STUDY OF EQUILIBRIUM PRICES, USING GENERAL EQUILIBRIUM MODELS

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Abstract: *The first part of the paper presents some theoretical aspects related to general equilibrium models and the motivation of formulating the general equilibrium models as mixed complementary ones. Then we present a general equilibrium application using GAMS software (General Algebraic Modelling Systems). The application is a general equilibrium model for the Romanian energetic system, considered as a component of the national economy.*

Key words: general equilibrium models; equilibrium prices; *General Algebraic Modelling Systems – GAMS*

1. Theoretical background

We use the complementarity format because: first - it facilitates the links between prices and equilibrium conditions on the market; second - it allows relaxing the 'integrability conditions', which is not possible in the standard formulation of the general equilibrium models. More than these, the complementarity format facilitates associating second order conditions of the dual problem, with real economical tools available for policy design (for instance related to taxation effects, or related to external effects, or market failures). [4]

The complementarity format is a suitable one for formulating mathematical models related to energy – environment, because it allows integrating 'bottom-up models', with 'top-down models'. More precisely, it facilitates integrating very technologically detailed models, but insufficient developed on the macroeconomic side, with models having a very strong macroeconomic component, but weak in describing the energy and environment technologies on the production side.[2]

The complementarity format for energy – environment models is specifically useful because: frequently, the regulation authorities impose price constraints (i.e. upper bounds), or quantities constraints (i.e. a given percentage out of the total produced energy in the economy to come from renewables energy sources, or to be of nuclear type).

The general equilibrium model Arrow-Debreu is the starting point for any rigorous general equilibrium study.[1]

The model' hypothesis, which ensure the existence of the equilibrium in an economy, are very general and related to initial endowments, consumers' preferences, production and consumption possibilities. Arrow and Debreu have proved the existence of the equilibrium in such a model using the *fixed point theorem*, issued by Kakutani.

The producers are characterized by all production possibilities $Y_f \subset R^n, \quad f = 1,2,...,m$, while the consumers by the utility functions – convex and belonging to C^2 group of functions, $U_h : R_+^n \to R, \quad h = 1,2,...,k$ and by the initial endowments $\overline{x_h} \in R^n$. The market is responsible for the price vector $p \in R_+{}^n$.

<u>Definition 1</u>: *An Arrow – Debreu model state* is given by a production and consumption allocation, as well as by a price vector:

$$\omega = (x, y, p), \quad where:$$

$$x = (x_{1,} x_2, ... x_h, ..., x_k) \subset R_+^n,$$

$$y = (y_1, y_2, ..., y_f, ..., y_m) \subset Y_1 x Y_2 x ... x Y_f x ... x Y_m = \underset{f=1}{\overset{m}{X}} Y_f \ and \ p \in R_+^n$$

<u>Definition 2</u>: *An Arrow – Debreu equilibrium* state represent a consumption allocation $x^* = (x^*{}_{1,} x^*{}_2, ..., x^*{}_k)$, a production allocation $y^* = (y^*{}_1, y^*{}_2, ..., y^*{}_m)$, and a price vector $p^* = (p^*{}_1, p^*{}_2, ..., p^*{}_n)$, verifying the following:

$$a) \sum_{i=1}^{m} y_i^* + \sum_{j=1}^{k} \overline{x_j} \ge \sum_{j=1}^{k} x_j^*$$

$$b) p^* y_i^* = \max_{y_i \in Y_i} p^* y_i, \quad i = 1,2,...,m$$

$$c) U_j(x_j^*) = \max\{U_j(x_j) / p^* x_j \le p^* \overline{x_j} + \sum_{i=1}^{m} d_{ji} p^* y \ \}$$

In the following the Arrow – Debreu conditions will be rewritten.

Let us consider an economy with *n* goods (including production factors), *m* production sectors, and *k* consumers (households). Let *p* denote the prices vector ($p \ge 0$), *y* – the production vector based on constant returns to scale technologies ($y \ge 0$), M – the income vector, and a fictive agent which will be called 'market', establishing the prices.

The Arrow – Debreu equilibrium conditions in the considered abstract economy are as follows:

- the zero profit condition (no production sector has a positive profit)

  $$-\Pi_j(p) \geq 0 \quad \forall j \in \overline{1,m}$$

- the feasability condition for supply and demand:

  $$\sum_j y_j \frac{\partial \Pi_j(p)}{\partial p_i} + \sum_h \bar{x}_{ih} \geq \sum_h D_{ih}(p, M_h) \quad \forall i \in \overline{1,n}$$

where: $\bar{x}_{ih}$ - the initial endowment of household *h* with the good *i*

$D_{ih}(p, M_h)$ - the utility maximizing demand, for household *h,* the good *i*

- the income condition (the income of household *h* equals the value of initial endowments)

  $$M_h = \sum_i p_i \bar{x}_{ih} \quad \forall h \in \overline{1,k}$$

At equilibrium, the following relations will be satisfied:

$$y_j \Pi_j(p) = 0 \quad \forall j \in \overline{1,m} \qquad (1)$$

$$p_i \left[\sum_j y_j \frac{\partial \Pi_j(p)}{\partial p_i} + \sum_h \bar{x}_{ih} - \sum_h D_{ih}(p, M_h) \right] = 0 \quad \forall i \in \overline{1,n} \qquad (2)$$

So, at equilibrium, any production activity with a negative unitary profit is hidden, while the price of any good offered in excess in the economy is zero.

In a general equilibrium state, it cannot exist a consumption vector x_j so that:

$$x_j^q > \sum_{i=1}^{m} \max y_i^q + \sum_{j=1}^{h} \overline{x_j^q} = \overleftrightarrow{x_j^q} \qquad q = 1,2,....n$$

because the condition (a) cannot be fulfilled for the vector x_j, for any $y_{1,} y_2, ..., y_m$

It means the domain of the vectors $x_j, \quad j = 1,2,....k$ is not R_+^n, but only

$X = \{x \in R_+^n / 0 \leq x \leq \overleftrightarrow{x}\}$, where $\overleftrightarrow{x} = (\overleftrightarrow{x^1}, \overleftrightarrow{x^2}, ..., \overleftrightarrow{x^n})$.

To each state z of the model, we associate the following:

$Y_{1,} Y_2, ..., Y_m, X_1(z), X_2(z), ..., X_n(z), P$

where

$$X_j(z) = \begin{cases} \{x \in X / px \leq \sum_{i=1}^{m} d_{ji} p y_i + p\overline{x_j}\} & if \quad \sum_{i=1}^{m} d_{ji} p y_i + p\overline{x_j} \geq 0 \\ \{0\} & otherwise \end{cases}$$

We assume that the vector of initial endowments has positive components.

Then the functions z → $Xj(z), \quad j = 1,2,...k$ are continuous.

For a given z, $Xj(z)$ are convex, being associated to convex functions.

Let $f : R^{(m+k+1)n} \longrightarrow R^{(m+k+1)n}$ denote a function associated to the model:

$$f_i : R^{(m+k+1)n} \longrightarrow R^n, \quad i = 1,2,...m+k+1 \quad and$$
$$f_i(z) = -p, \quad i = 1,2,...,m$$
$$f_{m+j}(z) = -\nabla_{x_j} U_j(x_j), \quad j = 1,2,...,k$$
$$f_{m+n+1}(z) = \sum_{j=1}^{m} y_j + \sum_{j=1}^{k} \overline{x_j} - \sum_{j=1}^{k} x_j$$

where $\nabla_{x_j} U_j(x_j)$ represent the vector of the partial derivatives of U_j function against the components of the vector x_j.

Theorem 1: If Y_i are convex, compact and contain the origin point, and the utility functions are concave and belonging to C^2 class of functions, then the stationary points of the pair $(f, Y_1 x Y_2 x .... Y_m x X_1(z) x ... x X_k(z) x P)$ represent equilibrium states of the Arrow Debreu model. [

Proof:

Let $\overleftrightarrow{z} = (\overleftrightarrow{Y_{1,}} \overleftrightarrow{Y_2},..., \overleftrightarrow{Y_m, X_1(z), X_2(z)},..., \overleftrightarrow{X_n(z), P})$ denote a stationary point of the pair mentioned within theorem1. The points $\overleftrightarrow{Y_i}, \quad i = 1,2,...,m$, $\overleftrightarrow{X_j}, \quad j = 1,2,...,k$ are stationary points for the pairs $(f_i(\overleftrightarrow{z}), Y_i)$, $\overleftrightarrow{X_j}$ are stationary points for the pairs $(f_{m+j}(\overleftrightarrow{z}), Y_i)$, while P is a stationary point for the pair $(f_{m+k+1}(\overleftrightarrow{z}), P)$. Based on the definition of f function, we conclude that for i=1,2,…,m,

$$f_i(z) = \nabla_{y_i}(p y_i)$$
$$f_{m+k+1}(z) = -\nabla_p (p(\sum_{j=1}^{k} x_j - \sum_{j=1}^{m} y_j - \sum_{j=1}^{k} \overline{x_j}))$$

Condition (b) is a result of the properties of the stationary points for which $\overleftrightarrow{p y_i} = \max_{y_i \in Y_i} \overleftrightarrow{p} y_i, \quad i = 1,2,...,m$.

But $U_j(\overleftrightarrow{x_j}) = \max_{x_j \in X_j(\overleftrightarrow{z})} U_j(x_j), \quad j = 1,2,...,k$, and having the hypothesis $0 \in Y_i$, we get:

$$\overleftrightarrow{p y_i} \geq 0 \quad and \quad \sum_{i=1}^{m} d_{ji} \overleftrightarrow{p y_i}$$

so the condition (c) and the income one are fulfilled as well.

We will prove that the stationary point $\overleftrightarrow{z}$ verifies condition (a) as well.

We assume that the goods $k_1, k_2,..., k_r$ exist so that

$$\sum_{i=1}^{m} \overleftrightarrow{y}_i^{k_q} + \sum_{j=1}^{k} x_j^{k_q} < \sum_{j=1}^{k} \overleftrightarrow{x}_j^{k_q} \quad for \quad q = 1,2,...r$$

Then:

$$\sum_{j=1}^{n}\overset{\leftrightarrow}{x}_j^{k_q}-\sum_{j=1}^{k}x_j^{k_q}-\sum_{i=1}^{m}\overset{\leftrightarrow}{y}_i^{k_q}>0\geq\sum_{j=1}^{n}\overset{\leftrightarrow}{x}_j^{k_q}-\sum_{j=1}^{k}x_j^{k_q}-\sum_{i=1}^{m}\overset{\leftrightarrow}{y}_i^{k_q}\quad for\quad q=1,2,...,r\quad and\quad k\in\{1,2,...n\}\setminus\{k_1,k_2,...k_r\}$$

Because $\overset{\leftrightarrow}{P}$ is minimizing the function $P(\sum_{j=1}^{k}\overset{\leftrightarrow}{x}_j^{k_q}-\sum_{i=1}^{m}\overset{\leftrightarrow}{y}_i^{k_q}-\sum_{j=1}^{k}x_j^{k_q})$, then: $\sum_{q=1}^{r}\overset{\leftrightarrow}{P}_{k_q}=1$.

By summing in the inequalities above, we get a contradiction, so the condition (a) is verified.

Theorem 2: If Arrow – Debreu model has the properties (a), (b), (c), then it exists an equilibrium state.

Proof:

Let *T* denote the cartesian multiplication of the m+k+1 convex and compact results

$$T=Y_1xY_2x....Y_mxX_1x...xX_kxP$$

Using Theorem 1 above, it is sufficient to prove that exists one stationary point, z, of the pair $(f,Y_1xY_2x....Y_mxX_1(z)x...xX_k(z)xP)$

We take *g* function, g:T → T,

$$g(z)=Arg\min\|u-z+f(z)\|$$

$$u\in\prod_{j=1}^{m}Y_jx\prod_{i=1}^{k}X_i(z)xP$$

This function is continuous, and *T* is convex and compact. Using the Brower's Theorem for fixed points, we deduce that *g* has a fixed point, so the pair above has a stationary point.

We still need to prove that *g(z)* is a function, meaning $\|u-z+f(z)\|$ has a unique minimum on $Y_1xY_2x....Y_mxX_1(z)x...xX_k(z)xP,\quad\forall z\in T$

Because *T* is compact and $X_j(z),\quad\forall z\in T,\quad j=1,2,...,k$ are closed and bounded, then $Y_1xY_2x....Y_mxX_1(z)x...xX_k(z)xP,\quad\forall z\in T$ is also convex and compact.

If the production possibilities are specified explicitly, then can be established a link between Arrow – Debreu equilibrium states and the solutions of a complementary nonlinear problem.

Let

$$Y_i=\{y_i\in R^n/-k_i^1\leq(y_i)^Td_i\leq k_i^2\},$$

$$where\quad d_i\in R_+^n,\quad d_i\geq 0,\quad k_i^1,k_i^2\in R_+,\quad i=1,2,...,m$$

Corresponding to the *m* possibilities of production, we choose *m* scalars, which to fulfil the following condition:

$$m_i\geq\max_{y\in Y_i,k=1,2,...,n}(|\min y^k|),\quad i=1,2,...,m$$

Using the elements of Arrow – Debreu model, we consider the function $h:R^M\to R^M$, where M=m(n+2)+k(n+1)+n+2 with the components:

$$h = (h_1^1,...,h_m^1,h_1^2,...,h_n^2,h^3,...,h^8)$$
$$h_i^1(t) = -p + (z_i^1 - z_i^2)d_i, \quad i = 1,2,....,m$$
$$h_j^2(t) = -\nabla_x U_j(x_j) + z_j^3 p, \quad j = 1,...,n$$
$$h^3(t) = \sum_{i=1}^{m}(v^i - m_i e) + \sum_{j=1}^{k}(\overline{x}_j - x_j) + (z^5 - z^4)e$$
$$h_i^4(t) = k_i^2 - (v^i - m_i e)^T d_i, \quad i = 1,2,...,m$$
$$h_i^5(t) = k_i^1 - (v^i - m_i e)^T d_i, \quad i = 1,2,...,m$$
$$h_j^6(t) = \sum_{i=1}^{m} d_{ji} p(v^i - m_i e) + p(\overline{x}_j - x_j), \quad j = 1,2,...,k$$
$$h^7(t) = pe - 1$$
$$h^8(t) = 1 - pe$$

where

$$t = (v^1,v^2,...,v^m,x_1,x_2,...,x_k,p,z^1,...,z^5) \quad and \quad z^1,z^2 \in R_+^m, \quad z^3 \in R_+^k, \quad z^4,z^5 \in R_+,$$
$$e \in R_+^n, \quad e = (1,1,...,1)$$

Theorem 3: The allocation $\overline{z} = (\overline{y_1},\overline{y_2},....\overline{y_m},\overline{x_1},...,\overline{x_k},\overline{P})$ represents an equilibrium state of the Arrow – Debreu model if and only if it exist $\overline{z^1},\overline{z^2} \in R_+^m$, $\overline{z^3} \in R_+^k$, $,\overline{z^4},\overline{z^5} \in R_+$ so that $\overline{t} = (\overline{v^1},\overline{v^2},...,\overline{v^m},\overline{x_1},...,\overline{x_k},\overline{P},\overline{z^1},...,\overline{z^5})$ to represent a solution of the following non-linear complementary problem:

$$t \geq 0$$
$$h(t) \geq 0$$
$$t^T h(t) = 0$$

where $y_i = v^i - m_i e$

Proof:

We apply successively Kuhn-Tucker conditions to the following mathematical programming problems:

$$\min_{v_i \in V_i}(v^i)^T f_i^u(\overline{v^i}), \quad i = 1,2,...,m$$
$$\min_{p \in P} P^T f_{m+k+1}^u(\overline{P})$$
$$\min_{x_j \in X_j(u)}(x_j)^T f_{m+j}^u(\overline{x_j}), \quad j = 1,2,...,k$$

where $\overline{u} = (\overline{v^1},\overline{v^2},...,\overline{v^m},\overline{x_1},...,\overline{x_k},\overline{P})$ is fixed, and V_i are obtained based on Y_i, by changing $y_i = v^i - m_i e, \quad i = 1,2,...,m$ and forcing $u = \overline{u}$ to be an optimum point.

For revealing the link between the general equilibrium conditions, briefly mentioned above, with the complementarity format, we consider in the following the energy sector described by the model:

$$\begin{cases} \min \sum_t \overline{c_t} y_t \\ \sum_t a_{jt} y_t = \overline{d_j} \quad \forall j \\ \sum_t b_{kt} y_t \le s_k \quad \forall k \\ y_t \ge 0 \quad \forall t \end{cases}$$

where:

t - index for the possible energy production possibilities
j - index for the energy goods
k - index for the energy resources
y_t the production level based on technology *t*
a_{jt} - the net quantity of good *j*, produced based on technology *t*
$\overline{d_j}$ - the market demand for good *j*
$\overline{c_t}$ - the marginal cost associated to production technology *t*
b_{kt} - the unitary demand for the resource *k*, used with technology *t*
s_k - the aggregate supply of the energetic resource *k*

The above model is a costs minimization problem/model for the energy sector, so that the demand to be covered, and the technological constraints to be fulfilled.

For solving it, we apply Kuhn – Tucker method. So we associate positive multipliers λ_j and μ_k to the constraints. The optimum conditions are as follows:

$$\lambda_j (\sum_t a_{jt} y_t - \overline{d_j}) = 0 \qquad (1')$$

$$\mu_k (\sum_t b_{kt} y_t - s_k) = 0 \qquad (2')$$

$$\sum_j \lambda_j \ \overline{d_j} = \sum_t c_t y_t + \sum_k \mu_k s_k \qquad (3')$$

It can be noticed it exist similarities between the general equilibrium relations (1) and (2), and the relations obtained by applying Kuhn-Tucker conditions (1′) and (2′). Also, there is a similarity between the duality relation (3′) and the zero profit condition from the general equilibrium model.

2. Relevant aspects from the electricity market in Romania

The scope of making this insight into the Romanian electricity market is to facilitate the understanding of the entry data used in the applicative part of this paper, the last one.

Nuclear energy currently represent in Europe one of the most important energy resources without CO_2 emmissions. The nuclear plants ensure one third of the total electricity production in EU, so bringing a real contribution to sustainable development in the Union. In Romania, Unit 1 Cernavoda covers almost 10% of total electricity production. This year is expected to start to be used Unit 2 Cernavoda, and so to double the contribution of nuclear resource in covering electricity needs in Romania. The Romanian nuclear program is based on a secure tehnology, with a good recognition at international level, and being well perceived by the public opinion.

Based on some predictions for the evolution of several macroeconomic indicators between 2007 – 2020[4] (population, gross domestic product, energy intensity), the Economy and Commerce Ministery estimated the electricity needs for the same period (2007 – 2020), as well as the contribution of each energetic resources for covering those electricity needs [9]. Selected results[5] are displayed in the table bellow:

Table 1. Structure of electricity domestic production

	2003	2004	2005	2006	Estimated for 2010	Estimated for 2020
Electricity production for internal consumption coverage, TWh	54.55	55.3	56.48	58.99	66.1	85
Electricity export, TwH	2.08	1.18	2.93	3.41	4.5	15
Total electricity production, TwH	56.63	56.48	59.41	62.4	70.6	100
Electricity production in hydro plants + renewables, TwH	13.57	16.83	20.21	17.75	21.7	32.5
Electricity production in hydro plants + renewables, %	23.96%	29.80%	34.02%	28.45%	30.74%	32.50%
Electricity production in nuclear plants, Twh	4.9	5.55	5.54	5.55	10.8	21.6
Electricity production in nuclear plants, %	8.65%	9.83%	9.33%	8.89%	15.30%	21.60%
Electricity production in thermo plants, coal, Twh	23.34	21.47	21.66	27.1	27.1	34.9
Electricity production in thermo plants, coal, %	41.21%	38.01%	36.46%	43.43%	38.39%	34.90%
Electricity production in thermo plants, gas, Twh	11.19	10.46	10	10	9.5	9.5
Electricity production in thermo plants, gas, %	19.76%	18.52%	16.83%	16.03%	13.46%	9.50%
Electricity production in thermo plants, oil, Twh	3.63	2.17	2	2	1.5	1.5
Electricity production in thermo plants, oil, %	6.41%	3.84%	3.37%	3.21%	2.12%	1.50%

3. General equilibrium application

The application bellow is a general equilibrium model for the Romanian energetic system, considered as a component of the national economy. It is run with GAMS software (General Algebraic Modeling Systems), more specifically with the dedicated solver MPSGE (Mathematical Programming System for General Equilibrium). [3]

GAMS is a mathematical programming language, in use starting with 1987 in over 100 countries currently. It can solve complex problems of optimization and modelling: linear, linear mixed integer, non-linear, non-linear mixed integer, mixed complementary, general equilibrium, stochastic optimization, systems of non-linear simultaneously equations etc.

Let us consider a closed economy having:

- 2 goods being produced: electricity and the rest of the goods produced in the economy, aggregated (the non-energetic good)
- 3 production factors: labor, capital, and 5 types of energetic resources:
  - o resources thermo-energetic (coal, gas, and oil)
  - o resources hydro-energetic
  - o nuclear resources

  Note: the aggregate good plays a double role: output, as well as input
- 1 representative consumer, endowed with the production factors, and responsible for the consumption of the goods produced
- 3 production sectors: for the non-energetic good, for the electricity, and for the consumer satisfaction based on his final consumption

  Note: the third sector is chosen like this for allowing the analysis of some relevant indices related to consumer satisfaction.

With this specifications made, the general equilibrium model can lead to conclusions for both consumption and production activities in the economy, as well as for equilibrium prices.

Entry data for Scenario I :

We consider the following entry data for the production side:

- for the satisfaction of the representative consumer:

The total of 100% comes from 3 sources:
o usage of leisure (complementing the working time): 50%
o consumption of the non-energetic goods: 40%
o consumption of the electricity: 10%

Note: conforming with the Romanian National Institute of Statistics, the value of the electricity consumption in total consumption is 10%

- for the production of the aggregate non-energetic good:

The total of 40% is produced using the production factors as follows:
o capital: 18.65%
o labour: 18.65%
o energetic resources: 2.7%

Note: conforming with the Romanian National Institute of Statistics, the value of the production in the energetic sector in total value produced in all sectors of the economy is 6.8%, which leads to 2.7% input contribution, after weighting accordingly with the total

Note: we assume that the production factors capital and labour equally contribute in producing the non-energetic good

- for electricity production

The total of 10% is produced using the production factors as follows:

- capital: 2%
- labour: 2%
- non-energetic good: 1%
- energetic resources: 5%

Note: the energetic resources contribute to electricity production following exactly the distribution of electricity production at national level, as presented above in Table 1. For Scenario I, according to 2006 data, the weights should be:

- resources thermo-energetic, coal : 43.4 %
- resources thermo-energetic, gas: 16.0 %
- resources thermo-energetic, oil: 3.2 %
- resources hydro-energetic: 28.4 %
- nuclear resources: 8.9 %

These contributions are weighted before being input into the final model, so that to sum 5% at the end, and so to reach the assigned quota for energetic resources.

Note: it is assumed as a working hypothesis that labour and capital production factors equally contribute to production of the energetic good.

Entry data for the consumption side are presented in the following. It is assumed that all production factors, which are not intermediary goods, represent the endowment of a single consumer, a representative one in the economy considered.

- capital: 20.65% (used in producing the aggregated non-energetic good, as well as in producing the energetic good, as it can be noticed from the entry data above)
- human resource: 70.65% (this factor is used in goods production, as well as in consumer satisfaction production)
- energetic resources: 5% (this factor is used only in producing the energetic good)

At equilibrium, the following results are obtained:

- the optimal production levels:
  - satisfaction from final consumption: 1.034
  - aggregated non-energetic good: 1.006
  - energetic good, split by production technologies:
    - resources thermo-energetic, coal : 0.613
    - resources thermo-energetic, gas: 0.649
    - resources thermo-energetic, oil: 0.015
    - resources hydro-energetic: 0.687
    - nuclear resources: 0.389
- relative equilibrium prices:
  - for the goods:

- utility index of final consumption: 1.125
- aggregated non-energetic good: 1.224
- energetic good: 0.514

- for the production factors:
  - human resources usage: 1.215
  - capital usage: 1.362
  - usage of resources thermo-energetic, coal: 0.191
  - usage of resources thermo-energetic, gas: 0.092
  - usage of resources thermo-energetic, oil: 0.004
  - usage of resources hydro-energetic: 0.156
  - usage of nuclear resources: 0.033

Entry data for Scenario II:

We consider the same entry data as for Scenario I, except for the electricity production structure. In this case, estimated data for 2010 are used:

- resources thermo-energetic, coal: 38.3 %
- resources thermo-energetic, gas: 13.0 %
- resources thermo-energetic, oil: 2.1 %
- resources hydro-energetic: 30.7 %
- nuclear resources: 15.3 %

The difference comes from increased contribution of the nuclear resource (almost doubled, compared to the previous scenario), so less contributions assigned to the fossil fuels based technologies. The motivation for simulating this scenario stands in the previous chapter of this paper.

The following results are obtained at equilibrium:

- the optimal production levels:

  - satisfaction from final consumption: 1.035
  - aggregated non-energetic good: 1.007
  - energetic good, split by production technologies:
    - resources thermo-energetic, coal: 0.619
    - resources thermo-energetic, gas: 0.524
    - resources thermo-energetic, oil: 0.035
    - resources hydro-energetic: 0.650
    - nuclear resources: 0.584

- relative equilibrium prices:

  - for the goods:
    - utility index of final consumption: 1.124
    - aggregated non-energetic good: 1.223
    - energetic good: 0.507

o	for the production factors:
	▪ human resources usage: 1.215
	▪ capital usage: 1.362
	▪ usage of resources thermo-energetic, coal: 0.174
	▪ usage of resources thermo-energetic, gas: 0.061
	▪ usage of resources thermo-energetic, oil: 0.007
	▪ usage of resources hydro-energetic: 0.155
	▪ usage of nuclear resources: 0.079

4. Conclusions

The most important value in the economy considered is that of the aggregated non-energetic good (1.224), followed by the satisfaction of the representative consumer for the final consumption, and followed then by the energetic resources. The less important value is obtained for the energetic resource least used in the economy (oil, with 0.004).

By changing the structure of energetic resources usage, the equilibrium values are affected. It was shown how the equilibrium usage of the nuclear resource is changing from 0.033 to 0.079, once the nuclear contribution is almost doubling. Minor influences can be noticed at macroeconomic level as well, such as a decrease of the final consumption value and of the consumer's satisfaction (with 0.001 each of them).

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