RESOURCE REALLOCATION MODELS FOR DETERMINISTIC NETWORK CONSTRUCTION PROJECTS

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Abstract: *Hierarchical budget reallocation models for a portfolio of construction network projects with deterministic activity durations are considered. Optimal reallocation models both at the company level and at the project level are developed.*

Key words: *network construction project; hierarchical budget reallocation models; time-cost optimization problems; R&D network projects*

1. Introduction

It can be well-recognized nowadays that the broad variety of network projects is subdivided into two classes:

- R&D projects which are carried out under random disturbances, and
- Construction projects to be identified by an essentially lower level of indeterminacy.

R&D projects are characterized by the following properties [1]:

a) the level of indeterminacy is extremely high, moreover, random parameters are implemented in the nature of the project's elements, e.g., project's activities are of random duration and an essential part of R&D projects are of random branching type. In the latter case the direction of realizing the project is of random nature;

b) R&D projects are usually realized by means of networks of acyclic type;

c) monitoring R&D projects is carried out through probability control, i.e., by implementing chance constraint models;

d) R&D projects are facilitated by both cost- and non-cost detailed resources.

Construction projects are usually characterized by:

a) operations (activities) of deterministic nature and corresponding networks of deterministic structure. The level of indeterminacy is, thus, caused not by the network, but by disorder of the project's functioning. This results in generally higher levels of determinacy;

b) networks with both acyclic and cyclic graphs. Construction network projects comprise more complicated logical relations, than R&D networks;

c) standard resource units (i.e., in the form of construction teams). Individual resources are less frequent, than for the case of R&D projects;

d) monitoring by means of scheduling network activities. Probability control is practically not used;

e) control actions are usually determined by optimal cost reallocation models at the company's or project levels.

In the paper under consideration hierarchical budget reallocation models for a portfolio of construction network projects with deterministic activity durations are suggested. Optimal reallocation models both at the company level and at the project level are developed.

2. Notation

Let us introduce the following terms:

$G_k(N,A)$ - construction network projects, $1 \le k \le n$;

n - number of projects;

C - budget value assigned to carry out all projects (to be optimized);

C_{kt} - budget assigned to $G_k(N,A)$ at moment t;

$(i,j)_k$ - activity (i,j) entering project $G_k(N,A)$;

$c_{ijk}^{(r)}$ - budget value which can be assigned to $(i,j)_k$ to operate and realize the latter ($1 \le r \le n_{ijk}$, both $c_{ijk}^{(r)}$ and $n_{ijk}^{(r)}$ are deterministic and pregiven);

n_{ijk} - number of possible durations of activity $(i,j)_k$ by means of assigned budget $c_{ijk}^{(r)}$;

$t_{ijk}^{(r)}$ - deterministic duration of activity $(i,j)_k$ by means of assigned budget $c_{ijk}^{(r)}$ (pregiven). Note that relation $c_{ijk}^{(r_1)} < c_{ijk}^{(r_2)} \Rightarrow t_{ijk}^{(r_2)} < t_{ijk}^{(r_1)}$ holds;

S_{ijk} - moment activity $(i,j)_k$ starts to be operated (to be determined);

F_{ijk} - moment activity $(i,j)_k$ is finished (to be determined, depends on assigned budget $c_{ijk}^{(r)}$; note that relation $F_{ijk} = S_{ijk} + t_{ijk}^{(r)}$ holds);

C_t - total non-realized budget at moment t; $0 \le t$; $C_0 = C$;

F_k - actual moment project $G_k(N,A)$ is accomplished (to be determined);

D_k - the due date for project $G_k(N,A)$ (pregiven);

$G_{kt}(N,A)$ - a non-realized part of project $G_k(N,A)$ at moment t;

$T_{cr}\left\{G_{kt}(N,A)/c_{ijk}^{(r)}\right\}$ - critical path length of project $G_{kt}(N,A)$ subject to assigned budget values $c_{ijk}^{(r)}$.

3. The Problem

The optimization problem is as follows:

At any moment $t \ge 0$ determine both C_t and C_{kt}, $1 \le k \le n$, as well as values $c_{ijk}^{(r)}$ for all non-accomplished at moment t activities $(i,j)_k$, to minimize objective

$$\underset{\left\{C_{kt},\, c_{ijk}^{(r)}\right\}}{Min}\ C_t \tag{1}$$

subject to

$$\sum_{k=1}^{n} C_{kt} = C, \tag{2}$$

$$\sum_{\{i,j\}_k^t} c_{ijk}^{(r)} \le C_{kt}, \tag{3}$$

$$\underset{r}{Min}\ c_{ijk}^{(r)} \le c_{ijk}^{(r)} \le \underset{r}{Max}\ c_{ijk}^{(r)}, \tag{4}$$

$$t + T_{cr}\left\{G_{kt}(N,A)/c_{ijk}^{(r)}\right\} \le F_k,\ 1 \le k \le n, \tag{5}$$

where $(i,j)_k^t$ denotes the set of activities entering $G_{kt}(N,A)$.

Note that since we deal only with budget values C_{kt}, and taking into account that all projects under consideration can be regarded as independent ones, problem (1-5) can be simplified and transformed to the case of one project only. Cancel index k of the project and formulate the amended problem as follows:

Given for each activity (i,j) a set of n_{ij} couples $\left\{c_{ij}^{(r)}, t_{ij}^{(r)}\right\}$, $1 \le r \le n_{ij}$, where each couple denotes the possible assigned budget value with the corresponding activity duration, together with the pregiven due date D of a deterministic network project $G(N,A)$, determine budget values c_{ij} as well as the total budget $C = \sum_{\{i,j\} \in G(N,A)} c_{ij}$, which results in minimizing value C

$$\underset{\{c_{ij}\}\subset G(N,A)}{Min} \quad C = C^* \tag{6}$$

subject to

$$\sum_{(i,j)\subset G(N,A)} c_{ij} \le C^* \tag{7}$$

and

$$T_{cr}\left\{G(N,A)/c_{ij}\right\} \le D\,. \tag{8}$$

It can be well-recognized that determining C <u>for each project $G_k(N,A)$ independently</u>, results in providing objectives (1-2) using relation (3).

To solve problem (6-8), we require solving an auxiliary problem (AP).

4. Auxiliary Problem (AP)

Problem AP [2] may be considered as follows:

Given a deterministic PERT graph $G(N,A)$ together with pregiven functions $t_{ij}^{(r)}$ and $c_{ij}^{(r)}$,

$t_{ij}^{(r)} = f_{ij}\left(c_{ij}^{(r)}\right),\ 1 \le r \le n_{ij}$,

number n_{ij} pregiven for each activity $(i,j)\subset G(N,A)$, determine:

• the minimal total project direct costs C,

$$Min\ C, \qquad \text{and} \tag{9}$$

• the optimal assigned budget values c_{ij}^{opt}, subject to

$$T_{cr}\left\{t_{ij} = f_{ij}\left(c_{ij}^{opt}\right)\right\} \le D, \tag{10}$$

$$\sum_{\{i,j\}} c_{ij}^{opt} = C, \tag{11}$$

$$c_{ij\ \min} \le c_{ij}^{opt} \le c_{ij\ \max}, \tag{12}$$

where D stands for a pregiven due date.

Problem AP is solved in [2] by means of heuristic methods based on transferring the possible budget ΔC from non-critical activities (which have practically no influence on the project's critical path duration) to critical activities either belonging to the critical path or being very close to the latter.

The corresponding algorithm is described in [2].

5. Problem's (6-8) Solution

The step-wise procedure of solving problem (6-8) is as follows:

<u>Step 1</u>. Determine the minimal budget value C_1,

$$C_1 = \sum_{(i,j)\subset G(N,A)} \left(\min_r c_{ij}^{(r)}\right), \tag{13}$$

which by no means can be diminished - otherwise problem (6-8) has no solution.

<u>Step 2</u>. Determine the maximal budget value C_2

$$C_2 = \sum_{(i,j)\subset G(N,A)} \left(\max_r c_{ij}^{(r)}\right), \tag{14}$$

which by no means should be increased - otherwise solving problem (6-8) results in redundant budget spending.

Step 3. For both cases considered in Steps 1-2, calculate critical path lengths for graph $G(N,A)$:

$$t_{ij}(C_1) = \min_r t_{ij}^{(r)}, \ (i,j)\subset G(N,A), \tag{15}$$

$$t_{ij}(C_2) = \max_r t_{ij}^{(r)}, \ (i,j)\subset G(N,A), \tag{16}$$

Call henceforth T_1 the critical path length of $G(N,A)$ in case (15) and T_2 - in case (16). It can be well-recognized that when $D < T_1$ problem (6-8) has no solution. Taking into account [1,3] the obvious relation

$$c^{(r_1)}(i,j) > c^{(r_2)}(i,j) \Rightarrow t^{(r_1)}(i,j) < t^{(r_2)}(i,j) \tag{17}$$

for any $(i,j)\subset G(N,A)$, a conclusion can be drawn that relation $D \geq T_2$ should result in $C^* \leq C_2$.

Assume, further [1,3], that the critical path length T_{cr} of any project $G(N,A)$, designated henceforth as $T_{cr}(C)$, depends linearly on budget C assigned to that project; in other words,

$$\frac{T_{cr}(C') - T_{cr}(C'')}{C'' - C'} \approx const \tag{18}$$

holds.

Step 4. Calculate, by means of (18), the preliminary (non-minimal!) value C corresponding to the due date D. Using

$$\frac{T_{cr}(C_1) - T_{cr}(C_2)}{C_2 - C_1} \approx \frac{T_{cr}(C_1) - D}{C - C_1},$$

we finally obtain

$$C = C_1 + \frac{(T_{cr}(C_1) - D)\cdot(C_2 - C_1)}{T_{cr}(C_1) - T_{cr}(C_2)}. \tag{19}$$

Step 5. Solve subsidiary Problem A:

Given budget C assigned to project $G(N,A)$, determine the minimal critical path length $T_{\min}^{(\nu)}$ by means of redistributing C among activities $(i,j)\subset G(N,A)$. Let ν be the number of the current iteration.

Step 6. Compare values $T_{\min}^{(\nu)}$ and D. If $T_{\min}^{(\nu)} > D$, go to 7. Otherwise apply the next step.

Step 7. Set $T_{\min}^{(\nu)} \Rightarrow T_{cr}(C_1)$, $C \Rightarrow C_1$, $\nu + 1 = \nu$. Go to Step 4.

Step 8. Compare $T_{\min}^{(\nu)}$ with $T_{\min}^{(\nu-1)}$. If $\left|T_{\min}^{(\nu)} - T_{\min}^{(\nu-1)}\right| < \varepsilon$, where ε stands for the problem's accuracy, apply the next step. Otherwise go to Step 11.

Step 9. Budget value C referring to value $T_{\min}^{(\nu)}$ at Step 5, is considered to be the optimal (minimal) value C^* with local budget values $\left\{c_{ij}^{(r)}\right\}$ obtained in the course of solving Problem A at Step 5.

Step 10. The problem's solution terminates.

Step 11. Set $T_{\min}^{(\nu)} \Rightarrow T_2$, $C \Rightarrow C_1$, $\nu+1=\nu$. Go to Step 4.

The solution of the global problem (1-4) can be obtained by summarizing values C_{kt} calculated independently for each k at any moment $t \geq 0$. If $t > 0$ we take into account the updated graph $G_t(N, A)$ instead of $G(N, A)$ (for a single project) and determine value C_t^* instead of value C^*. After the algorithm's (6-8) termination all optimal values C_{kt}^* are summarized in order to obtain the updated total value C_t^*.

6. Conclusions

1. The newly developed algorithm is easy in usage and effective in practice. Its implementation requires mostly no more than 3÷5 iteration.

2. The algorithm has been widely used both for medium- and large-scale projects with the number of activities exceeding 50÷100. In all cases the algorithm performed well and the number of iterations did not exceed 5.

3. The algorithm can be realized on the basis of classical algorithms which are widely used in network planning and are described in many textbooks on project management.

4. The model suggested in this paper is open for various modifications: e.g., instead of purely deterministic values defining the closeness of activities to the critical area and, thus, the level of their influence on the project's duration, other terms may be implemented. However, those modifications are not essential from the principal point of view.

5. In our opinion, the developed research can be widely used for network construction projects of acyclic type.

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