

HAZARD MORAL MODELS WITH THREE STATES OF NATURE

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Abstract:

The paper analyses a moral hazard model with three states of nature. The model is solved using as variables the informational rents and effort levels. Finally, we determine the features of the optimal contracts in asymmetric information.

Key words: *moral hazard; asymmetric information; informational rents; optimal contract*

Despite 30 years of studies in economics of information, the effects of asymmetric information on different markets are far to be complete known. In fact, this asymmetric information constitutes the central point in economics of information and corresponds to the situation where a contractual partner has more information or better information than the other partner about the transaction characteristics. The economics of information concentrates on studying the incentives to get some potential gains from having private information in a transaction. The incentives are present in almost all economic activities: there are incentives to work with high productivity, to produce good quality products, incentives to study, incentives to invest or to save money.

A different part of economics of information corresponds to moral hazard models. This type of models analyses the economic agents' behavior when acting on different markets: labor market, financial markets, insurance markets, agriculture contracting etc. The macroeconomic literature about the problem of efficient wages correlated with the Agent's effort started with the papers of Solow and Salop (1979), Shapiro and Stiglitz (1984) and

was later developed by Carmichael (1985), McLeod and Malcomson (1987), Saint-Paul (1996), Krishnan (2007). Holmstrom and Tirole (1994) developed a credit rationing theory based on moral hazard models. Dave and Kaestner (2009) analyzed the effects of pure ex-ante moral hazard on health insurance market, and Duhnam (2003) proposed a moral hazard model for the leasing market.

Recent research shows that the models became more and more complex, most of them being mixed models with moral hazard, signaling or screening. Fudenberg and Tirole (1990) proposed a mixed model where the Agent's actual choice regarding the effort is an endogenous adverse selection variable at the renegotiation stage and this aspect generates inefficiency. This problem was partially solved by Matthews (1995) and Ma (1994). Page (1991, 1997) presented a mixed model with moral hazard and adverse selection, and Jullien and Salanie (2007) extended the moral hazard model for the situation where the Agent's risk aversion constitutes his private information, such that the model presents also an adverse selection problem. Such approach was also used by Reichlin and Siconolfi (2004); they generalized the pure adverse selection model of Rothschild and Stiglitz, including some moral hazard variables. Mylovanov and Schmitz (2008) studied a two period moral hazard model, where the Agents are risk neutral, with limited liability and three identical activities.

Introduction

The most used models of hazard moral are the models where the Principal doesn't have direct control on the Agent's effort. There are also some models of hazard moral, not so used in the literature – the Agent's behavior constitutes hidden information either because this behavior is not observable, or, even it is observable, the Principal can not know exactly which is the best Agent's decision regarding the level of effort. [2]

In the later situation (the second type of moral hazard), once the contract is signed, the Agent gets information about the states of nature and knows which is the best choice regarding the effort he exerts. This information is not observable or verifiable by the Principal.

From this point of view, there are two types of hazard moral models:

- the models with an ex-ante participation constraint. In this type of models, the Agent has a given expected utility when signing the contract, and, if he accepts the Principal's offer, he can not breach the contract in the future.

- the models with ex-post participation constraints (the number of constraints is equal to the number of unknown or unpredicted situations), such that, the Agent gets an expected utility which is always equal or greater than his reservation utility, for such unpredicted situation.

We will analyze a model from the first type presented above (this model is not so often discussed in the literature) with three states of nature. The structure of the paper is as follows. Section 1 presents the model. In Section 2 we transform the model using a well known concept in economics of information literature – informational rents. Section 3 studies the optimal contract in the situation of asymmetric information and in the last part (Section 4) we present the features of the optimal contract and some concluding remarks.

1. The model

We suppose that after signing the contract, the Agent observes (knows) the market conditions – if these conditions are good or bad.

We denote by θ the parameter that characterizes the market conditions, with $\theta \in \{\theta^G, \theta^M, \theta^B\}$. A high level of this variable, $\theta = \theta^G$, indicates a favorable situation for the business, while $\theta = \theta^M$ corresponds to a medium situation (a medium state of nature) and $\theta = \theta^B$ (bad situation of unfavorable situation on the market) implies some decisions regarding the effort with a higher cost than the other ones. It is obvious then that $\theta^G > \theta^M > \theta^B$.

We also suppose that the Agent will exert a total level of effort denoted by E , but this effort level costs more when the market conditions are bad.

We consider that $E = \theta + e$, where the Agent's decision regarding the effort level e is costly, but θ doesn't. The Agent will choose the costly effort e , with respect to the information he gets from θ .

The Agent, after signing the contract, observes the true value of the variable θ (θ^B, θ^M or θ^G). The Principal observes the total decision E ; because he cannot distinguish between the market conditions, the Principal doesn't know the effort level exerted by the Agent. This means that the later could exerts a high level of effort or a medium or low level of effort.

The Principal faces six types of incentives constraints (3 pairs of constraints), some of them being local constraints (4 upward and downward incentive constraints), and the other two constraints being global incentive constraints (one upward constraint and one downward constraint). [2, 7]

The first type of constraints shows that the Agent does not pretend that the market conditions are G (or M, or B) when the true conditions are M or B (or (G or B) or (G or M)).

The second type of constraints shows that the Agent does not announce that the market conditions are M (or G, or B) when the true conditions correspond to the other types.

Subject to these constraints, the Principal will offer a menu of contracts $\{(e^G, W^G), (e^M, W^M), (e^B, W^B)\}$, where e and W represent the costly effort and the Agent's wage for each state of nature (favorable, medium or unfavorable), with $\theta^B < \theta^M < \theta^G$.

We consider that the respective probabilities of the three nature states are π^B, π^M and π^G (strictly positive), with $\pi^B + \pi^M + \pi^G = 1$.

If the Principal is risk neutral and the Agent is risk adverse, than - using the usual notations - the mathematical model (P) for deriving the optimal contract in the situation of asymmetric information is:

$$\text{Max}_{\{(e^G, W^G), (e^M, W^M), (e^B, W^B)\}} \left\{ \pi^G [e^G + \theta^G - W^G] + \pi^M [e^M + \theta^M - W^M] + \pi^B [e^B + \theta^B - W^B] \right\}$$

s.t.

$$\pi^G [U(W^G) - V(e^G)] + \pi^M [U(W^M) - V(e^M)] + \pi^B [U(W^B) - V(e^B)] \geq \underline{u} \quad (2)$$

$$U(W^G) - V(e^G) \geq U(W^M) - V(e^M + \theta^M - \theta^G) \quad (3)$$

$$U(W^G) - V(e^G) \geq U(W^B) - V(e^B + \theta^B - \theta^G) \quad (4)$$

$$(P) U(W^M) - V(e^M) \geq U(W^B) - V(e^B + \theta^B - \theta^M) \quad (5)$$

$$U(W^M) - V(e^M) \geq U(W^G) - V(e^G + \theta^G - \theta^M) \quad (6)$$

$$U(W^B) - V(e^B) \geq U(W^M) - V(e^M + \theta^M - \theta^B) \quad (7)$$

$$U(W^B) - V(e^B) \geq U(W^G) - V(e^G + \theta^G - \theta^B) \quad (8)$$

Remarks

The objective function maximizes the Principal's expected net profit. The expression $e + \theta - W$ represents the difference between the total revenue $e + \theta$ (the equivalent of the total effort $e + \theta = E$) and the wage received by the Agent (paid by the Principal) if the state of nature is characterized by the parameter θ .

The constraints given by (3), (5) and (6), (7) are local constraints (upward and downward constraints), and the constraints (4) and (8) are global constraints (one upward constraint and one downward constraint).

The utility function $U(\cdot)$ characterizes the Agent's risk aversion and has the following properties: $U'(\cdot) > 0$ and $U''(\cdot) < 0$ (strictly increasing and strictly concave).

The function $V(\cdot)$ represents the cost function of the effort (the effort disutility) and has the following properties: $V'(\cdot) > 0$ and $V''(\cdot) > 0$ (strictly increasing and strictly convex). For example, the term $V(e^B + \theta^B - \theta^M)$ represents the cost of effort when the total effort is $E^B = e^B + \theta^B$ and the state of nature is described by the parameter's value θ^M .

The transformed model – using the variables: informational rents and costly effort levels

Let U^G, U^M, U^B be the Agent's utility levels obtained in each state of nature. Therefore, we can express these informational rents as:

$$U^G = U(W^G) - V(e^G)$$

$$U^M = U(W^M) - V(e^M)$$

$$U^B = U(W^B) - V(e^B)$$

We also consider the function $f : R \rightarrow R, f(e) = V(e + \Delta\theta) - V(e)$, with (for simplicity and without any loss of generality) $\Delta\theta = \theta^M - \theta^B = \theta^G - \theta^M$ (the spread of uncertainty on the market conditions).

Now, the constraints (3)-(8) become:

$$U^G \geq U^M + f(e^M - \Delta\theta) \quad (9)$$

$$U^G \geq U^B + f(e^B - \Delta\theta) + f(e^B - 2\Delta\theta) \quad (10)$$

$$U^M \geq U^B + f(e^B - \Delta\theta) \quad (11)$$

$$U^M \geq U^G - f(e^G) \quad (12)$$

$$U^B \geq U^M - f(e^M) \quad (13)$$

$$U^B \geq U^G - f(e^G) - f(e^G + \Delta\theta) \quad (14)$$

These new constraints are easy to derive. For example, the constraint (10) is a transformation of the relation (4), as we can see below:

$$\begin{aligned} U^G &= U(W^G) - V(e^G) \geq U(W^B) - V(e^B + \theta^B - \theta^G) = \\ &= U(W^B) - V(e^B) + V(e^B) - V(e^B - \Delta\theta) + V(e^B - \Delta\theta) - V(e^B - 2\Delta\theta) = \\ &= U^B + f(e^B - \Delta\theta) + f(e^B - 2\Delta\theta) \end{aligned}$$

Or, the constraint (14) (the global downward constraint) is a transformation of (8), as we can see below:

$$\begin{aligned} U^B &= U(W^B) - V(e^B) \geq U(W^G) - V(e^G + \theta^G - \theta^B) = \\ &= U(W^G) - V(e^G) + V(e^G) - V(e^G + \Delta\theta) + V(e^G + \Delta\theta) - V(e^G + 2\Delta\theta) = \\ &= U(W^G) - V(e^G) - [V(e^G + \Delta\theta) - V(e^G)] - [V(e^G + \Delta\theta + \Delta\theta) - V(e^G + \Delta\theta)] = \\ &= U^G - f(e^G) - f(e^G + 2\Delta\theta) \end{aligned}$$

We must note, for the following propositions, that the function $f(\cdot)$ has the properties:

- i) $f(e) > 0$
- ii) $f'(e) > 0, \forall e$.

These features are easy derived using the effort cost function.

Proposition 1. If the set of feasible solutions of the program (P) is nonempty, then the following inequalities are satisfied:

- i) $e^G + \theta^G \geq e^M + \theta^M \geq e^B + \theta^B$;
- ii) $W^G \geq W^M \geq W^B$.

Proof

i) We use the local upward and downward constraints. Summing up the relations (9) and (12) we get:

$$U^G + U^M \geq U^M + f(e^M - \Delta\theta) + U^G - f(e^G)$$

or:

$$f(e^G) \geq f(e^M - \Delta\theta)$$

From the properties of the function $f(\cdot)$ it follows that:

$$e^G \geq e^M - \theta^G + \theta^M \text{ or } e^G + \theta^G \geq e^M + \theta^M$$

Next, from the constraints (11) and (13), by summing up we get:

$$U^M + U^B \geq U^B + f(e^B - \Delta\theta) + U^M - f(e^M)$$

or:

$$f(e^M) \geq f(e^B - \Delta\theta)$$

Using the monotonicity of the function $f(\cdot)$, the later inequality yields to:

$$e^M \geq e^B - \Delta\theta = e^B - \theta^M + \theta^B \text{ or } e^M + \theta^M \geq e^B + \theta^B.$$

The condition $e^G + \theta^G \geq e^M + \theta^M \geq e^B + \theta^B$ represents the *implementability condition (or monotonicity constraint)* for the second best contracts (in the situation of asymmetric information).

ii) Now, using the constraint from (3) and the implementability condition we obtain:

$$U(W^G) - U(W^M) \geq V(e^G) - V(e^M + \theta^M - \theta^G) \geq 0$$

Then, $U(W^G) \geq U(W^M)$ and so $W^G \geq W^M$.

From (5) we get:

$$U(W^M) - U(W^B) \geq V(e^M) - V(e^B + \theta^B - \theta^M) \geq 0$$

It is obvious now that $W^M \geq W^B$.

To conclude, we can state that $W^G \geq W^M \geq W^B$.

The optimal contract in the situation of asymmetric information

Coming back to our settings from Section 1, we are now interested in solving the incentive problem (P). To simplify the analysis and find the relevant binding constraints we proceed as follows. First, we ignore for the moment the local and global downward incentive constraints given by (6), (7) si (8). It is almost obvious that the most efficient types would want to lie upward and claim that they are less efficient. Second, in the final step we check ex post that the incentive constraints are indeed not binding (nonrelevant) and are satisfied by the optimal solution.

We need first to prove the following proposition.

Proposition 2. The global upward constraint (4) is implied by the two local upward constraints (3) and (5), when the monotonicity constraint holds.

Proof

To show this result, we use the constraints (9) and (11), which are equivalent with the constraints (3) and (5) and were obtained using the change of variables.

Suppose that the following inequalities

$$U^G \geq U^M + f(e^M - \Delta\theta)$$

and

$$U^M \geq U^B + f(e^B - \Delta\theta)$$

are satisfied.

Summing up the above two relations we get:

$$U^G + U^M \geq U^M + U^B + f(e^M - \Delta\theta) + f(e^B - \Delta\theta)$$

or

$$U^G \geq U^B + f(e^M - \Delta\theta) + f(e^B - \Delta\theta).$$

It easy to show that $f(e^M - \Delta\theta) + f(e^B - \Delta\theta) \geq f(e^B - \Delta\theta) + f(e^B - 2\Delta\theta)$.

This is because $e^M - \Delta\theta \geq e^B - 2\Delta\theta$ or $e^M \geq e^B - (\theta^M - \theta^B) = e^B - \theta^M + \theta^B$ or $e^M + \theta^M \geq e^B + \theta^B$. This last expression corresponds exactly to the implementability condition, assumed to be true

With this simplification of the Principal's program, the only remaining relevant constraints are (2), (3) and (5).

The corresponding Kuhn-Tucker multipliers for the constraints (2), (3) and (5) are denoted by α, λ and μ . Therefore, the Lagrange function it is written as:

$$\begin{aligned} L(e^G, W^G, e^M, W^M, e^B, W^B; \alpha, \lambda, \mu) = & \pi^G (e^G + \theta^G - W^G) + \pi^M (e^M + \theta^M - W^M) + \\ & + \pi^B (e^B + \theta^B - W^B) + \alpha \{ \pi^G [U(W^G) - V(e^G)] + \pi^M [U(W^M) - V(e^M)] + \\ & + \pi^B [U(W^B) - V(e^B)] - u \} + \lambda [U(W^G) - V(e^G) - U(W^M) + V(e^M + \theta^M - \theta^G)] + \\ & + \mu [U(W^M) - V(e^M) - U(W^B) + V(e^B + \theta^B - \theta^M)] \end{aligned}$$

The first order (the optimality conditions) Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial e^G} = 0 \Rightarrow \frac{\pi^G}{V'(e^G)} = \alpha\pi^G + \lambda \quad (15)$$

$$\frac{\partial L}{\partial e^M} = 0 \Rightarrow \frac{\pi^M}{V'(e^M)} = \alpha\pi^M + \mu - \lambda \frac{V'(e^M + \theta^M - \theta^G)}{V'(e^M)} \quad (16)$$

$$\frac{\partial L}{\partial e^B} = 0 \Rightarrow \frac{\pi^B}{V'(e^B)} = \alpha\pi^B - \mu \frac{V'(e^B + \theta^B - \theta^M)}{V'(e^B)} \quad (17)$$

$$\frac{\partial L}{\partial W^G} = 0 \Rightarrow \frac{\pi^G}{U'(W^G)} = \alpha\pi^G + \lambda \quad (18)$$

$$\frac{\partial L}{\partial W^M} = 0 \Rightarrow \frac{\pi^M}{U'(W^M)} = \alpha\pi^M - \lambda + \mu \quad (19)$$

$$\frac{\partial L}{\partial W^B} = 0 \Rightarrow \frac{\pi^B}{U'(W^B)} = \alpha\pi^B - \mu \quad (20)$$

Proposition 3. The participation constraint (2) and the local upward constraints (3) and (5) are binding at the optimum.

Proof

Adding up the relations (18)-(20) :

$$\frac{\pi^G}{U'(W^G)} + \frac{\pi^M}{U'(W^M)} + \frac{\pi^B}{U'(W^B)} = \alpha(\pi^G + \pi^M + \pi^B) = \alpha.$$

From this it results that $\alpha > 0$ and so the ex-ante participation constraint is binding. Therefore, we get:

$$\pi^G [U(W^G) - V(e^G)] + \pi^M [U(W^M) - V(e^M)] + \pi^B [U(W^B) - V(e^B)] = \underline{u} \quad (2')$$

Next, we analyze the optimal value of the two variables (λ, μ) and we consider the following cases:

Case 1. $\lambda = \mu = 0$. This is not an interesting situation, since it corresponds to the case of symmetric information.

Case 2. $\mu = 0$ and $\lambda > 0$

The first order conditions from (18), (19) and (20) yield to the inequality:

$$\pi^G \left(\frac{1}{U'(W^G)} - \alpha \right) = \lambda > 0 \text{ and so } \alpha < \frac{1}{U'(W^G)}.$$

In the same way we also obtain $\alpha > \frac{1}{U'(W^M)}$ and $\alpha = \frac{1}{U'(W^B)}$.

We can write then:

$$\frac{1}{U'(W^M)} < \frac{1}{U'(W^B)} < \frac{1}{U'(W^G)} \text{ or } W^G > W^B > W^M, \text{ but this contradicts the}$$

result $W^M \geq W^B$ from Proposition 1.

Case 3. $\mu > 0$ and $\lambda = 0$

Using the same relations as above we obtain:

$$\frac{1}{U'(W^B)} < \frac{1}{U'(W^G)} < \frac{1}{U'(W^M)}$$

or $W^B < W^G < W^M$, being a contradiction of the result $W^G \geq W^M$ from Proposition 1.

Case 4. The first three cases are not possible solutions. Therefore, the only possible case corresponds to $(\lambda > 0$ and $\mu > 0)$. The immediate consequence is that the local upward incentive constraints are binding. Another consequence follows: it is impossible that the global upward incentive constraint to hold with equality (to be binding).

More, using the implementability condition and the previous results, we can state that the downward incentive constraints hold strictly. We proof this statement in the next proposition.

Proposition 4. If the multipliers λ and μ are strictly positive, then the following are true:

- i) $U^M \geq U^G - f(e^G)$
- ii) $U^B \geq U^M - f(e^M)$

$$\text{iii) } U^B \geq U^G - f(e^G) - f(e^G + \Delta\theta)$$

Proof

i) If $\lambda > 0$ and $\mu > 0$, then the corresponding constraints are binding:

$$U^G = U^M + f(e^M - \Delta\theta) \text{ and } U^M = U^B + f(e^B - \Delta\theta).$$

Using the first equality we obtain:

$$U^M = U^G - f(e^M - \Delta\theta) \geq U^G - f(e^G)$$

This is true due to the implementability condition $e^G \geq e^M - \theta^G + \theta^M$ or equivalently $e^G \geq e^M - \Delta\theta$.

Remark:

More than it was shown, the constraint holds strictly, meaning that the Agent is not interested to claim (to announce) the best state of nature when the true state is the medium one.

Indeed, if $e^G = e^M - \theta^G + \theta^M$, then from (3) or from the equivalent relation (9) we get $W^G = W^M$.

From (16) and using (19) it results:

$$\frac{\pi^M}{V'(e^M)} > \alpha\pi^M - \lambda + \mu$$

and so:

$$V'(e^M) < U'(W^M)$$

or

$$V'(e^M) < U'(W^M) = U'(W^G) = V'(e^G)$$

Therefore, we have $e^M < e^G$. But $\theta^G > \theta^M$ and this implies that $e^M + \theta^M < e^G + \theta^G$, which is a contradiction to $e^G = e^M - \theta^G + \theta^M$.

The conclusion is immediate, $U^M > U^G - f(e^G)$.

ii) The binding constraint (11) yields to:

$$U^B = U^M - f(e^B - \Delta\theta) \geq U^M - f(e^M)$$

The latter inequality is true because we know from the implementability condition that $e^M \geq e^B - \Delta\theta = e^B - \theta^M + \theta^B$.

We have already proved that the relations (i) and (ii) are satisfied. Summing up the terms from the two sides we get:

$$U^B > U^G - f(e^G) - f(e^M) \geq U^G - f(e^G) - f(e^G + \Delta\theta)$$

where $f(e^G + \Delta\theta) \geq f(e^M)$, corresponding to the implementability condition $e^G + \theta^G \geq e^M + \theta^M$ or $e^G + \Delta\theta \geq e^M$.

4. Conclusions

We derived in the last section the optimal solution of the Principal's problem. We can now summarize the characteristics of this optimal solution in the following theorem:

Theorem. The main features of the optimal contract in the situation of asymmetric information are:

A. The Agent's expected utility is exactly the outside opportunity level of utility, \underline{u} (the reservation utility level).

B. If the market conditions corresponds to θ^G (the most favorable situation), the contract is Pareto-optimal, i.e. $V'(e^G) = U'(W^G)$. In this case, there is no distortion with respect to the first best solution.

C. For the other two market conditions (states of nature), the contract is no longer Pareto-optimal. In this case, the following relations are satisfied:

$$V'(e^M) < U'(W^M) \text{ and } V'(e^B) < U'(W^B).$$

Indeed, using the relations (16) and (19) and the above result $\lambda > 0$ we get:

$$\frac{\pi^M}{V'(e^M)} = \alpha\pi^M + \mu - \lambda \frac{V'(e^M + \theta^M - \theta^G)}{V'(e^M)} > \alpha\pi^M + \mu - \lambda = \frac{\pi^M}{U'(W^M)}$$

or $V'(e^M) < U'(W^M)$.

On the other hand, using (17), (20) and $\lambda > 0$ we get:

$$\frac{\pi^B}{V'(e^B)} = \alpha\pi^B - \mu \frac{V'(e^B + \theta^B - \theta^M)}{V'(e^B)} > \alpha\pi^B - \mu = \frac{\pi^B}{U'(W^B)}$$

or $V'(e^B) < U'(W^B)$.

D. If the state of nature is θ^G , the Agent gets positive informational rents with respect to the states θ^M and θ^B . The Agent gets also a positive informational rent in the state θ^M with respect to the least favorable state of nature θ^B .

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