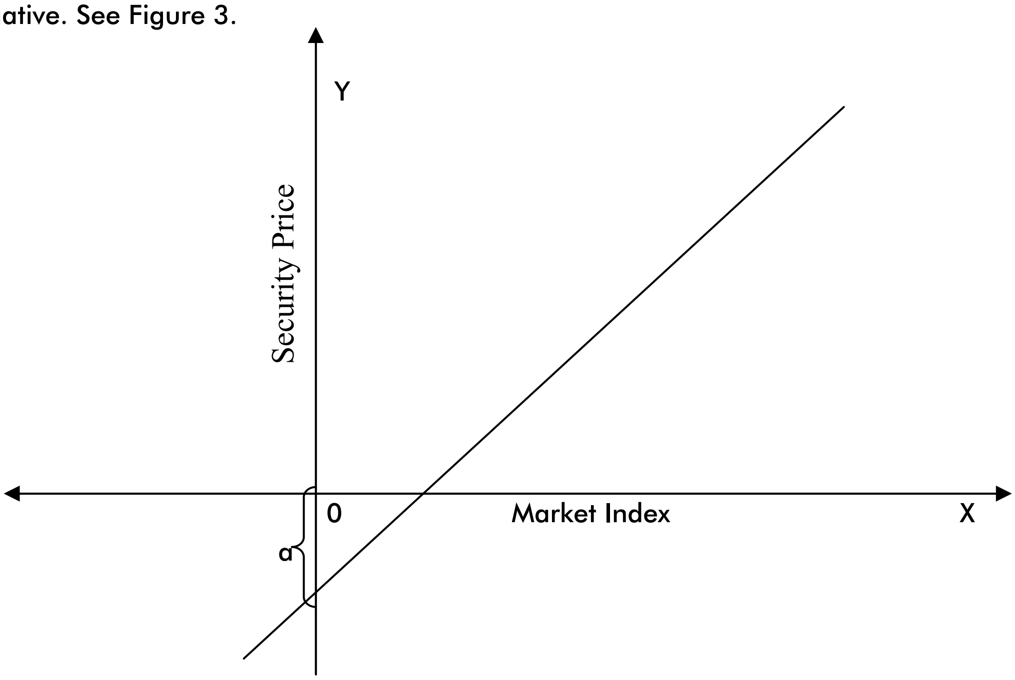

Figure 3. The regression of price of a stock on market index when $\eta > 1$.

Case (3). $\eta < 1$.

This is the case when the price return of a stock is less than proportional to market return. That is, the price of the security increases (decreases) by less than1 % when the share price index increases (decreases) by 1%. Also, $\eta < 1$ only if 'a' is positive. See Figure 4.

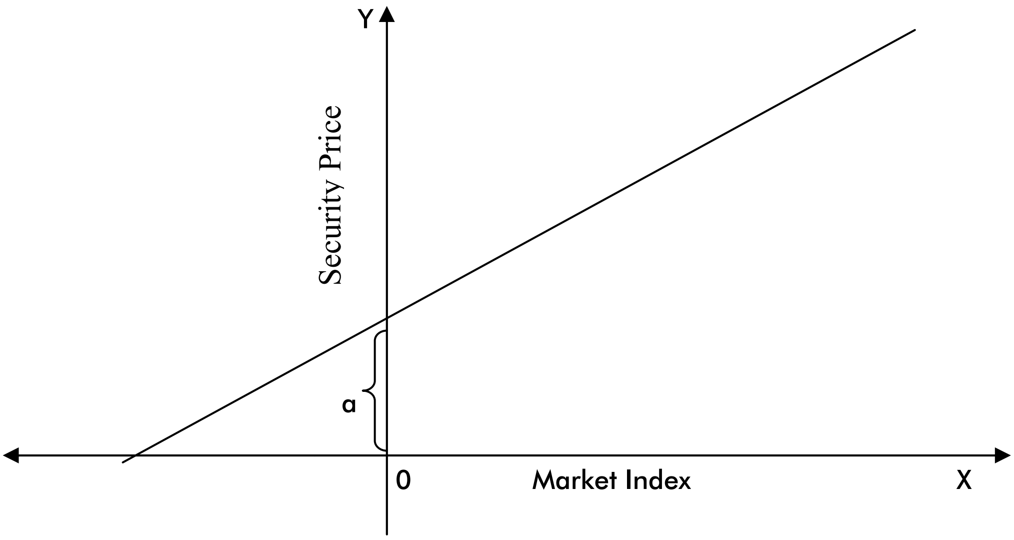

Figure 4. The regression of price of a stock on market index when $\eta < 1$.

4. Modified CAPM

Since elasticity measures the sensitivity of a security's price movement relative to the market movement, it will be more meaningful to replace the beta coefficient in the CAPM by the elasticity of the security. Then the modified CAPM takes the form

$$Rs = R_f + \eta (R_m - R_f) \qquad (3)$$

Where

Rs = the return required on investment

R_f = the return that can be earned on a risk-free investment

R_m = the return on the market index for a given index

η = the security's sensitivity (elasticity) to market movement for a given index

In the present form of the CAPM, beta remains the same irrespective of the sensitivity of the security with respect to market index. It depends only on the return on market and not on the index level. But, in the modified form, importance is given to η that varies as index varies. This is the striking advantage of the modified CAPM over the present form.

5. Illustration

This study is based on prices of shares of Reliance Industries Listed (RIL) at Mumbai Stock Exchange (BSE) in India and the BSE's benchmark price index Sensex during the period March 1996 to March 2006.Data relating to the financial year closing prices (Y) of (RIL) Shares and the Sensex (X) are given in Table 1. The yearly returns of X and Yare denoted by x and y respectively.

The regression of Y on X is given by

$$Y = -110.839 + 0.098918\,X \qquad (4)$$

Table 1. Yearly returns of RIL shares and Sensex (Price in Indian Rupees)

Period	X	Y	x	y
Mar-96	3367	104.97		
Mar-97	3361	154.45	-0.18	47.14
Mar-98	3893	177.20	15.83	14.73
Mar-99	3740	130.50	-3.93	- 26.35
Mar-00	5001	318.70	33.72	144.21
Mar-01	3604	391.20	-27.93	22.75
Mar-02	3469	398.40	-3.75	1.84
Mar-03	3158	278.50	-8.97	- 30.10
Mar-04	5613	538.20	77.74	93.25
Mar-05	6679	546.20	18.99	1.49
Mar-06	11603	1033.40	73.72	89.20

Source: Annual reports of Reliance Industries Limited and BSE publications

The elasticity of the price of the stock is

$$\eta = \frac{0.0989\,X}{-110.839 + 0.0989\,X}$$

When X = 9000, η = 1.14. This implies that when the price index is 9000, 1 % increase (decrease) in the index is followed by 1.14 % increase (decrease) and 10% increase (decrease) in the index is followed by 11.4 % increase (decrease) in the price of the share.

When X = 3500, η = 1.47. This shows that the sensitivity is varying at different levels of the index. It can also be seen that the sensitivity of the price of the RIL stock is very high for low values of the Sensex and comparatively low for higher values of the Sensex. Also, since the intercept is negative, η >1.

Now the characteristic equation based on the returns x and y is

$$y = 16.15 + 1.122\,x \tag{5}$$

When the market return is x = 1 %, y = 17.37 % which is an exaggerated value. Also, when x = 10 %, y = 27.37 %. In this case, the proportionality is not maintained as in the case of η. This implies that relation (5) has no meaning. Therefore the beta value β = 1.122 has no role in measuring the market sensitivity. Further, the different points (x, y) in the scatter diagram exhibit no functional relationship. For example, the points corresponding to March-2000 (33.72,144.21) and March-2005 (18.99,1.49) are not comparable. In fact, the Sensex and the price of the stock registered 33.55 % and 71.38 % in 2005 compared to the values in 2000. Since the coordinates (x, y) of a period are based on the values of the previous period, there is no common origin of measurements for the different points. As the different points in the scatter diagram will have no relationship between themselves, the concept of regression cannot be used for estimation purposes. This problem will be more aggressive if beta is estimated on the basis of a random sample.

Table 2. gives the returns of RIL shares (x) and Sensex (y) on arbitrary selected periods. X_1 and Y_1 are the values of X and Y arranged in another order. The returns of X_1 and Y_1 are denoted by x_1 and y_1 respectively.

Table 2. Returns of RIL shares and Sensex on arbitrary periods

X	Y	x	y	X_1	Y_1	x_1	y_1
3367.00	104.97			3740.00	130.50		
3361.00	154.45	-0.18	47.14	5001.00	318.70	33.72	144.21
3893.00	177.20	15.83	14.73	3604.00	391.20	-27.93	22.75
3740.00	130.50	-3.93	-26.35	3367.00	104.97	-6.58	-73.17
5001.00	318.70	33.72	144.21	3361.00	154.45	-0.18	47.14
3604.00	391.20	-27.93	22.75	3893.00	177.20	15.83	14.73
3469.00	398.40	-3.75	1.84	6679.00	546.20	71.56	208.24
3158.00	278.50	-8.97	-30.10	5713.00	540.30	-14.46	-1.08
5613.00	538.20	77.74	93.25	5786.00	576.50	1.28	6.70
6679.00	546.20	18.99	1.49	5186.00	486.00	-10.37	-15.70
3469.00	293.90	-48.06	-46.19	4756.00	423.30	-8.29	-12.90
3554.00	334.30	2.45	13.75	5677.00	534.90	19.37	26.36
3238.00	271.50	-8.89	-18.79	3469.00	398.40	-38.89	-25.52
4311.00	410.50	33.14	51.20	3158.00	278.50	-8.97	-30.10
5186.00	486.00	20.30	18.39	5613.00	538.20	77.74	93.25
5713.00	540.30	10.16	11.17	6325.00	514.80	12.68	-4.35
5786.00	576.50	1.28	6.70	8649.00	829.30	36.74	61.09
5677.00	534.90	-1.88	-7.22	3469.00	293.90	-59.89	-64.56
6325.00	514.80	11.41	-3.76	3554.00	334.30	2.45	13.75
4756.00	423.30	-24.81	-17.77	3238.00	271.50	-8.89	-18.79
8649.00	829.30	81.85	95.91	4311.00	410.50	33.14	51.20

The regression equation of y on x is

$$y = 8.20 + 1.167\ x \quad (6)$$

The regression equation of y_1 on x_1 is

$$y_1 = 12.45 + 1.617\ x_1 \quad (7)$$

The beta coefficients of (6) and (7) are considerably different. Further, the correlation coefficients between x and y is different from that between x_1 and y_1. This shows that beta cannot be estimated on the basis of a random sample. But, the regression equation of Y on X is the same as that of Y_1 on X_1. Similarly, the correlation coefficients between X and Y and that between X_1 and Y_1 are also the same. The elasticity also remains the same.

6. Conclusion

According to Nelson(2006), any asset pricing model should consider two areas of concern. The first concern is whether the model is well specified in random samples. The second concern is whether the model is powerful enough to explain stock returns. Our concept of elasticity and the modified CAPM address these two concerns. Also, this paper exposes the misconception of beta for measuring the sensitivity of the price of a security with respect to market movements. It also presents the mathematical logic against the fitting of a linear relation using regression method to a set of points that are not measured from an origin.

Our results show that the sensitivity of a stock is generally a varying aspect. Since beta is a constant as it is the slope of a straight line, it cannot be used to measure the sensitivity of a stock corresponding to market changes. The strong evidence in favour of time-varying betas (Bollerslev et al. 1992) highlights the limitations of ordinary least square betas. Further, the sensitivity of scrip's return depends largely on the elasticity of the price of the security. There are some securities that are more sensitive when the market index is at a peak level and less sensitive when the market index is at the moderate level. The expected rate of return varies from point to point. So, to estimate the expected rate of return corresponding to market returns, the use of the elasticity of price of the stock is advisable. Asset pricing based on elasticity will be more appealing than the one based on constant beta.

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