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VALUE-AT-RISK ESTIMATION ON BUCHAREST STOCK EXCHANGE

Olivia Andreea BACIU
PhD Candidate, Babes Bolyai University, Cluj Napoca, Romania

E-mail: oli_baciu@yahoo.com

Abstract
As an important tool in risk management, Value-at-Risk is estimated on the Romanian stock market based on single assets and a weighted portfolio of them. Because this is a measure of the extreme tails, several approaches are used to compute Value-at-Risk by taking in consideration the distribution of the data, namely the generalized hyperbolic distribution, Normal-Inverse Gaussian and asymmetric t-Student in comparison with the normal distribution. The considered period is divided into an analyze period and a test one, where based on the rolling windows approach are estimated the VaR values and then tested with the help of Kupiec's and Christoffersen's backtests. The choice of the time period affects the estimation due to the events that took place on the market and the approach based on the normal distribution predicted best the VaR values by underestimating the risk compared to the other distributions. This approach fits better the considered period because the analyzed period covers moments of severe economical crisis while the test period goes over a period of recovery.

Keywords: *Value-at-Risk; generalized hyperbolic distributions; heavy tails; stock market data*

1. Introduction

The instability of financial markets in the recent years has led to the need of a general accepted measure that quantifies risk of the loss when looking to invest or holding a portfolio. This measure of risk management has known various forms over time. A measure accepted and used both in practice and theory is value-at-risk (VaR). VaR was easily accepted for two reasons: the ease of computations and the weight of the information brought by a single number.

As defined by J.P. Morgan (1996), VaR represents "a measure of the maximum potential change in value of a portfolio of financial instruments over a pre-set horizon. VaR answers the question: how much can I lose with x% probability over a given time horizon?". In other words, VaR defines through a single number the maximum loss we should expect over a period of time except for x% of the cases, when the loss can exceed VaR. Due to this two inconveniences, the arbitrary choice of the probability and of the time interval, authors like Einhorn (2008) consideres VaR as being inefficient because it ignores mainly those x%

extreme situations, which represent the highest looses and compares the method with "an airbag that works all the time, except when you have a car accident".

Prause (1997) used the family of generalized hyperbolic distributions to model the returns of the German stocks and US index and based on them evaluated the risk of loss corresponding to each of the two markets. The analysis revealed that the family of generalized hyperbolic distributions provide a better fit to the empirical VaR. Bauer (2000) performed VaR computations on German stocks and DAX, Dow Jones and Nikkei market indexes over 1987- 1997 and found that the symmetric hyperbolic distribution outperforms the normal model. Huang et al. (2014) evaluated the performance of the hyperbolic distribution, Normal- Inverse Gaussian and asymmetric t- Student through VaR estimates on daily JSE Mining Index. All distributions outperformed the normal one both through means of goodness-of-fit and VaR estimates.

To overcome the shortcomings of the method, in this study are used daily returns and are applied several methods to compute VaR by taking into consideration the distribution of the returns. VaR estimates are computed using the rolling window approach and the performance of each method is tested through backtests. As shown in Prause (1999), Sognia and Wilcox (2014), Baciu (2014), appropriate distributions in modeling financial data are the generalized hyperbolic distribution or Normal- Inverse Gaussian distribution which account for the heavier tails and asymmetric distribution that characterize stock market returns. Although Vee et al. (2012) concluded that one specific distribution won't describe well indices from different markets and the choice of the distribution depends on the market, the performance of the generalized hyperbolic distribution on the index of the Romanian market was shown in Necula (2009) or Baciu (2014). This study uses the findings of Baciu (2014) in modeling financial returns and fitting the distribution to data, which show that the generalized hyperbolic distribution approximates the best the distribution of the returns of the five Romanian Investment Funds, followed by the Normal-Inverse Gaussian and asymmetric t-Student distributions. The purpose of this paper is to develop the previous study, in which the performance of each distribution was characterized through plots and goodness-of-fit means and focus on the distribution of the tails.

The remainder of this paper is organized as follows. Section 2 links the study with a previous research in which has been shown the performance of the generalized hyperbolic distribution on Romanian stock data in comparison to the normal distribution. Section 3 provides a description of the methodology used to compute VaR and the backtests of the VaR estimates. Section 4 introduces the data and presents the descriptive statistics of it. The VaR estimates and the results of the backtests are discussed in Section 5. Finally, Section 6 concludes the paper with the main findings and directions to develop the study.

2. Choice of the distribution

As documented in Baciu (2014), among the distributions that approximate well stock market returns is the family of the generalized hyperbolic distributions. Studies such as Prause (1997), Necula (2009), Rege and Menezes (2012), Sognia and Wilcox (2014) have shown the performance of this family of distributions. Because VaR accounts only for the tails, such a distribution should be more appropriate than the normal one to estimate the potential risk of loss. Based on the results in Baciu (2014) on the same data set as in the present paper, VaR will be computed on three of the generalized hyperbolic family of distributions: the generalized hyperbolic distribution, Normal- Inverse Gaussian and assymetric t- Student. The parameters of these distributions are estimated using the maximum likelihood estimation, implemented based on the EM scheme of Dempster et al (1977) in R software.

For a vector of observations $x_1, x_2, \dots, x_n$, the maximum likelihood estimation of the parameters $\lambda, \alpha, \beta, \delta, \mu$ is obtained by maximizing the log-likelihood function:

$$L(x_1, x_2, \dots, x_n;\ \lambda, \alpha, \beta, \delta, \mu) = loga + \frac{\lambda - \frac{1}{2}}{2}\sum_{i=1}^{n} \log(\delta^2 + (x_i - \mu)^2) +$$
$$+ \sum_{i=1}^{n} logK_{\lambda-\frac{1}{2}}(\alpha\sqrt{\delta^2 + (x_i - \mu)^2} + \sum_{i=1}^{n} \beta(x_i - \mu),$$

where a and K_λ are defined as:

$$a(\lambda, \alpha, \beta, \delta) = \frac{(\alpha^2 - \beta^2)^{\frac{\lambda}{2}}}{\sqrt{2\pi}\alpha^{\left(\lambda - \frac{1}{2}\right)}\delta^\lambda K_\lambda(\delta\sqrt{\alpha^2 - \beta^2})}$$
$$K_\lambda(x) = \frac{1}{2}\int_0^\infty y^{\lambda-1} \exp\left(-\frac{1}{2}x(y + y^{-1})\right) dy.$$

In the previous study, the choice among distributions was done based on plots and goodness-of-fit measures: Kolmogorov-Smirnov distance, Log-likelihood and Akaike Information Criterion. From the beginning, plots ruled out the Variance-Gamma distribution and have shown that the family of the generalized hyperbolic distributions offers a better and more appropriate fit to the Romanian market data than the normal distribution. Based on the goodness-of-fit measures, the generalized hyperbolic distribution seems to represent the best fit to the given data.

3. Methodology

3.1. VaR estimation methods

The VaR methodology has been easily adopted and accepted both in theory and practice due to the ease of implementation and interpretation. Among the most popular methods of computing VaR can be mentioned the methods based on the historical data, Monte Carlo simulations or the assumption of normal distribution.

A survey realized by Perignon and Smith (2006) has shown that 73% of the interogated banks are using the historical method for computing the risk of loss. This method estimates the future loss based on past returns. Besides its simplicity, among the benefits of this method is that it allows for skewed and leptokurtic distributions of data. On the other side, the method takes into consideration only events that took place in the analyzed period of time and can not capture the effect of any other type of events. If the analyzed period has gone only through low fluctuations of volatility, then the computed loss will be too small in case the fluctuations grow.

The normal VaR estimation method assumes a normal distribution of the returns. But it is a known fact that equity returns exhibit heavier tails than in the case of the normal distribution. To avoid the shorcomings of the normal VaR method, it was introduced the Cornish-Fisher method, described in Favre and Galeano (2002) which accounts for distributions other than the normal one by considering the third and fourth moments.

If it is considered

$$CF_p = Q_p + \frac{S({Q_p}^2 - 1)}{6} + \frac{K({Z_p}^3 - 3Z_p)}{24} - \frac{S^2(2{Z_p}^3 - 5Z_p)}{36}$$

where S stands for assimetry , K for excess kurtosis, p the selected probability level and Q_p the corresponding quantile to the selected probability level, then, through the Cornish-Fisher method, VaR reduces to:

$$VaR_{CF} = -mean(R) - \sqrt{\sigma} CF_p.$$

Brown (2008) comments on the performance of VaR and sustains that the results are trustful only if they are backtested. For this reason, in this study data is divided into an analyze period and a test one. As sugested by Brown (2008), for a considered probability of 1% greater looses than estimated by VaR for daily returns, the analysis should be performed on 3 years of historical data.

Each VaR is computed based on a rolling windows approach on 750 historical daily returns, counting for about three years of daily trading data. Data is divided into two periods of time: an analysis one and a test one. The analysis period represent the windows of the rolling windows approach and at each step the oldest return is dropped and added the return of the following day of the last observation included in the previous window. The test period accounts for the remaining data of more than two years of tradings. Over this period are compared the number of days with excess loss than estimated through VaR with the expected number of such days, according to the considered probability.

Two of the most used backtests are the ones of Kupiec (1995) and Christoffersen (1998).

3.2. Backtesting

According to Christoffersen (2012), one can only estimate with the chosen probability if the actual loss will exceed the computed VaR or not. This set of events is similar with a Bernoulli trial, with a probability *p* that the loss exceeds VaR and *(1-p)* that it does not. The total number of days in which the loss is greater than VaR, denoted by *x*, follow a Binomial (n,p) distribution, where *n* is the total number of observations.

Kupiec backtest

The Kupiec backtest, introduced by Kupiec (1995), is a test of the failure rate. The null hypothesis assumes a rate of failures equal to the expected one.

$$H_0: p = \widehat{p},$$

where, $\hat{p} = x/n$.

As mentioned in Nieppola (2009), if the number of days with excess loss is too high compared to the chosen probability level, it indicates an underestimation of the risk, while a low number of exceptions suggest an overestimation of the risk.

The Kupiec backtest is constructed as a likelihood-ratio test, with the statistic given by:

$$TS_{exc} = \frac{p^x(1-p)^x}{(\frac{n-x}{n})^{n-x}(\frac{x}{n})^x}.$$

Under the null hypothesis $TS_{exc} \sim \chi^2_{(1)}$.

Christoffersen backtest

Aditional to Kupiec backtest, Christofferson (1998) highlights the importance of the exceptions to be independent. Otherwise, if the excess loss occurs in a short period of time the company might be affected unlike if they occur occasionally, over a longer period. The new test keeps track of both events: the number of exceptions and their independence.

As defined in Christoffersen (1998), Christoffersen (2012) or Nieppola (2009), let

$$I_{t+1} = \begin{cases} 1 & pentru\ R_{t+1} < -VaR_{t+1} \\ 0 & pentru\ R_{t+1} \geq -VaR_{t+1} \end{cases}$$

be the sequence of exceptions and

$$\pi_1 = P(I_{t+1} = 1 | I_t = 1)$$
$$\pi_0 = P(I_{t+1} = 1 | I_t = 0)$$

the probability that tomorrow's loss will exceed VaR knowing that today's loss exceeds VaR, respectively the probability that tomorrow's loss exceeds VaR knowing that today's loss does not exceed VaR.

Let:

- n_{00} the number of days in which $I_{t+1} = 0$ based on $I_t = 0$
- n_{01} the number of days in which $I_{t+1} = 1$ based on $I_t = 0$
- n_{10} the number of days in which $I_{t+1} = 0$ based on $I_t = 1$
- n_{11} the number of days in which $I_{t+1} = 1$ based on $I_t = 1$.

Then, the probabilities π_0 and π_1 are:

$$\pi_0 = \frac{n_{01}}{n_{00} + n_{01}}, \pi_1 = \frac{n_{11}}{n_{10} + n_{11}}, \pi = \frac{n_{01} + n_{11}}{n_{00} + n_{01} + n_{10} + n_{11}}.$$

Because the test combines the hypothesis of the Kupiec test with the one the independence of the exceptions, the test statistic is defined as:

$$TS = TS_{exc} + TS_{ind}$$

where,

$$TS_{ind} = -2\ln\left(\frac{(1-\pi)^{n_{00}+n_{10}}\pi^{n_{01}+n_{11}}}{(1-\pi_0)^{n_{00}}\pi_0^{\ n_{01}}(1-\pi_1)^{n_{10}}\pi_1^{\ n_{11}}}\right)$$

Under the null hypothesis, TS $\sim\chi^2_{(2)}$.

4. Data

The risk management measure, VaR is applied on the Romanian market on five individual equities, namely the Investment Funds: SIF1, SIF2, SIF3, SIF4, SIF5 and on an equally weighted portfolio of them (Ristea *et al.*, 2010; Dumitrana *et al.*, 2010).

The choice of these equities is based on the fact that they hold shares on the most important domains in the Romanian market, the interest of the investors in them and in Baciu (2014) has been shown the performance of the generalized hyperbolic distribution in approximating their returns.

Daily returns were gathered for more than five years, starting the day of the maximum closing price in 2007 until the last trading day of 2012. The choice of the time period was due to the economical crisis that affected Romania at the middle of 2007. The period of economical crisis brought changes in the characteristics of each time series.

The time period is divided into an analysis period of 750 days and a test period. The analysis is done using the rolling windows approach, in which at each step is created a window of 750 observations by dropping the oldest return and adding the return of the next day.

Returns are computed as the difference between natural logarithm of current day closing price and natural logarithm of previous day closing price:

$$R_t = \ln(\frac{Y_t}{Y_{t-1}})$$

Table 1 presents the descriptive statistics of the daily returns for the five investigated investment funds. It can be noticed that the returns are skewed and leptokurtic. The hypothesis that data is following a normal distribution is rejected for all equities, as given by the Jarque- Bera test results. All equities have a positive mean return.

Table 1. Descriprive statistics

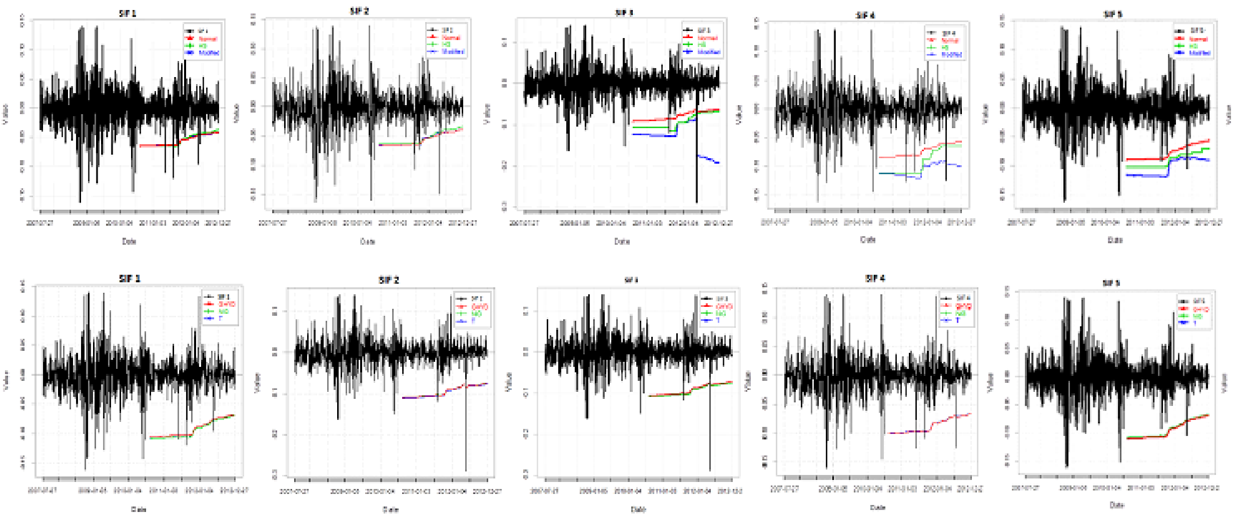
Equity	Sample size	Mean	Standard deviation	Skewness	Kurtosis	Jarque-Bera
SIF 1	1352	0.0946	3.2077	0.1802	4.1727	997,7*
SIF2	1353	0.0752	3.2264	0.2385	4.5521	1191.9*
SIF 3	1372	0.1053	3.3111	0.6811	7.5740	3460.8*
SIF 4	1362	0.0987	2.9963	0.0860	4.9256	1369.4*
SIF 5	1351	0.0924	3.1551	0.1243	4.1248	965.5*

Note: * denotes statistical significance at 5%

5. Results

VaR is computed for each asset for a period of about two years, based on three years of historical data. Several approaches were used to compute the loss of each equity and also of the portfolio: normal VaR, historical, Cornish- Fisher but also based on the distribution of the returns, namely, were used the generalized hyperbolic distribution, Normal- Inverse Gaussian and asymmetric t-Student.

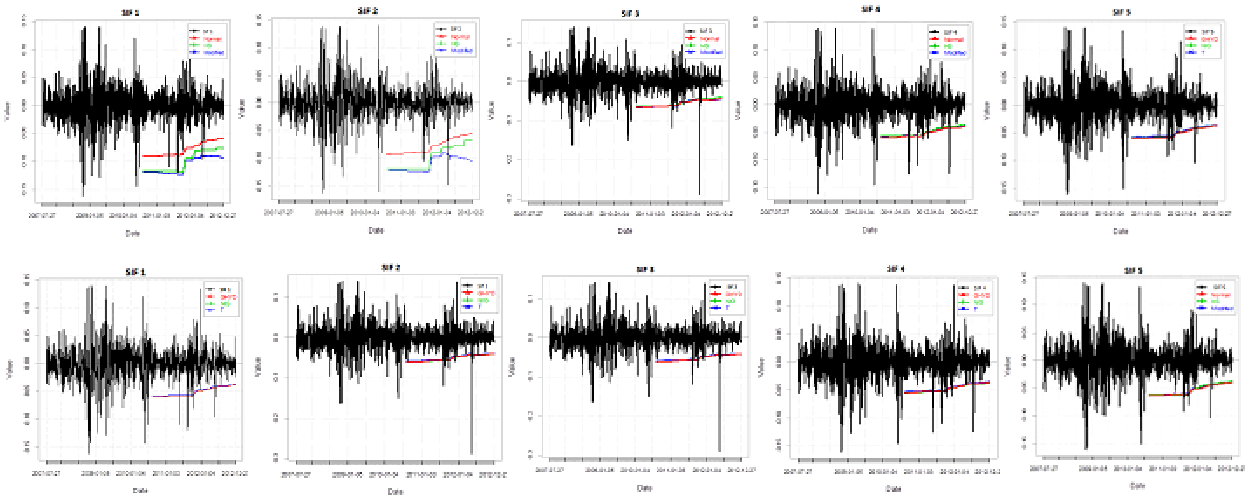
The below plots present estimated VaR values through all considered methods at probability levels of 1% and 5%.



Graph 1. 1% VaR estimates

For SIF1, normal, historical and Cornish- Fisher VaR return similar values at a 1% level of probability while at 5% level of probability normal VaR return smaller looses that the other two methods. When the distribution of the standardized returns is considered, the estimated VaR values are close for all three distributions at the two considered levels of probability.

In the case of SIF2, at 1% probability, all methods return close VaR values while at 5% probability, the Cornish- Fisher method returns the highest looses.



Graph 2. 5% VaR estimates

Estimated VaR for SIF 3 at 1% level of probability returns the smallest values through the Cornish- Fisher method. At 5% level of probability all estimated VaR values are similar. Based on the distribution of the standardized returns, VaR values obtained are similar at 1% and 5% levels of probability.

As in the case of SIF2 and SIF3, for SIF4, estimated VaR values for a 1% level of probability suggest higher looses through Cornish- Fisher method and the smallest ones through the normal method. For a level of probability of 5%, all methods return similar VaR values.

For a 1% level of probability, normal VaR, historical and Cornish- Fisher returns different VaR values for SIF5, while all methods return similar values at a 5% level of probability.

As mentioned by Brown (2008) that" Value-at-Risk is only as good as its backtest. When someone shows me a VaR number, I don't ask how it is computed, I ask to see the backtest" the below table presents the results of the two backtests: Kupiec and Christoffersen. There are also included the expected number of losses greater than VaR based on the chosen probability and the actual number of looses greater than VaR.

Table 2. Backtests results for normal VaR, historical and Cornish- Fisher

Activ	p	Normal				Historical				Cornish-Fisher			
		Expected	Actual	Kupiec p-value	Cristoffersen p-value	Expected	Actual	Kupiec p-value	Cristoffersen p-value	Expected	Actual	Kupiec p-value	Cristoffersen p-value
SIF1	0.05	29	5	0	-	29	5	0	-	29	5	0	-
	0.01	5	3	0.174	-	5	2	0.057	-	5	1	0.011	-
SIF2	0.05	29	4	0	-	29	5	0	-	29	4	0	-
	0.01	5	2	0.057	-	5	1	0.011	-	5	1	0.011	-
SIF3	0.05	29	10	0	0	29	9	0	0	29	10	0	0
	0.01	5	2	0.057	-	5	2	0.057	-	5	1	0.011	-
SIF4	0.05	29	7	0	0	29	7	0	0	29	7	0	0
	0.01	5	3	0.174	-	5	2	0.057	-	5	2	0.057	-
SIF5	0.05	29	8	0	0	29	8	0	0	29	8	0	0
	0.01	5	2	0.057	-	5	1	0.011	-	5	1	0.011	-

For normal VaR, Kupiec backtest rejects the hypothesis that the expected and actual number of looses are equal for a 5% level of probability while at 1% level of probability, Kupiec test fails to reject it. At 1% probability, the loss was overvalued for the equities SIF2 and SIF5 when VaR was computed using the historical method. As it was the case for normal and historical VaR, for Cornish- Fisher VaR, the loss is overvalued at a probability of 5%. At 1% level of probability only for SIF4 the expected number of cases of a greater loss than VaR was well estimated.

Christoffersen backtest does not offer any additional information. In all cases when it was computed, it rejects the hypothesis that the number of days when the loss exceeded VaR was well estimated and these exceedences are independent of each other.

Table 3. Backtests results based on generalized hyperbolic distribution, Normal- Inverse Gaussian and asymmetric t-Student

Activ	p	GHYD				NIG				T			
		Expected	Actual	Kupiec p-value	Cristoffersen p-value	Expected	Actual	Kupiec p-value	Cristoffersen p-value	Expected	Actual	Kupiec p-value	Cristoffersen p-value
SIF1	0.05	29	7	0	0	29	7	0	0	29	8	0	0
	0.01	5	3	0.174	-	5	3	0.174	-	5	3	0.174	-
SIF2	0.05	29	11	0	0	29	10	0	0	29	13	0	0
	0.01	29	2	0	-	29	2	0	-	29	2	0	-
SIF3	0.05	29	10	0	0	29	10	0	0	29	12	0	0
	0.01	5	2	0.057	-	5	2	0.057	-	5	2	0.057	-
SIF4	0.05	29	9	0	0	29	9	0	0	29	10	0	0
	0.01	5	3	0.1742	-	5	3	0.174	-	5	3	0.174	-
SIF5	0.05	21	9	0	0	29	9	0	0	29	9	0	0
	0.01	5	1	0.011	-	5	1	0.011	-	5	1	0.011	-

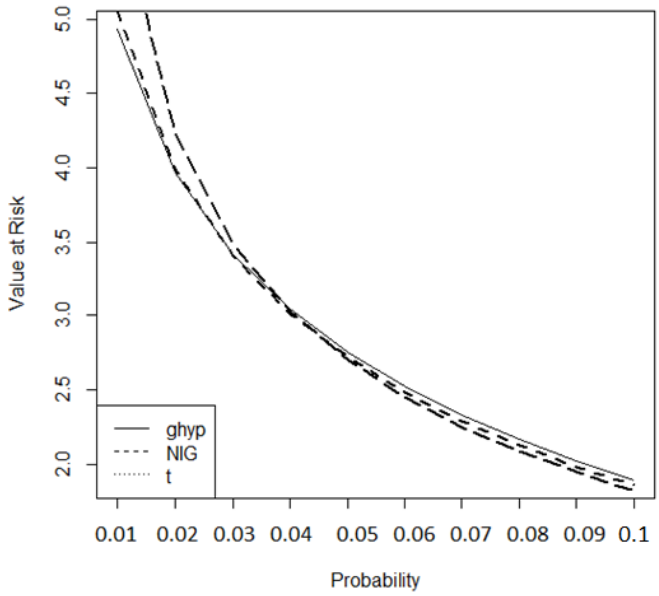
According to Kupiec backtest, VaR estimated based on the distribution of the returns, gives proper estimations of the loss for SIF1, SIF3 and SIF4 at a probability of 1%.

Table 4 presents the results of the backtests for the estimated loss of the portfolio of the five equally weighted equities. The best performance in estimating the loss of the portfolio is of the normal method, for which the loss excess situations are well estimated and are independent at 1%. Also, VaR computed through the historical method or based on the generalized hyperbolic distribution and Normal-Inverse Gaussian is well estimated at 1% probability, while VaR estimated through Cornish-Fisher method or based on asymmetric t-Student distribution overestimates the loss.

Table 4. Backtests for portfolio VaR

Method	P	Expected	Actual	Kupiec p-value	Cristoffersen p-value
Normal	0.05	29	14	0	0
	0.01	5	6	0.99	0.131
Historical	0.05	29	16	0.004	0
	0.01	5	2	0.057	-
Cornish-Fisher	0.05	29	15	0.002	0.001
	0.01	5	1	0.011	-
GHYD	0.05	29	17	0.008	0.007
	0.01	5	2	0.057	-
NIG	0.05	29	16	0.004	0.003
	0.01	5	2	0.057	-
T	0.05	29	18	0.015	0.002
	0.01	5	1	0.01	-

The interest of the investors in a portfolio increases when the loss minimizes. Based on VaR estimations, it can be created a weighted portfolio such that the loss to be minimum. Graph 3 presents the estimated VaR values based on the distribution of the returns at different levels of probability. Table 5 presents the weights of each equity in the portfolio such that the estimated VaR to be minimum at 1% and 5% levels of probability. Such a portfolio is an optimal one because it assumes the minimum risk of loss.



Graph 3. Minimum loss portfolio VaR estimation at different levels of probability

Table 5. Minimum risk portfolio weights

Distribution	p	VaR minimum	Weights SIF1	SIF2	SIF3	SIF4	SIF5
GHYD	0.05	-0.027	0.16	0.18	0.19	0.27	0.20
	0.01	-0.049	0.16	0.17	0.19	0.28	0.20
NIG	0.05	-0.027	0.16	0.18	0.19	0.27	0.2
	0.01	-0.05	0.16	0.17	0.19	0.27	0.21
T	0.05	-0.027	0.16	0.18	0.19	0.27	0.2
	0.01	-0.058	0.17	0.16	0.19	0.27	0.21

6. Conclusions

Value- at- Risk is one of the most used measures of risk management. This measure is widely accepted both in practice and theory because of the benefits it brings compared to the ease of the computation. In this study is compared the performance of six methods used to compute VaR, where three of them take into consideration the distribution of the returns, other than normal distribution. The analysis is performed on five of the most important equities traded at Bucharest Stock Exchange and on their weighted portfolio.

All methods used overestimated the loss by considering more days with higher looses than they actual were. This overreaction can be explained by the fact that the analysis period covered the years 2007-2008, when the market went through moments of severe instability and important looses.

At a 5% probability, the backtests reject all the methods for estimating correctly the number of days with looses higher than VaR. Contrary to the expectation, at 1% level of probability, the normal method works the best, as shown by the backtests, followed by the historical one and the methods based on the generalized hyperbolic distributions. The performance of the normal method under the considered period is because the events that take place in that period are the ones that influence VaR estimation and these years are characterized by frequent extreme values. A distribution like the generalized hyperbolic one, Normal- Inverse Gaussian or asymmetric t- Student catch in its tails these extreme values while the normal one does not and underestimates the loss. This is in the advantage of the normal method because the test period is characterized by less severe looses.

When it was considered the portfolio of the five equally weighted Investment Funds, the best performance was of the historical method, normal and based on generalized hyperbolic distribution and Normal- Inverse Gaussian distribution at 1% level of probability. At a probability of 5%, all methods overestimate risk.

Although the results revealed the performance of the normal distribution in VaR estimation, it brings into attention the sensibility of VaR to the events that take place in the analyzed period, which leads to over or underestimated looses.

The analysis should be extended to a longer period of time and a wider portfolio in order to confirm the performance of the generalized hyperbolic distribution and the weakness of VaR to the chosen time period.

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